Betonske konstrukcije 2

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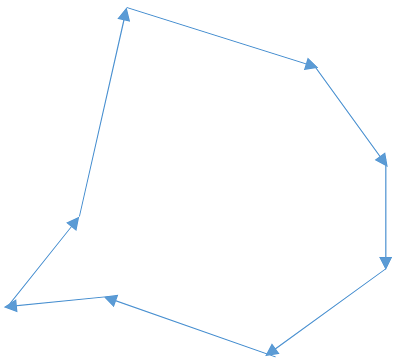
Ponavljanje

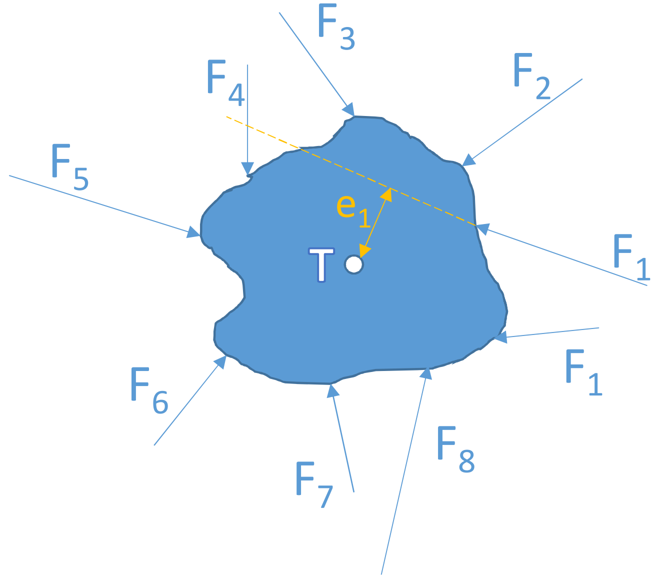
Jednoosno savijanje

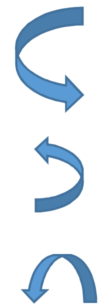
Ravnoteža tijela



$$\Sigma F_i^h = 0$$

$$\Sigma F_i^v = 0$$





$M_1 = F_1 \times e_1$

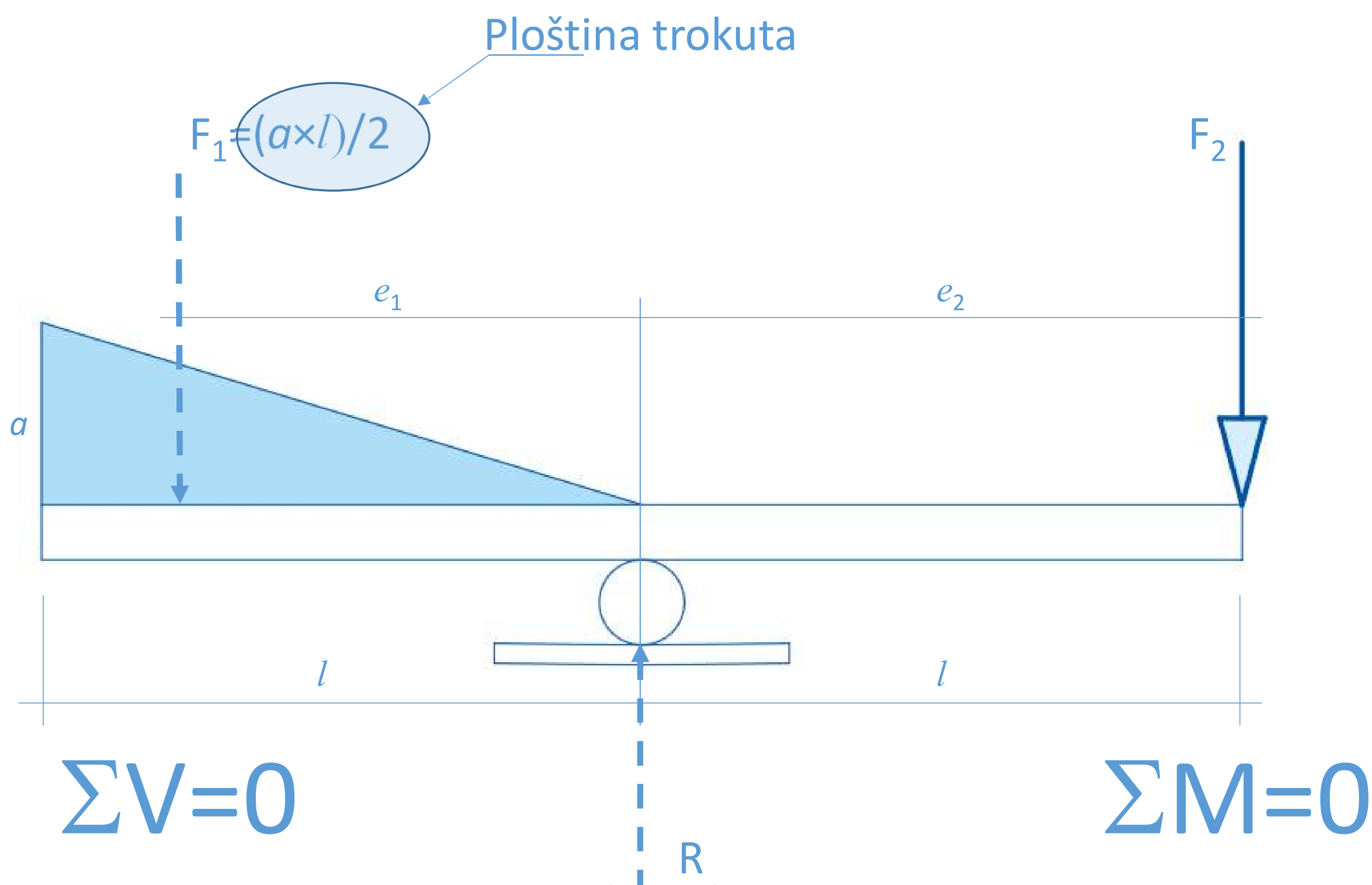
$M_2 = F_2 \times e_2$

$M_3 = F_3 \times e_3$

....

$$\Sigma M_i = 0$$

Ravnoteža tijela



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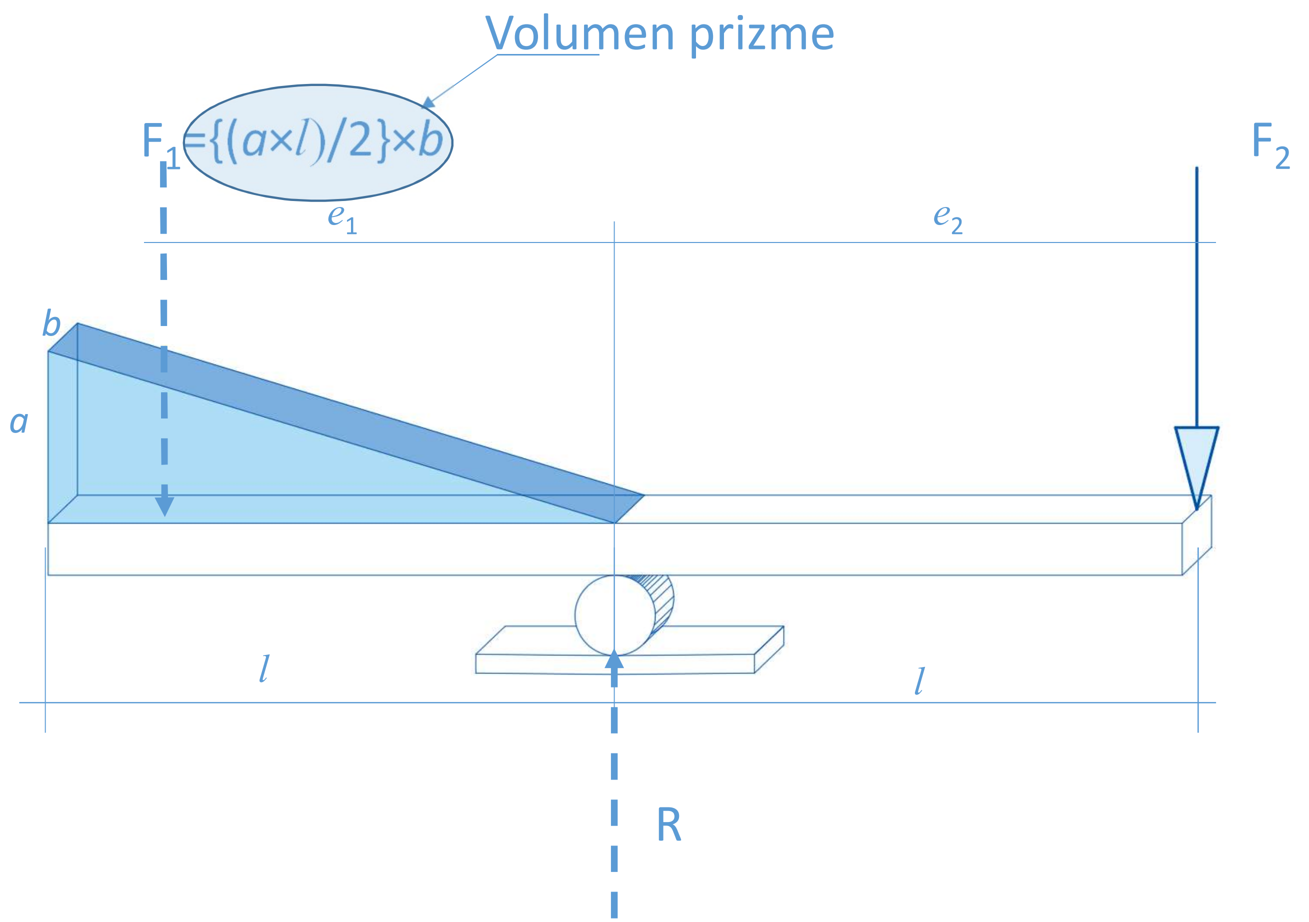

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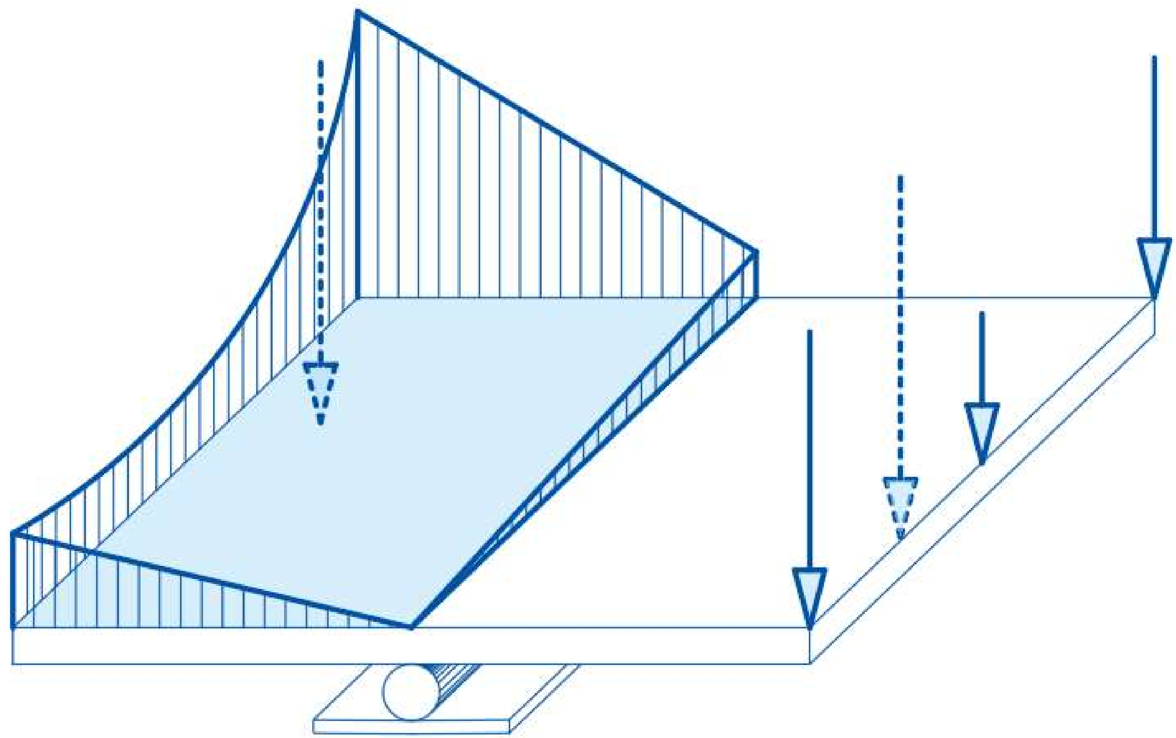

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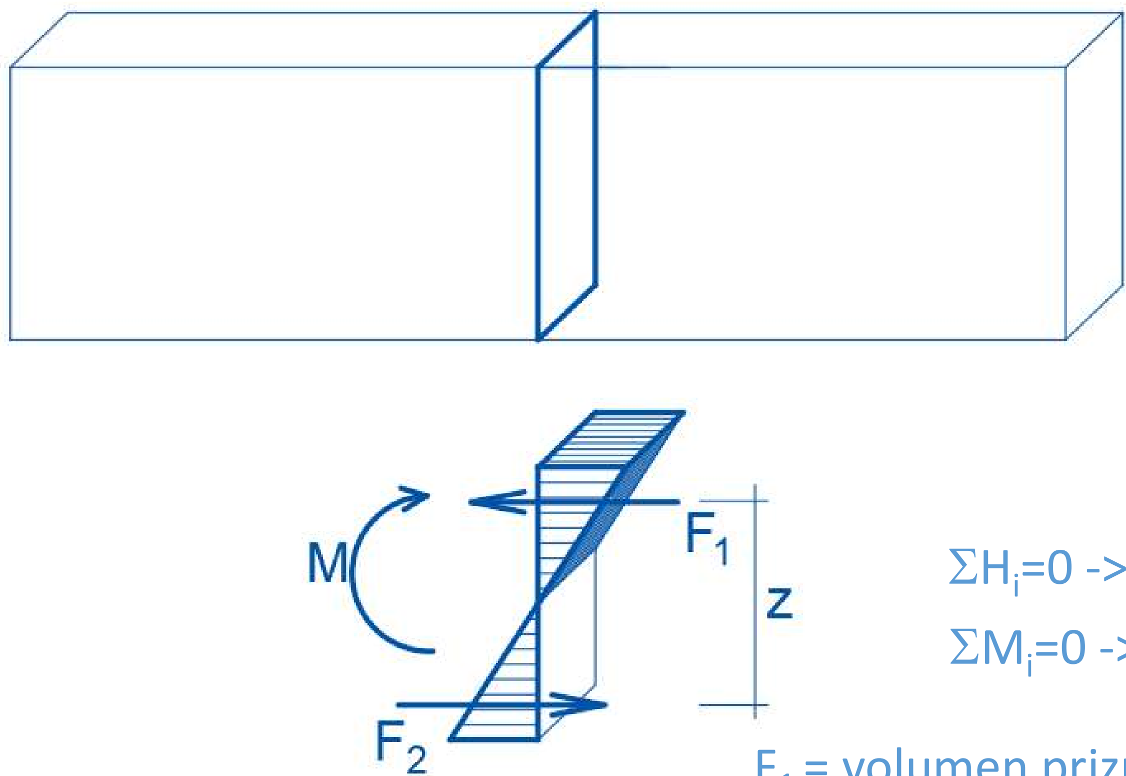
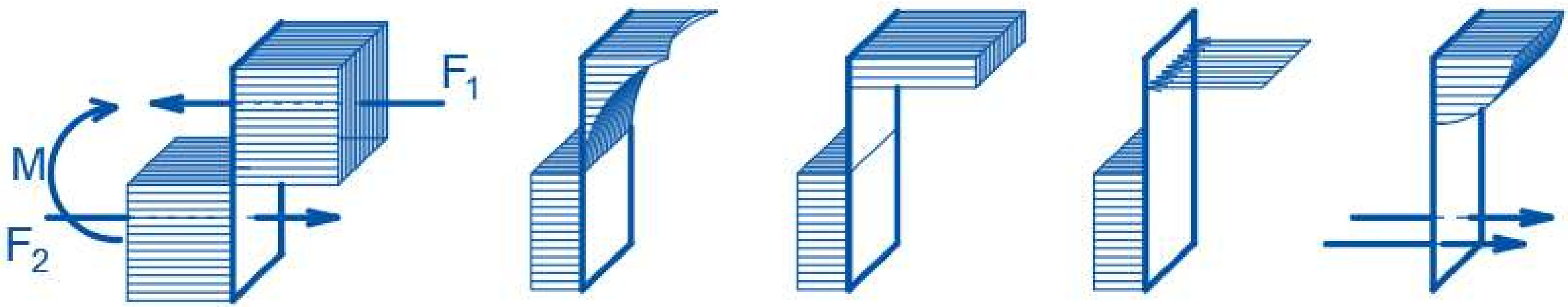


$\Sigma H_i = 0 \rightarrow F_1 = F_2$

$\Sigma M_i = 0 \rightarrow F_1 \times z = F_2 \times z = M$

F_1 = volumen prizme u gornjoj zoni
F_2 = volumen prizme u donjoj zoni
M = vanjski moment

Ravnoteža tijela



$\Sigma H_i=0 \rightarrow F_1=F_2$

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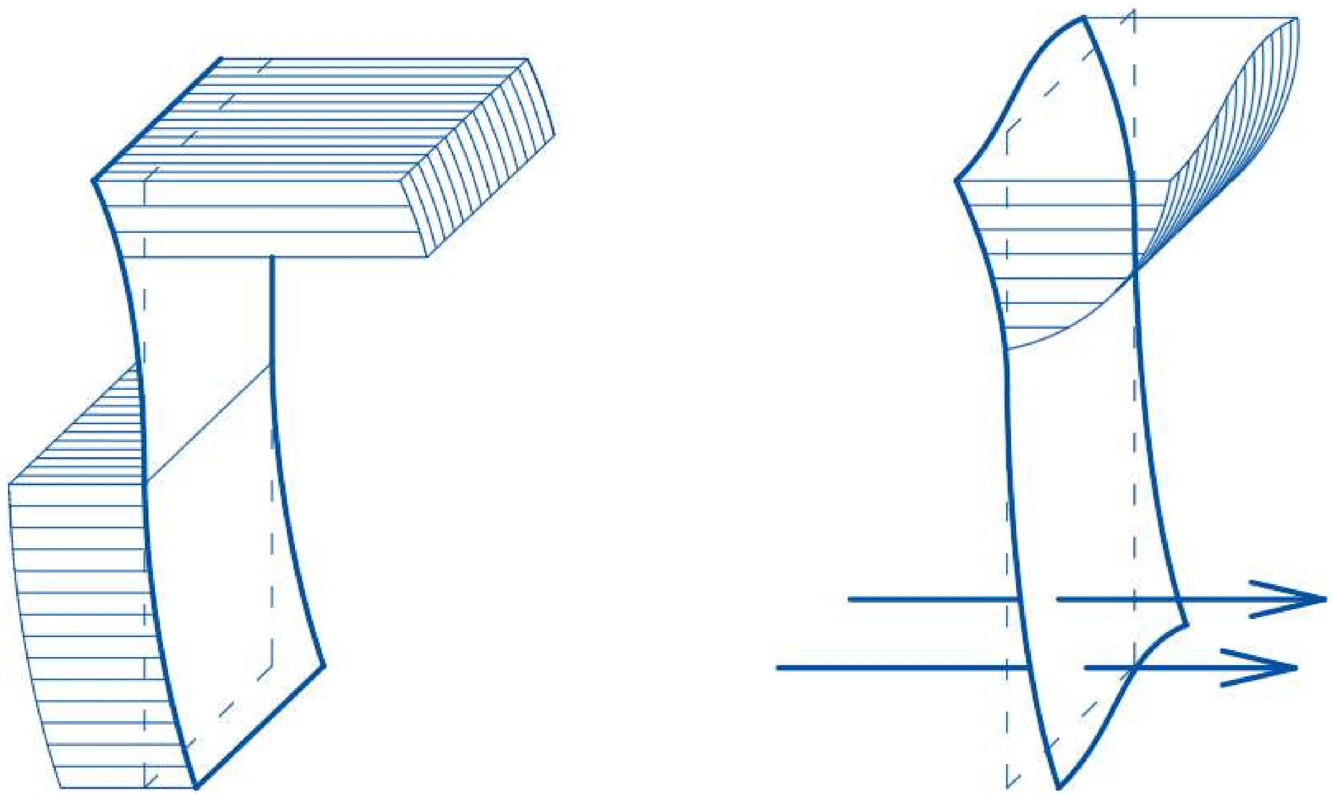


Granično stanje nosivosti - GSN

Osnovne pretpostavke za proračun prema graničnim stanjima nosivosti (GSN):

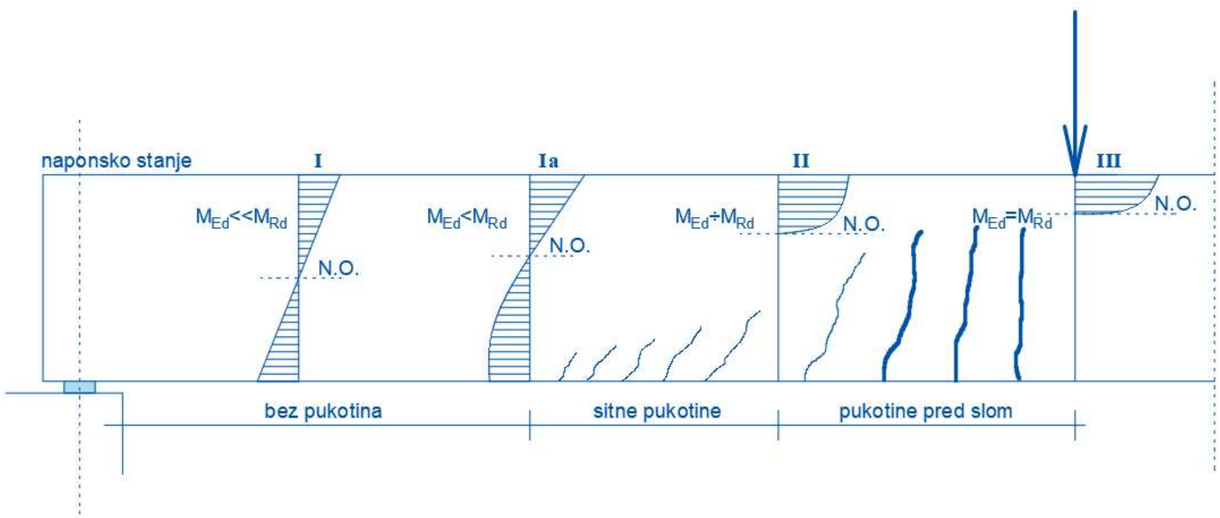
Bernoullyeva teorija ravnih presjeka: ravni presjeci ostaju ravni i poslije deformacije

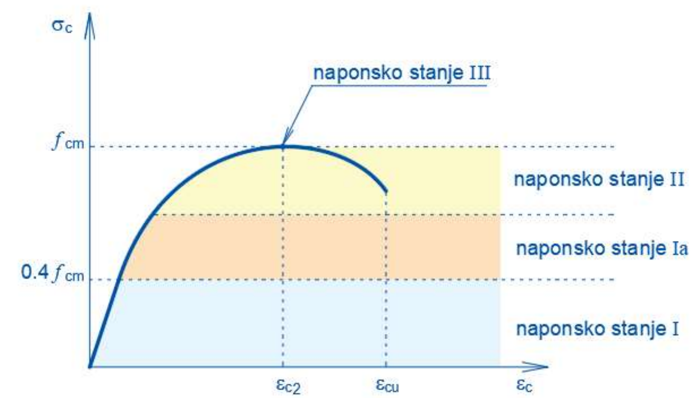
Beton i armatura djeluju zajedno do sloma

Deformacije betona i armature su jednake

Beton ne preuzima naprezanja u vlačnoj zoni (pukotine)

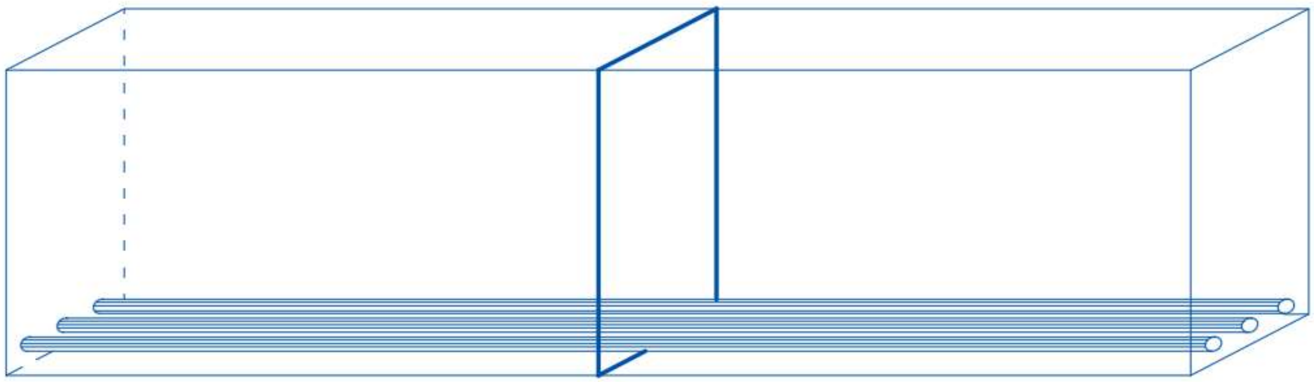
Granično stanje nosivosti – GSN
Čisto savijanje

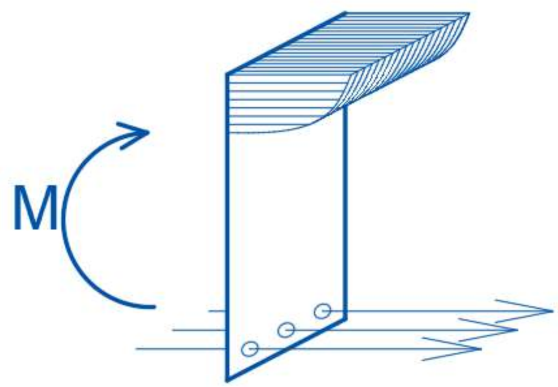




Granično stanje nosivosti – GSN

Čisto savijanje

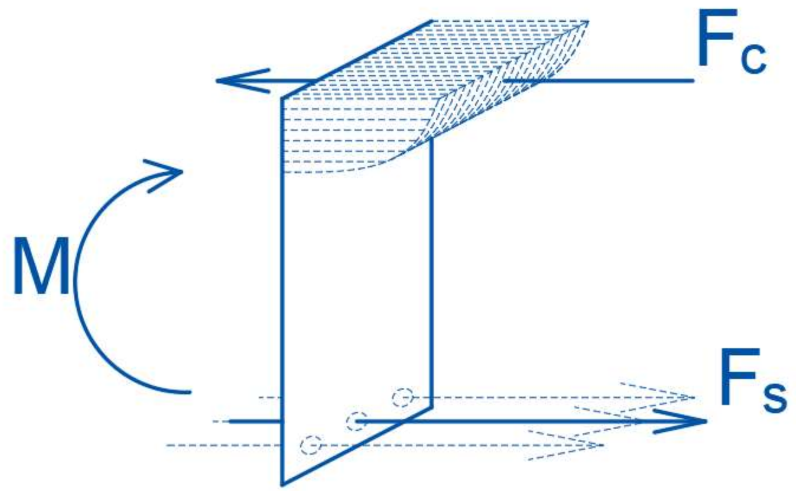




Naponsko stanje III:
Granično stanje nosivosti

Granično stanje nosivosti – GSN

Čisto savijanje



Zadatak dimenzioniranja: uspostaviti ravnotežu poprečnog presjeka (kao krutog tijela)!

POZNATO:

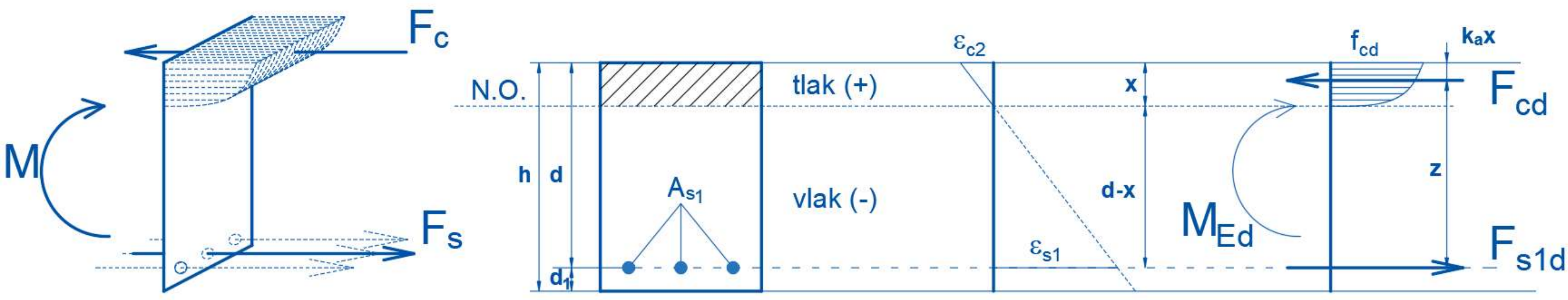
1. Vanjski moment savijanja – dobiven analizom opterećenja i proračunom statičkog sustava
2. Rezultantna (zamjenska) sila u betonu F_c – dobivena kao volumen prizme naprezanja. Pri tome moramo znati materijalne karakteristike betona kako bismo odredili ploštinu baze prizme (volumen se dobije množenjem ploštine baze sa širinom presjeka).

NEPOZNATO:

1. Rezultantna (zamjenska) sila u čeliku za armiranje F_s. Pri tome želimo ugrađenu armaturu iskoristiti najviše što je moguće pa je poznato najveće naprezanje (ovisno o klasi čelika). Stoga se zadatak svodi na određivanje potrebne ploštine poprečnog presjeka armature ($\sigma_s=F_s/A_s$ -> $A_s=F_s/\sigma_s$).

Granično stanje nosivosti – GSN

Čisto savijanje



Podsjetnik za oznake:

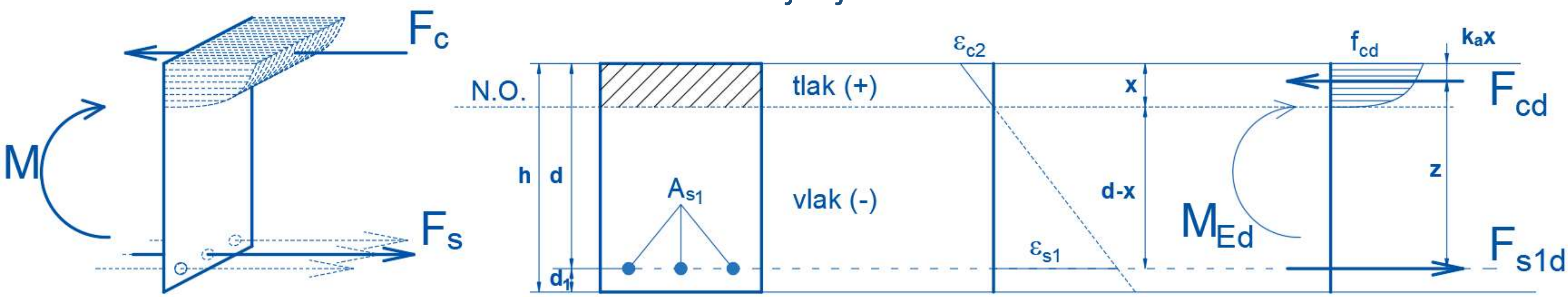
1. Indeks $_c$ uvijek se odnosi na beton
2. Indeks $_d$ uvijek se odnosi na računsku vrijednost (indeks k na karakterističnu)
3. Indeks $_1$ uvijek se odnosi na vlak, razvlačenje, produljenje
4. Indeks $_2$ uvijek se odnosi na tlak, skraćenje

Na primjer:

1. F_{s1d} čitamo: sila-čelik-vlak-računska (računska vlačna sila u čeliku)
2. ε_{c2} čitamo: relativna deformacija-beton-tlak (relativno skraćenje betona)

Granično stanje nosivosti – GSN

Čisto savijanje



Diskusija o dijagramu relativnih deformacija po visini presjeka:

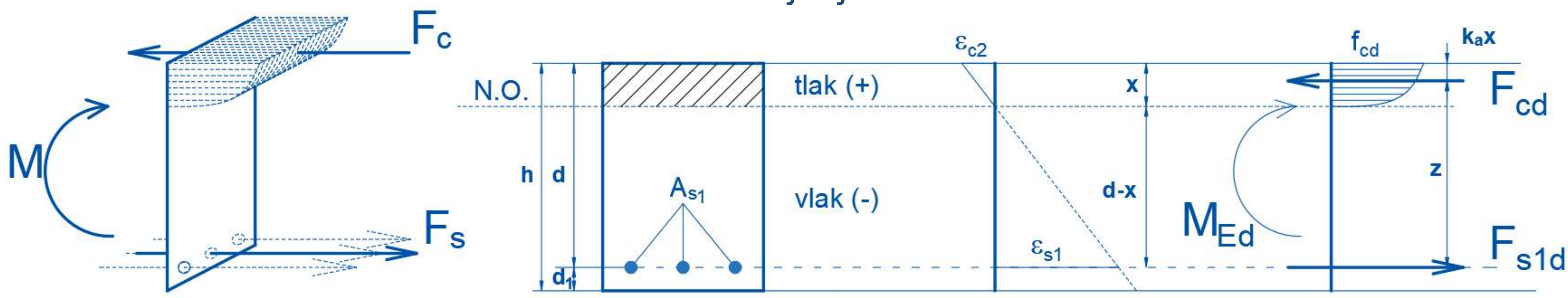
1. Zašto je dijagram linearan?
2. Što u stvarnosti predstavlja dijagram deformacija?
3. Zašto je vrijednost relativne deformacije nula na mjestu neutralne osi?
4. Zašto je deformacija čelika proporcionalna s deformacijama betona (proizlazi iz sličnosti trokuta)?

Diskusija o dijagramu naprezanja betona po visini presjeka:

1. Zašto je dijagram nelinearan?
2. Kako dobiti oblik krivulje?
3. Kako naći ploštinu i težište lika?
4. Gdje djeluje sila F_{cd}?
5. Gdje djeluje sila F_{s1d}?

Granično stanje nosivosti – GSN

Čisto savijanje



Uvjet ravnoteže: $M_{Ed} \leq M_{Rd} \quad \Rightarrow \quad M_{Ed} = M_{Rd} = F_{cd} \cdot z = F_{s1d} \cdot z$

Zadatak se svodi na određivanje veličine i položaja rezultantne sile F_{cd} i veličine sile F_{s1d}!

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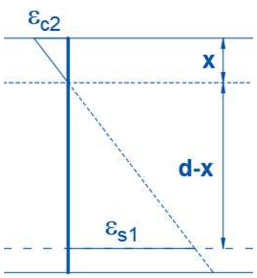
Granično stanje nosivosti – GSN

Čisto savijanje

$M_{Ed} \leq M_{Rd} \quad \Rightarrow \quad M_{Ed} = M_{Rd} = F_{cd} \cdot z = F_{s1d} \cdot \mathrm{z}$

Položaj neutralne osi: $\varepsilon_c : x = \varepsilon_{s1} : (d - x) \quad \Rightarrow \quad x = \dfrac{|\varepsilon_c|}{|\varepsilon_c| + \varepsilon_{s1}} \cdot d = \xi \cdot d$

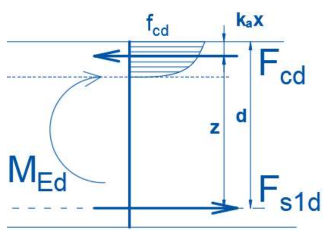
$$\xi = \frac{|\varepsilon_c|}{|\varepsilon_c| + \varepsilon_{s1}}$$



Krak unutarnjih sila: $z = d - k_a \cdot x = d - k_a \cdot (\xi \cdot d)$

$= \underbrace{(1 - k_a \cdot \xi)}_{\zeta} \cdot d$

$z = \zeta \cdot d$



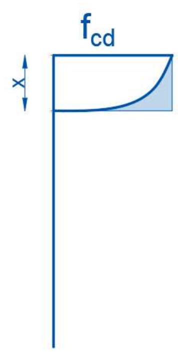
Granično stanje nosivosti – GSN

Čisto savijanje

$$M_{Ed} = M_{Rd} = F_{cd} \cdot z = F_{s1d} \cdot z$$

Rezultantna sila u betonu:

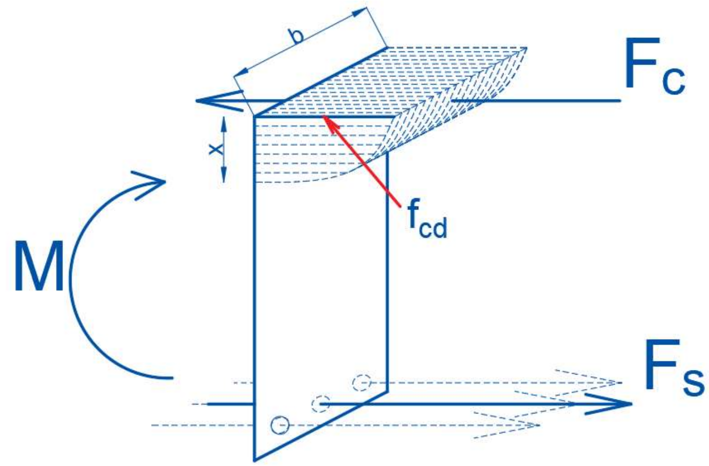
$$F_{cd} = f_{cd} \cdot \alpha_v \cdot x \cdot b$$



$$\xi \cdot d$$

$$F_{cd} = f_{cd} \cdot \alpha_v \cdot (\xi \cdot d) \cdot b$$

Volumen prizme naprezanja
(ploština baze × visina – *b*)



Rezultantna sila u armaturi:

$$F_{s1} = A_{s1} \cdot \sigma_{s1} \qquad \sigma_{s1} = \varepsilon_{s1} \cdot E_s \leq f_{yd}$$

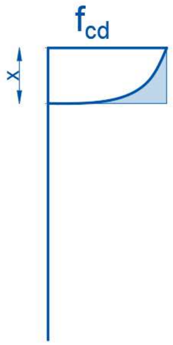
$$F_{s1} = A_{s1} \cdot f_{yd}$$

Diskusija o dijagramu naprezanja betona po visini presjeka:

1. O čemu ovisi oblik dijagrama?

Granično stanje nosivosti – GSN

Čisto savijanje



$$F_{cd} = \underbrace{f_{cd} \cdot \alpha_v \cdot (\xi \cdot d) \cdot b}$$

Volumen prizme naprezanja
(ploština baze × visina – *b*)

$0 < \varepsilon_c \leq 0{,}0020$

$$\alpha_v = \frac{1000 \cdot \varepsilon_c}{12} \cdot (6 - 1000 \cdot \varepsilon_c)$$

$$k_a = \frac{8 - 1000 \cdot \varepsilon_c}{4 \cdot (6 - 1000 \cdot \varepsilon_c)}$$

$0{,}0020 < \varepsilon_c \leq 0{,}0035$

$$\alpha_v = \frac{3000 \cdot \varepsilon_c - 2}{3000 \cdot \varepsilon_c} \cdot (6 - 1000 \cdot \varepsilon_c)$$

$$k_a = \frac{1000 \cdot \varepsilon_c \cdot (3000 \cdot \varepsilon_c - 4) + 2}{2000 \cdot \varepsilon_c \cdot (3000 \cdot \varepsilon_c - 2)}$$

Granično stanje nosivosti – GSN

Čisto savijanje

$$M_{Ed} = M_{Rd} = F_{cd} \cdot z = F_{s1d} \cdot \mathrm{z}$$

$$x = \xi \cdot d \qquad z = \zeta \cdot d$$

$$F_{cd} = f_{cd} \cdot \alpha_v \cdot (\xi \cdot d) \cdot b \qquad F_{s1d} = A_{s1} \cdot f_{yd}$$

$$F_{cd} \cdot z = f_{cd} \cdot \alpha_v \cdot (\xi \cdot d) \cdot b \cdot \zeta \cdot d = (b \cdot d^2 \cdot f_{cd}) \cdot (\xi \cdot \zeta \cdot \alpha_v)$$

$$M_{Ed} = (b \cdot d^2 \cdot f_{cd}) \cdot (\xi \cdot \zeta \cdot \alpha_v)$$

Ovisi o vanjskom opterećenju

Ovisi o odabranom presjeku i materijalu

Ovisi o odabranom paru deformacija ε_c i ε_{s1}

$$F_{s1d} \cdot \mathrm{z} = A_{s1} \cdot f_{yd} \cdot \mathrm{z} = A_{s1} \cdot f_{yd} \cdot (\zeta \cdot d)$$

$$M_{Ed} = A_{s1} \cdot f_{yd} \cdot (\zeta \cdot d)$$

Ovisi o odabranom presjeku i materijalu

Ovisi o odabranom paru deformacija ε_c i ε_{s1}

Granično stanje nosivosti – GSN
Čisto savijanje

$$M_{Ed} = (b \cdot d^2 \cdot f_{cd}) \cdot (\xi \cdot \zeta \cdot \alpha_v) \qquad \mu_{Ed} = \xi \cdot \zeta \cdot \alpha_v$$

$$\mu_{Ed} = \mu_{Rd} = \frac{M_{Ed}}{b \cdot d^2 \cdot f_{cd}}$$

$$M_{Ed} = A_{s1} \cdot f_{yd} \cdot (\zeta \cdot d)$$

$$A_{s1} = \frac{M_{Ed}}{f_{yd} \cdot (\zeta \cdot d)}$$

ε_c (‰)	ε_{s1} (‰)	ξ	ζ	μ_{Rd}
-3,5	15,0	0,189	0,921	0,120
-3,4	18,5	0,159	0,934	0,120
-2,6	10,0	0,206	0,919	0,120
-1,7	5,0	0,265	0,907	0,120

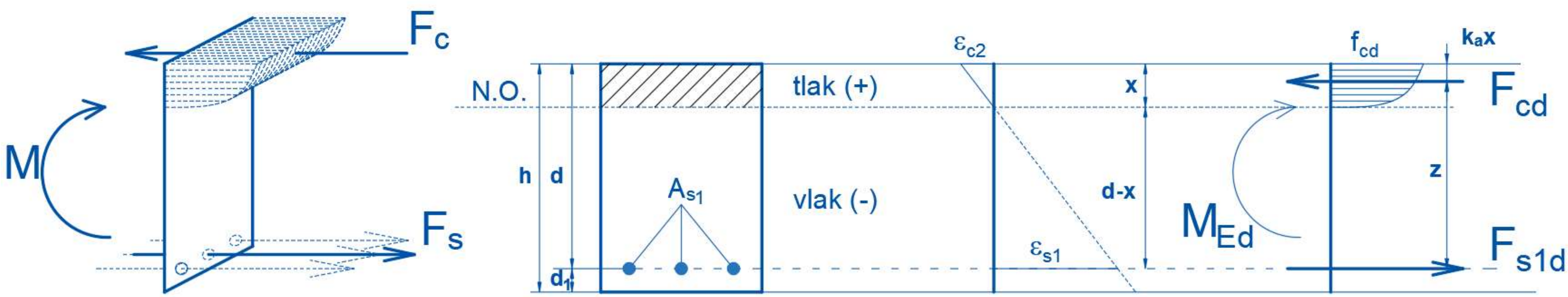
Diskusija:

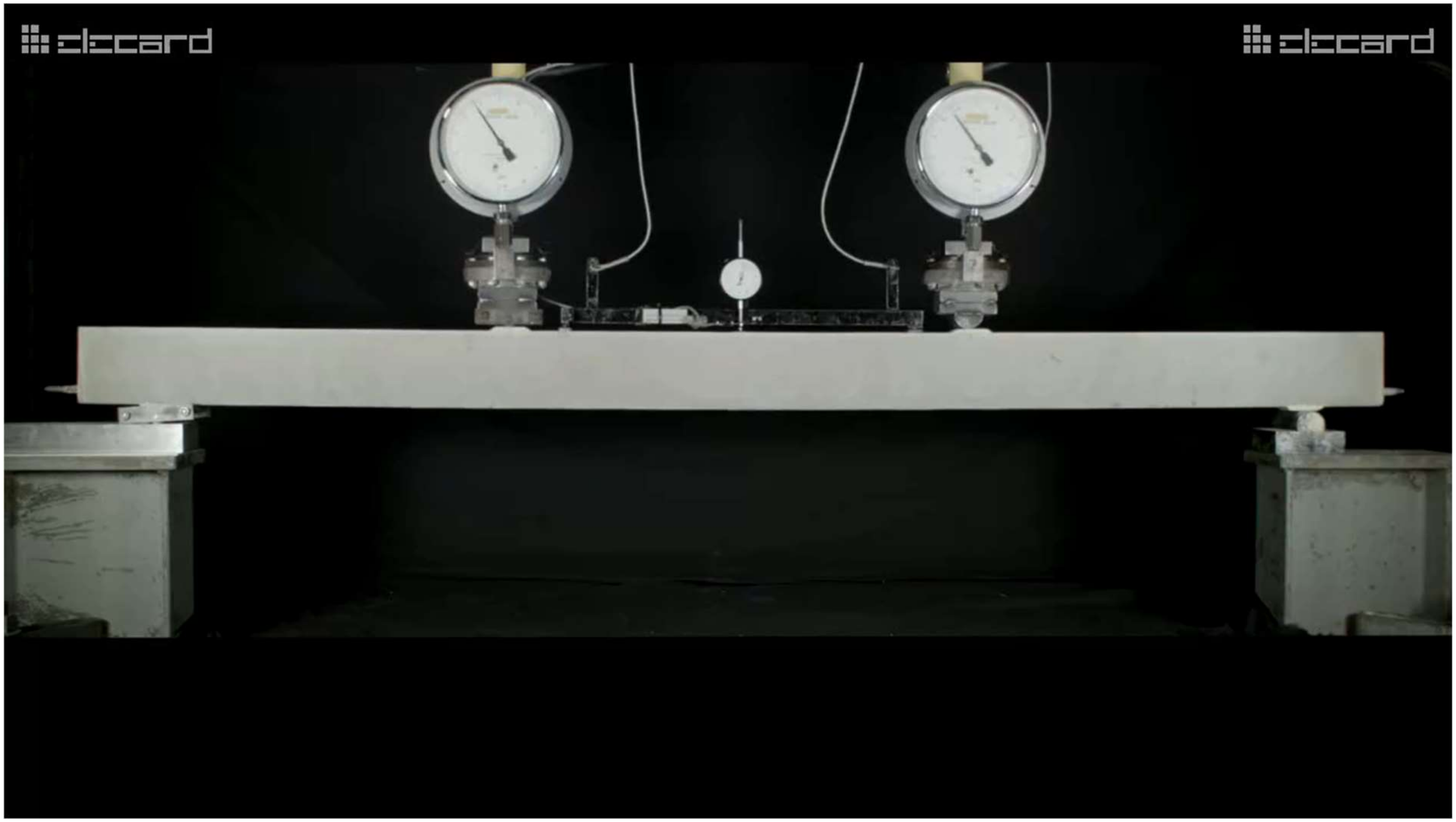
1. Koji par deformacija odabrati?

Granično stanje nosivosti – GSN
Čisto savijanje

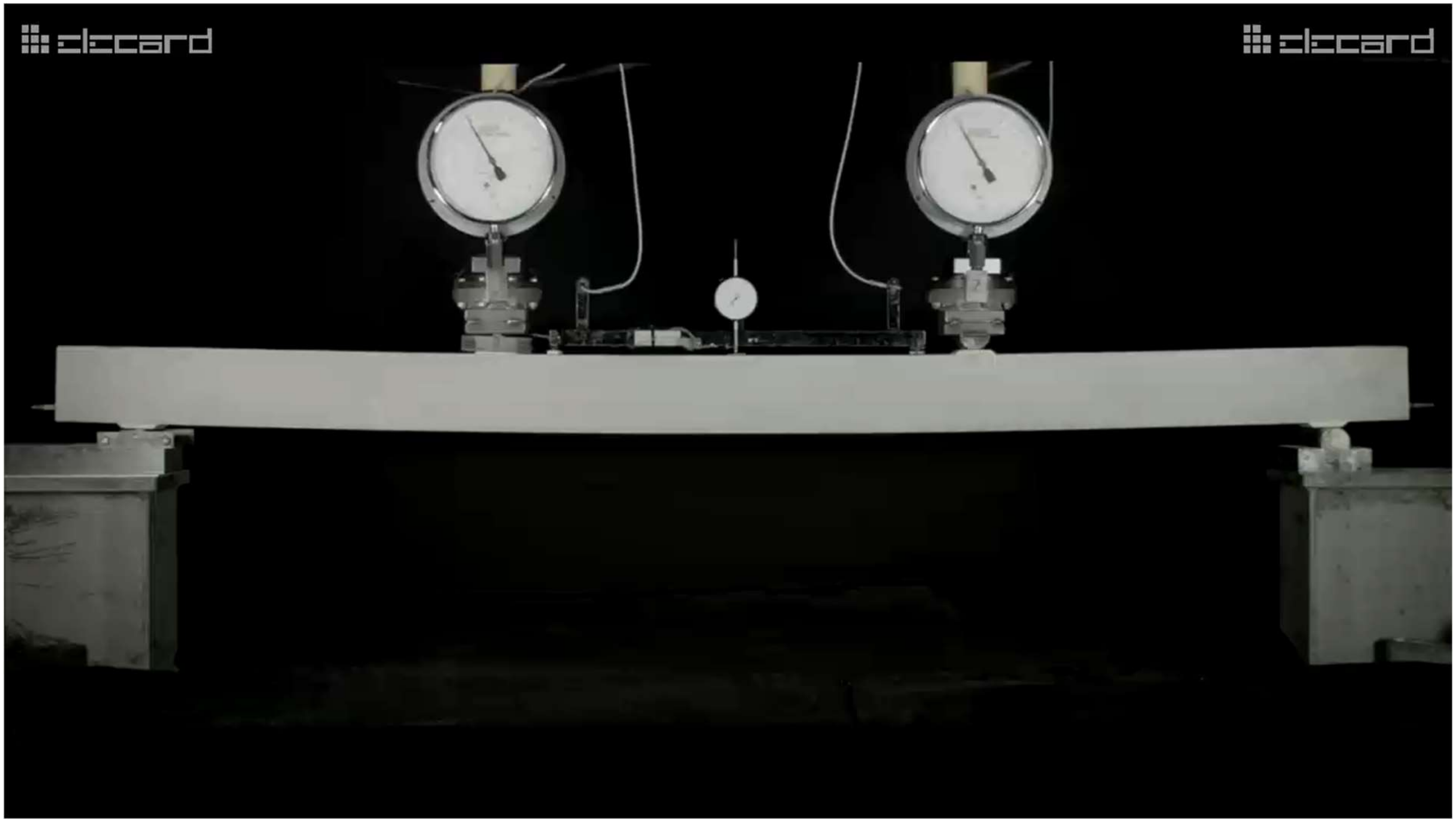
Diskusija:

1. Što se događa ako je premalo armature?
2. Što se događa ako je previše armature?





Izvor: Materials Lab Online (https://thinkup.org/innovation/materials-lab-online/)



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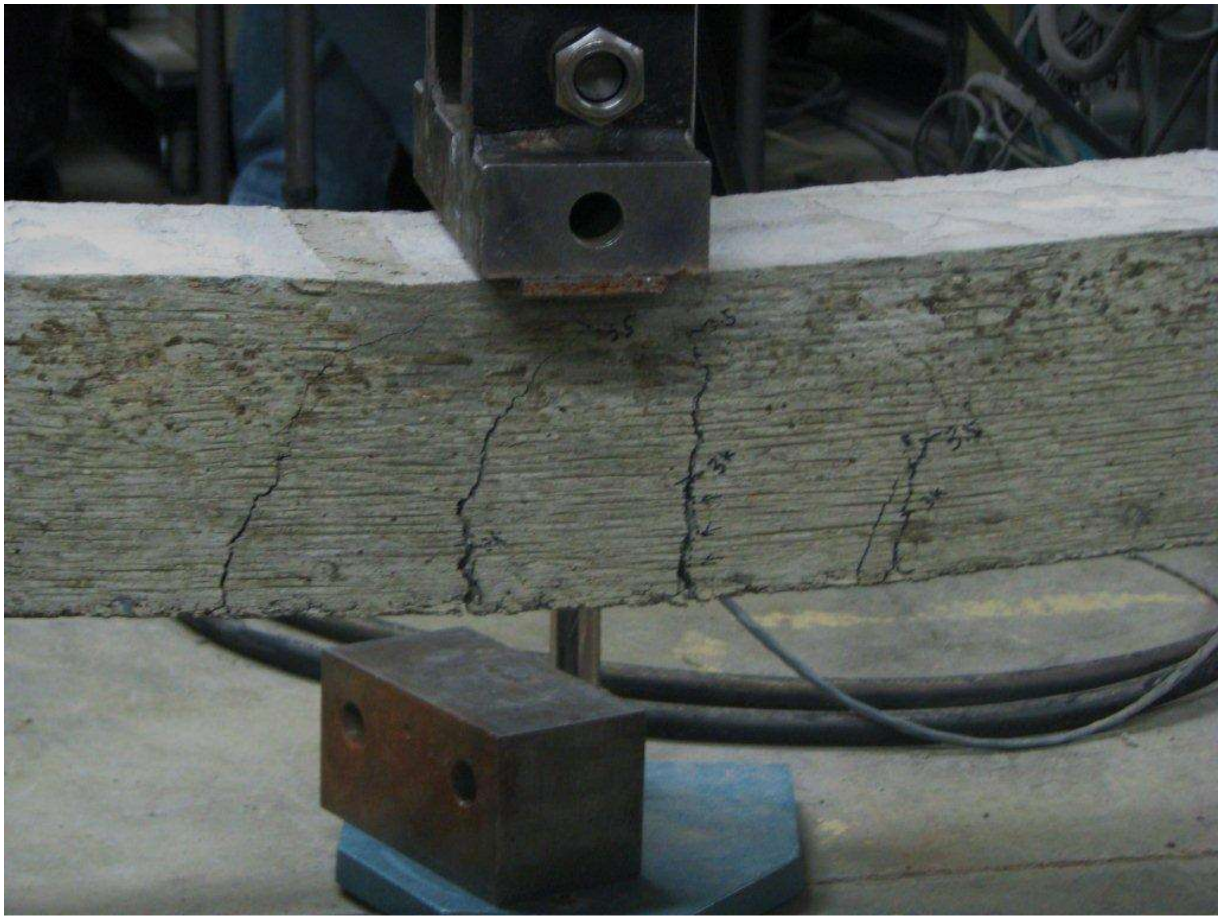

Izvor: http://reinforced-concrete.blogspot.com/

Ponavljanje

Centrična uzdužna sila

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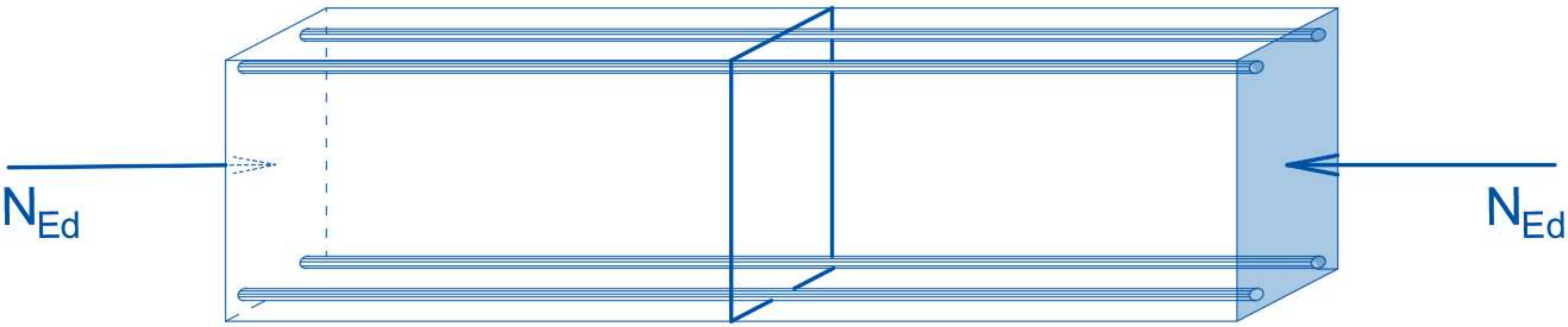
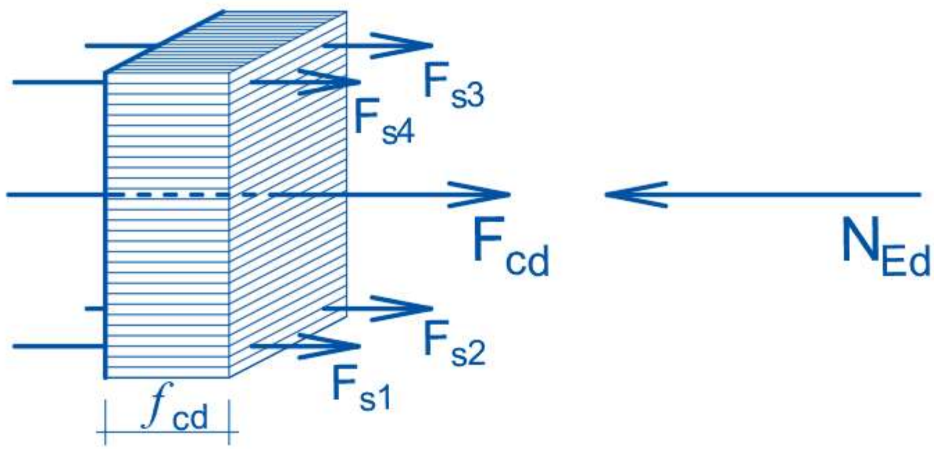
Granično stanje nosivosti – GSN

Centrična tlačna sila



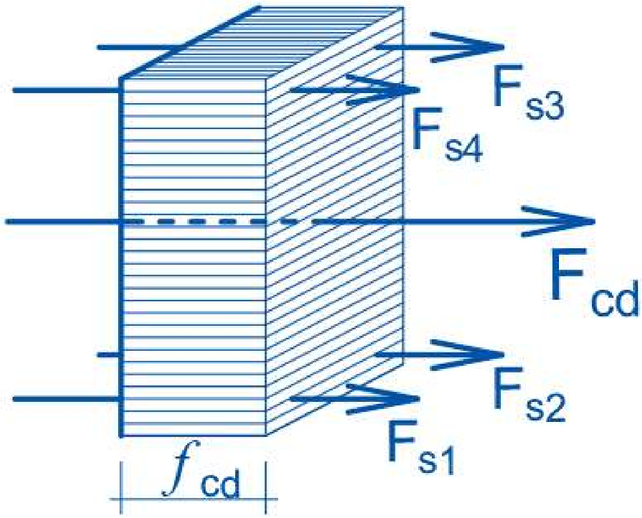
Uvjet ravnoteže:

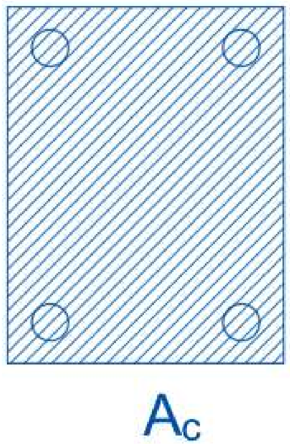
$$N_{Rd} = N_{Ed}$$

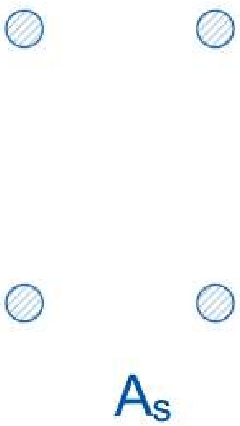


Granično stanje nosivosti – GSN

Centrična tlačna sila







$$N_{Rd} = N_{Ed}$$

$$N_{Rd} = F_{cd} + F_{s1} + F_{s2} + F_{s3} + F_{s4}$$

$$F_{cd} = f_{cd} \cdot (A_c - A_s)$$

$$F_{sd} = (A_{s1} + A_{s2} + A_{s3} + A_{s4}) \cdot f_{yd} = A_s \cdot f_{yd}$$

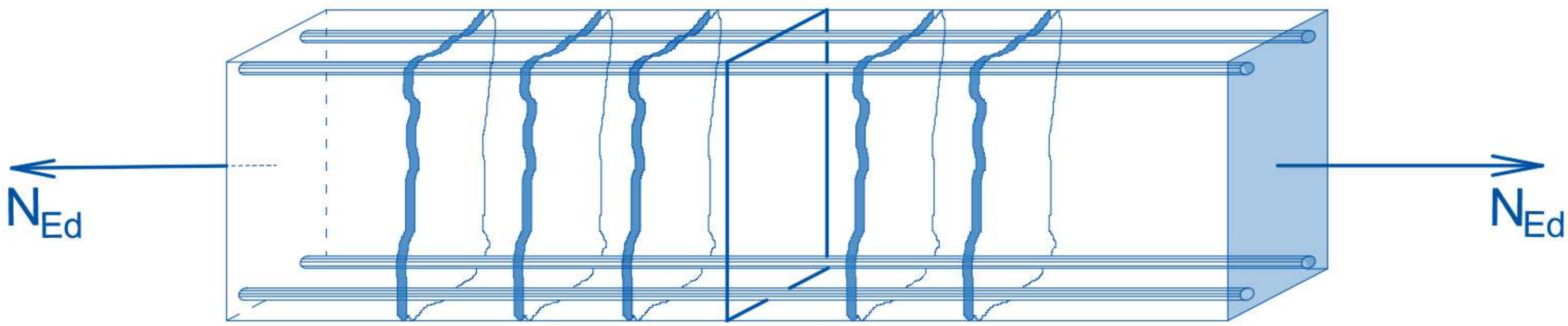
$$f_{cd} \cdot (A_c - A_s) + A_s \cdot f_{yd} = N_{Ed}$$

$$A_{s,req} = \frac{N_{Ed} - A_c \cdot f_{cd}}{f_{yd} - f_{cd}}$$

Granično stanje nosivosti – GSN

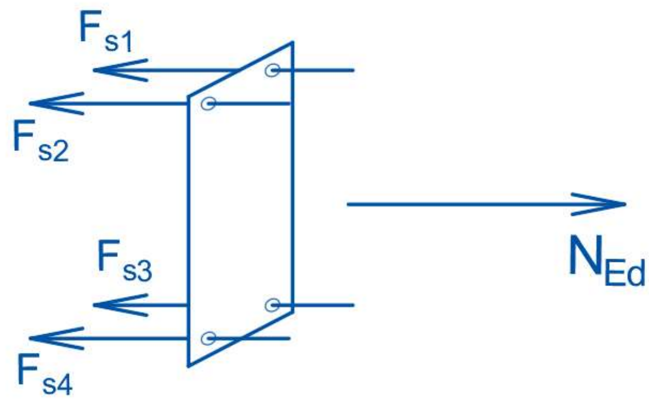
Centrična vlačna sila

Slučaj 1: dopuštaju se pukotine u betonu



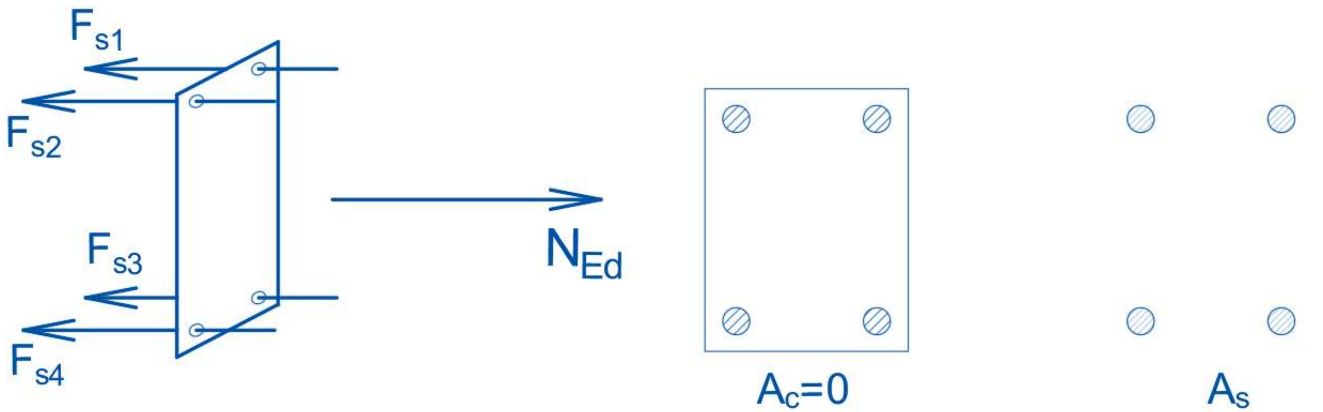
Uvjet ravnoteže:

$$N_{Rd} = N_{Ed}$$



Granično stanje nosivosti – GSN
Centrična vlačna sila

Slučaj 1: dopuštaju se pukotine u betonu



$$N_{Rd} = N_{Ed}$$

$$N_{Rd} = F_{s1} + F_{s2} + F_{s3} + F_{s4}$$

$$F_{sd} = (A_{s1} + A_{s2} + A_{s3} + A_{s4}) \cdot f_{yd} = A_s \cdot f_{yd}$$

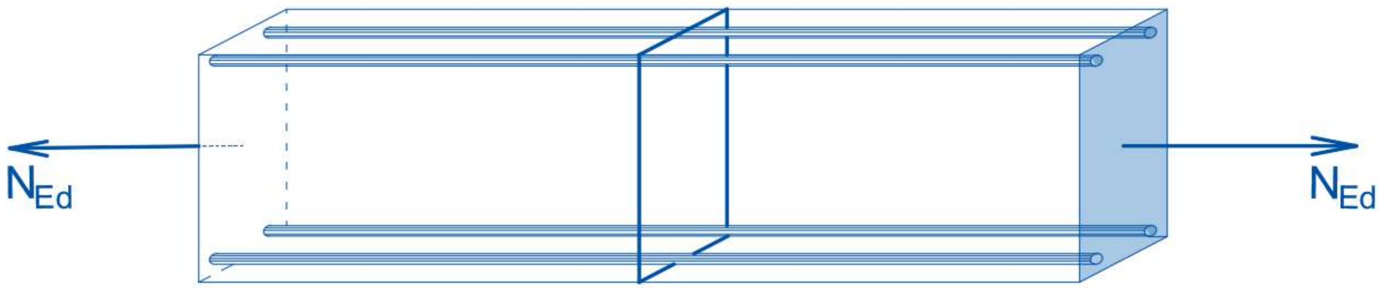
$$A_s \cdot f_{yd} = N_{Ed}$$

$$A_{s,req} = \frac{N_{Ed}}{f_{yd}}$$

Granično stanje nosivosti – GSN

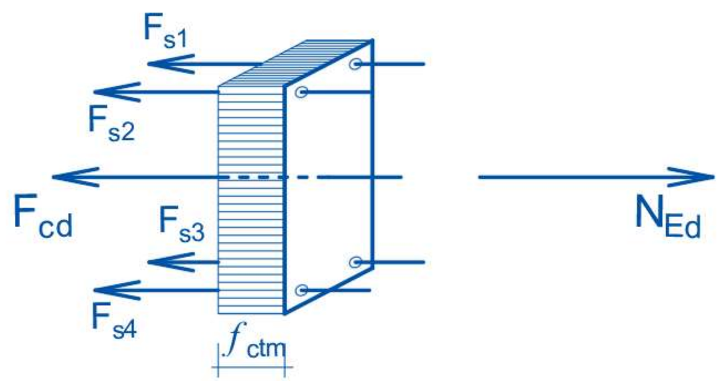
Centrična vlačna sila

Slučaj 2: ne dopuštaju se pukotine u betonu



Uvjet ravnoteže:

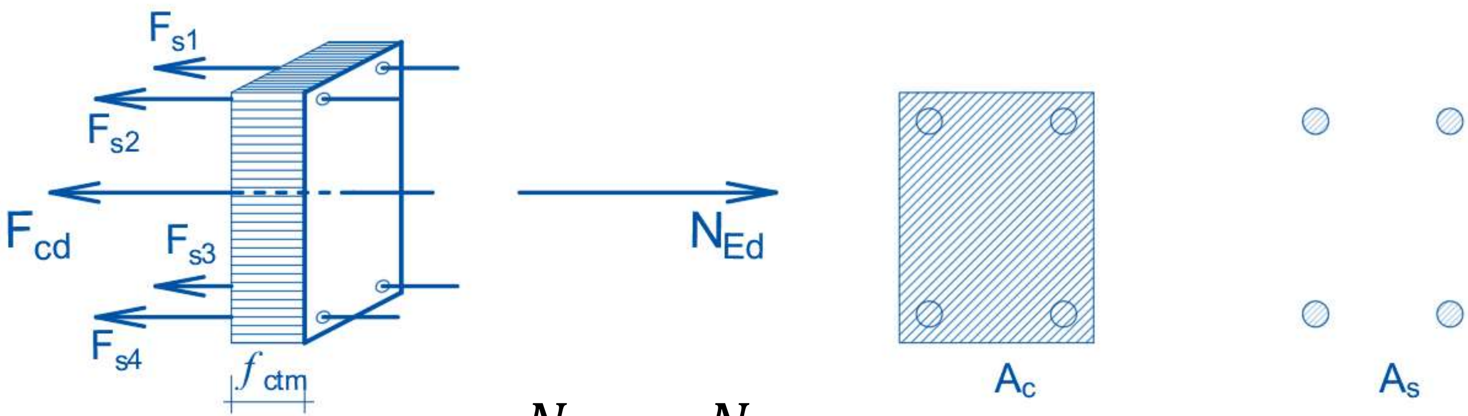
$$N_{Rd} = N_{Ed}$$



Granično stanje nosivosti – GSN

Centrična vlačna sila

Slučaj 2: ne dopuštaju se pukotine u betonu



$$N_{Rd} = N_{Ed}$$

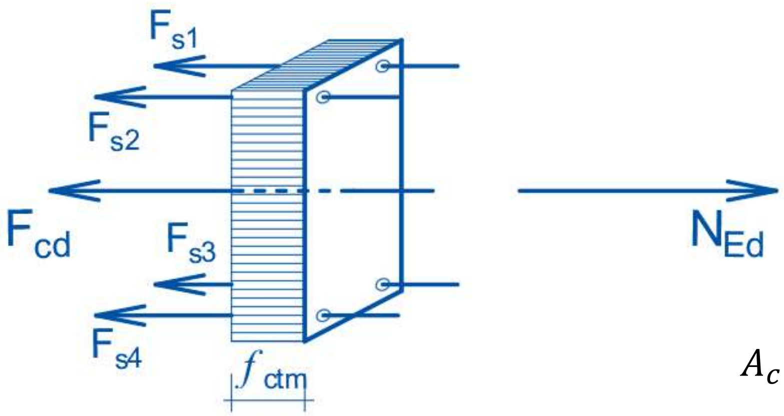
Ograničenja:

$$\varepsilon_{c,max} = \varepsilon_{s,max} = 0{,}1\ ^{\circ}/_{\circ\circ} = 0{,}0001$$

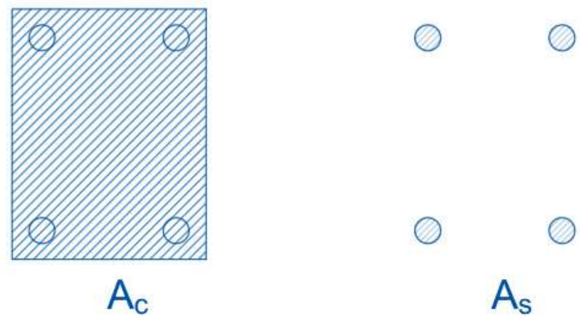
$$\sigma_s = E_s \cdot \varepsilon_{s,max} = 200000 \cdot 0{,}0001 = 20\ N/mm^2 = 2{,}0\ kN/cm^2$$

Granično stanje nosivosti – GSN

Centrična vlačna sila



$A_c \geq 2$



$$N_{Rd} = N_{Ed}$$

$N_{Rd} = A_c \cdot f_{ctm} + A_s \cdot \sigma_s$ $\quad N_{Ed} = N \cdot \gamma_I$ ⇨ $1{,}2 \leq \gamma_I \leq 1.5$

$A_s \cdot f_{yd} = N_G \cdot \gamma_G + N_Q \cdot \gamma_Q = N \cdot \gamma_F$ $\quad \gamma_F = \dfrac{N_G \cdot \gamma_G + N_Q \cdot \gamma_Q}{N_G + N_Q}$

$$A_{s,req} = \frac{N \cdot \gamma_I - A_c \cdot f_{ctm}}{\sigma_s}$$

$$A_c \geq \frac{A_s \cdot \gamma_I}{f_{ctm}} \left(\frac{f_{yd}}{\gamma_F} - \frac{\sigma_s}{\gamma_I} \right)$$