

# Betonske konstrukcije 2

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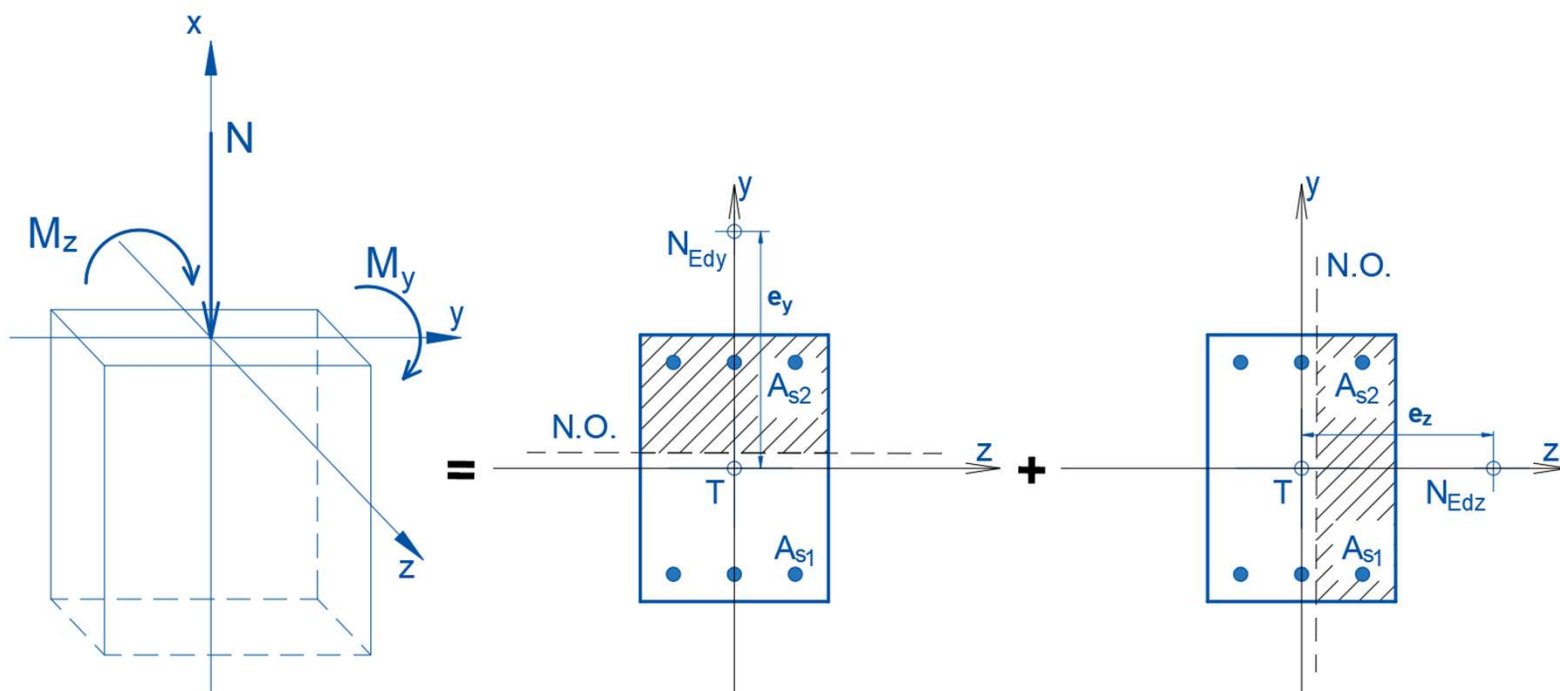
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# DIMENZIONIRANJE ELEMENATA OPTEREĆENIH DVOOSNIM SAVIJANJEM (I UZDUŽNOM SILOM)

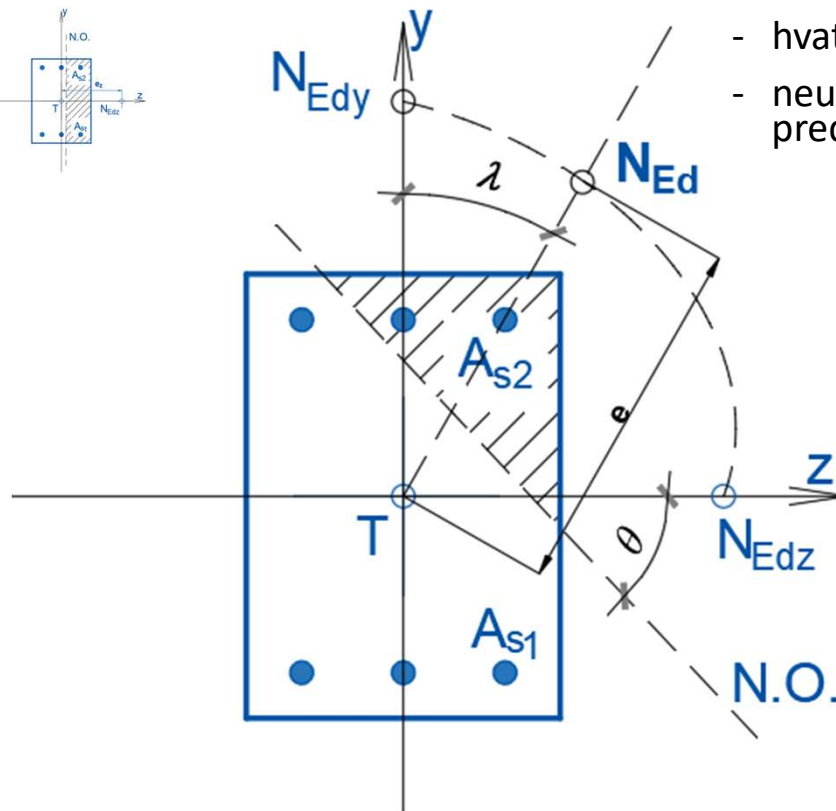
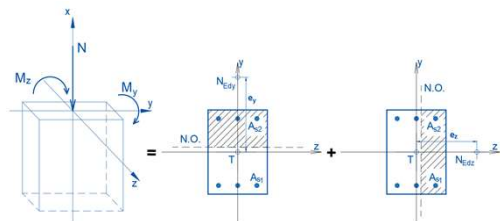
Z. Sorić, T. Kišiček: Betonske konstrukcije 2

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# Dvoosno savijanje



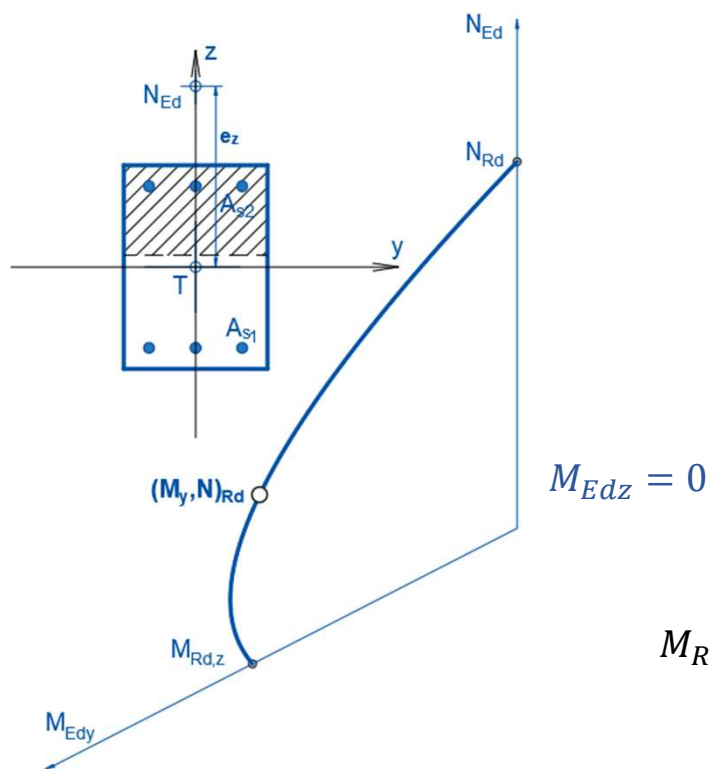
## Dvoosno savijanje



- hvatište sile uvijek izvan glavnih osi
- neutralna os nije okomita na ekscentricitet, već predstavlja os oko koje se štap izvija!

## Dvoosno savijanje

$$N_{Rd} = F_{c1d} + F_{sd} = f_{cd} \cdot (A_c - A_{s1} - A_{s2}) + f_{yd} \cdot (A_{s1} + A_{s2})$$



Savijanje oko osi y

$$M_{Rd} = F_{cd} \cdot z = f_{cd} \cdot \alpha_v \cdot \xi \cdot \zeta \cdot d^2 \cdot b$$

$$= f_{cd} \cdot \mu_{Rd} \cdot d^2 \cdot b$$

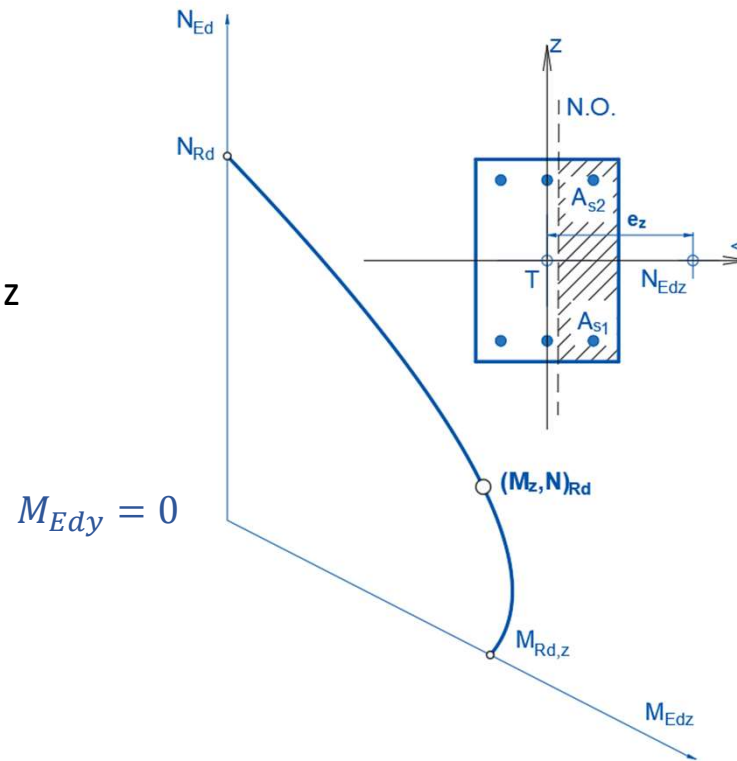
$\mu_{Rd}$

$$M_{Rd} = F_{s1d} \cdot z = A_{s1} \cdot \sigma_{s1} \cdot \zeta \cdot d$$

## Dvoosno savijanje

$$N_{Rd} = F_{c1d} + F_{sd} = f_{cd} \cdot (A_c - A_{s1} - A_{s2}) + f_{yd} \cdot (A_{s1} + A_{s2})$$

Savijanje oko osi z

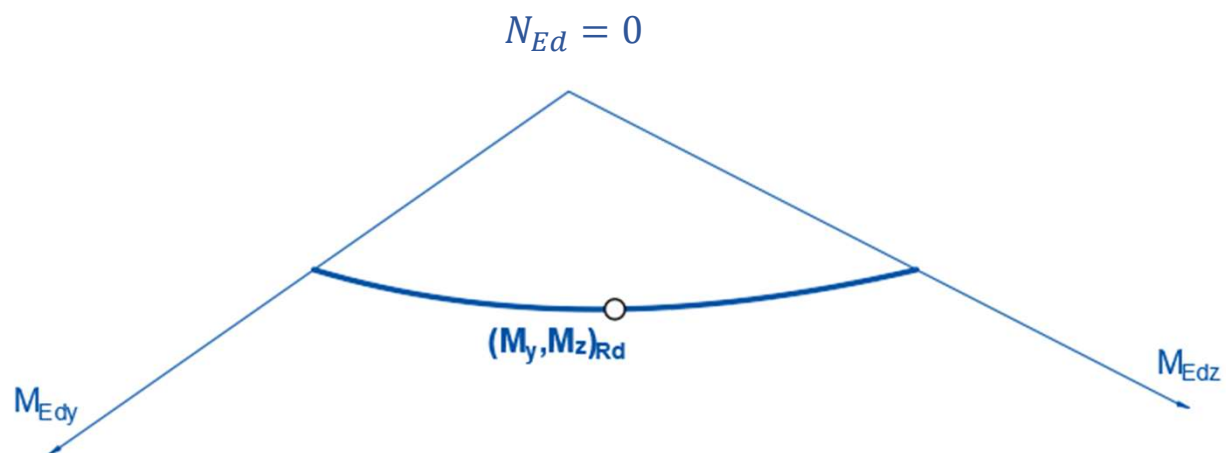


$$M_{Rd} = F_{cd} \cdot z = f_{cd} \cdot \alpha_v \cdot \xi \cdot \zeta \cdot d^2 \cdot b$$

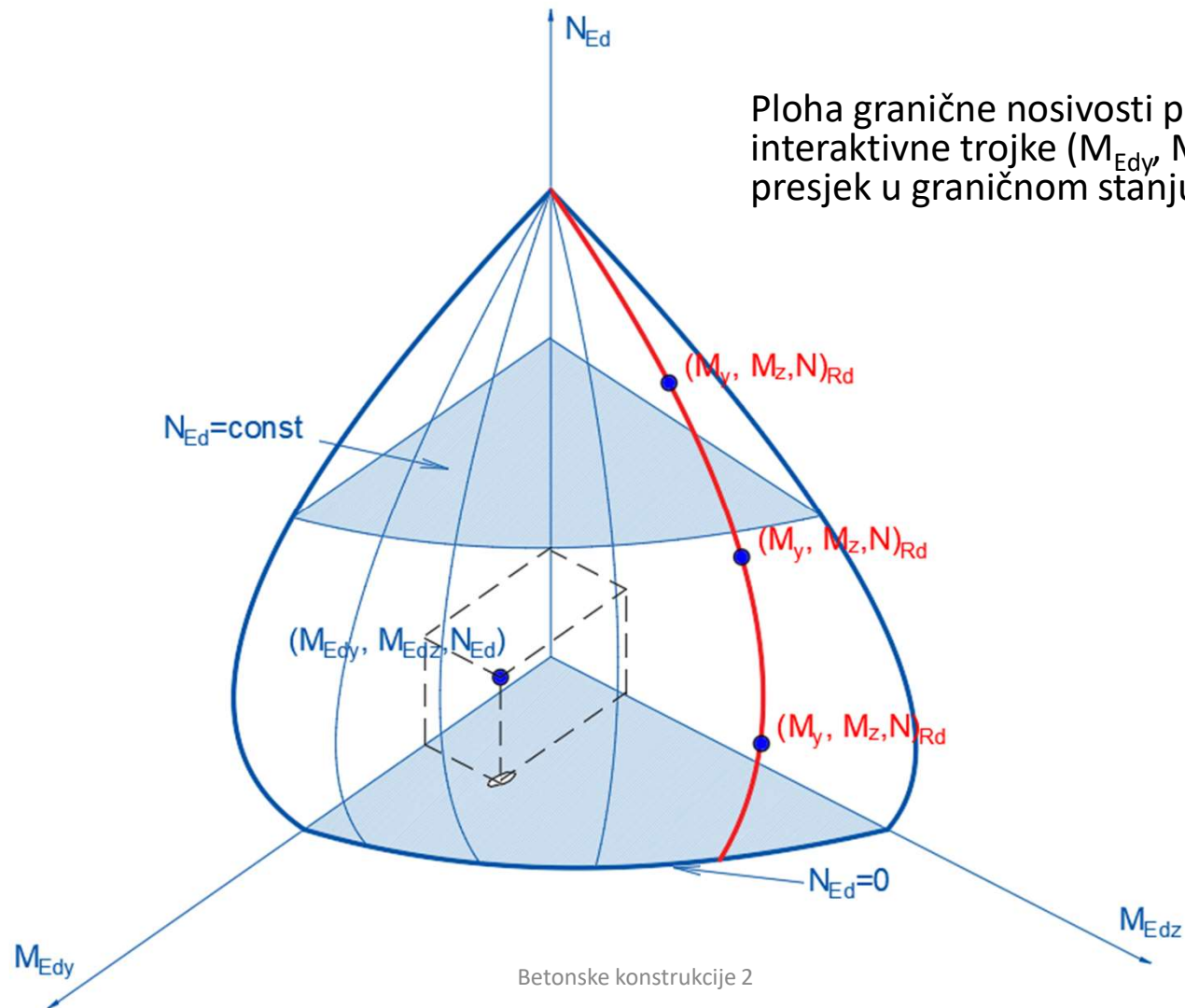
$$= f_{cd} \cdot \mu_{Rd} \cdot d^2 \cdot b$$

$$M_{Rd} = F_{s1d} \cdot z = A_{s1} \cdot \sigma_{s1} \cdot \zeta \cdot d$$

# Dvoosno savijanje

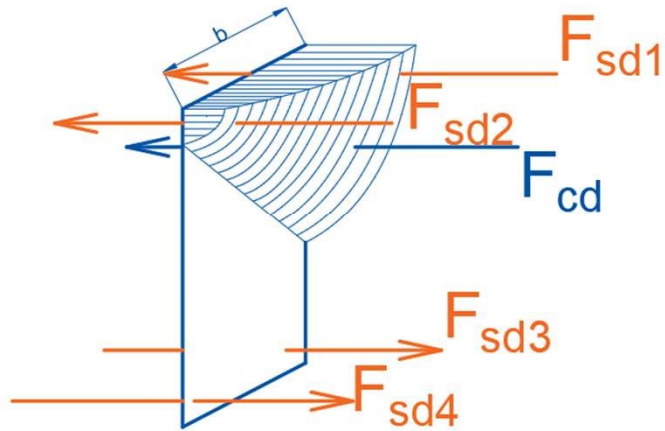


## Aproksimacija plohe i krivulja granične nosivosti





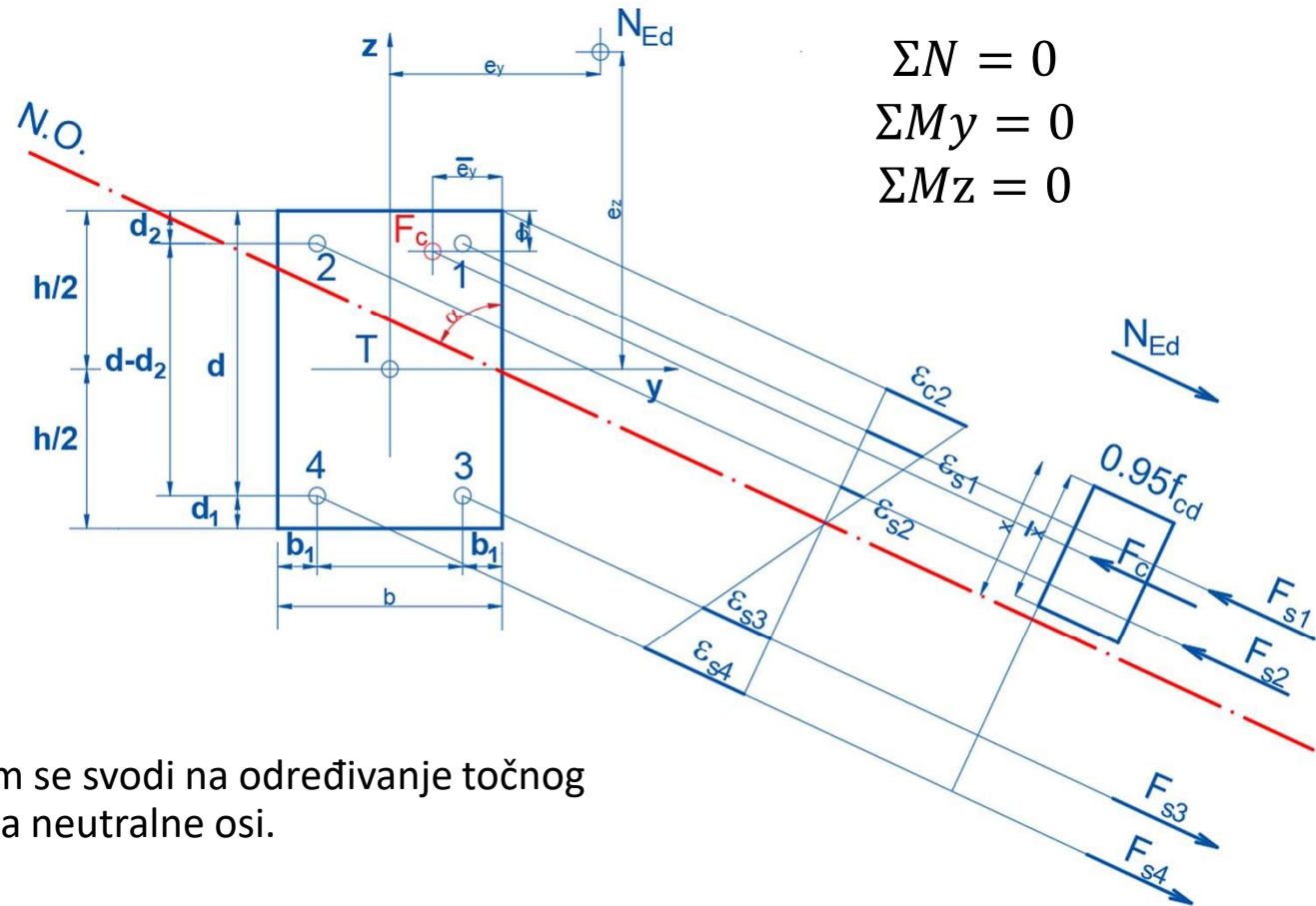
## Dvoosno savijanje



$$M_{E dy} = N_{Ed} \cdot e_z$$

$$M_{E dz} = N_{Ed} \cdot e_y$$

Problem se svodi na određivanje točnog položaja neutralne osi.

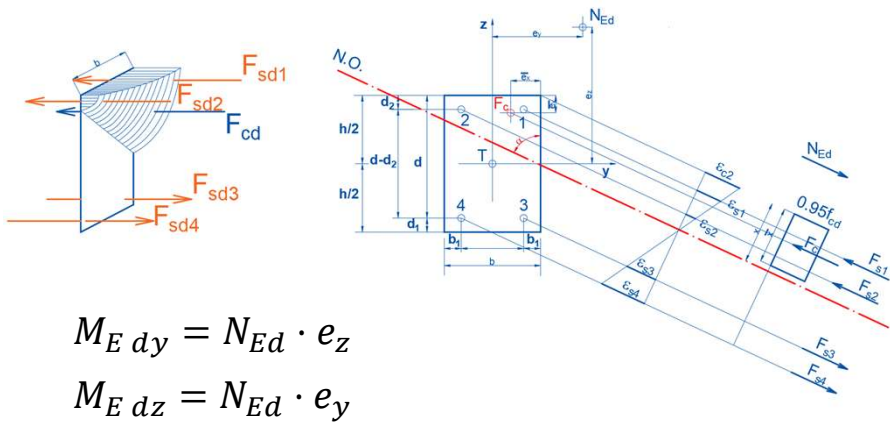


$$\Sigma N = 0$$

$$\Sigma M_y = 0$$

$$\Sigma M_z = 0$$

## Dvoosno savijanje



Simultano moraju biti zadovoljene sve tri  
jednadžbe ravnoteže:

$$\Sigma N = 0$$

$$\Sigma M_y = 0$$

$$\Sigma M_z = 0$$

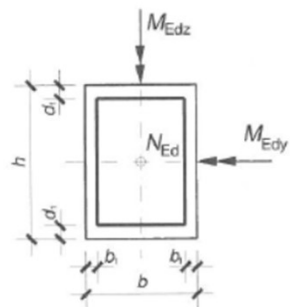
$$M_{E dy} = N_{Ed} \cdot e_z$$

$$M_{E dz} = N_{Ed} \cdot e_y$$

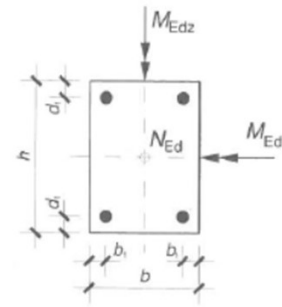
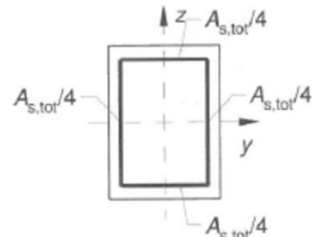
$$\begin{aligned} N_{Rd} &= F_{cd} + F_{sd1} + F_{sd2} - F_{sd3} - F_{sd4} \\ &= 0.95 \cdot f_{cd} \cdot \bar{x} \cdot b + f_{yd} \cdot (A_{sd1} + A_{sd2} - A_{sd3} - A_{sd4}) \end{aligned}$$

$$\begin{aligned} M_{Rdy} &= N_{Rd} \cdot e_z \\ &= F_{cd} \cdot \left(\frac{h}{2} - \bar{e}_z\right) + (F_{sd1} + F_{sd2}) \cdot \left(\frac{h}{2} - d_2\right) + (F_{sd3} + F_{sd4}) \cdot \left(\frac{h}{2} - d_1\right) \\ &= 0.95 \cdot f_{cd} \cdot \bar{x} \cdot b \cdot \left(\frac{h}{2} - \bar{e}_z\right) + (A_{s1} \cdot \sigma_{s1} + A_{s2} \cdot \sigma_{s2}) \cdot \left(\frac{h}{2} - d_2\right) + (A_{s3} \cdot \sigma_{s3} + A_{s4} \cdot \sigma_{s4}) \cdot \left(\frac{h}{2} - d_1\right) \end{aligned}$$

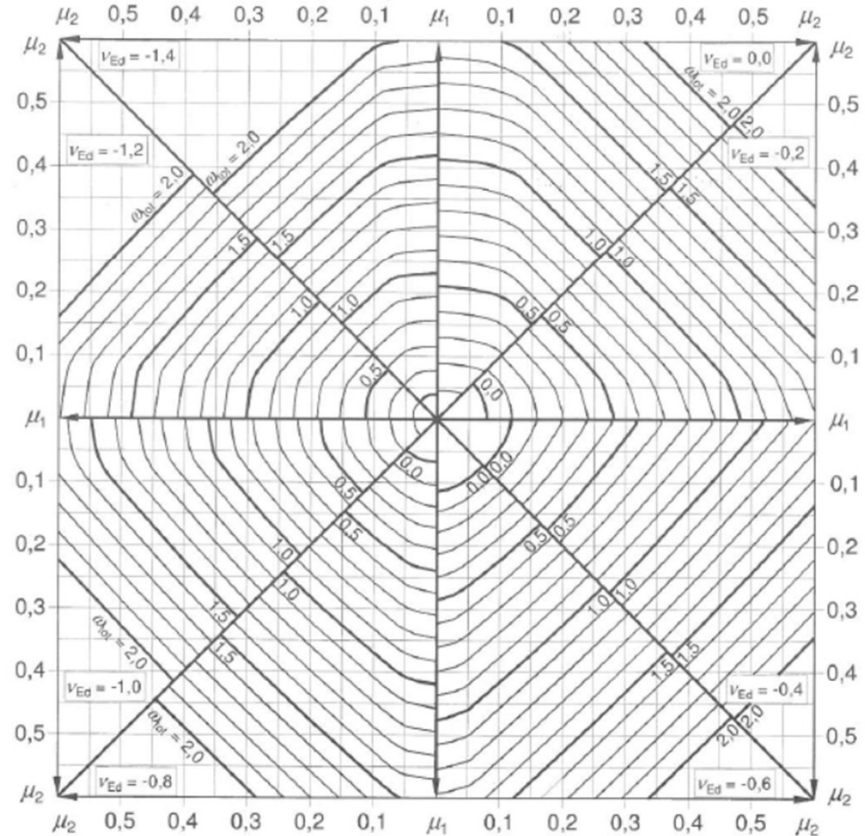
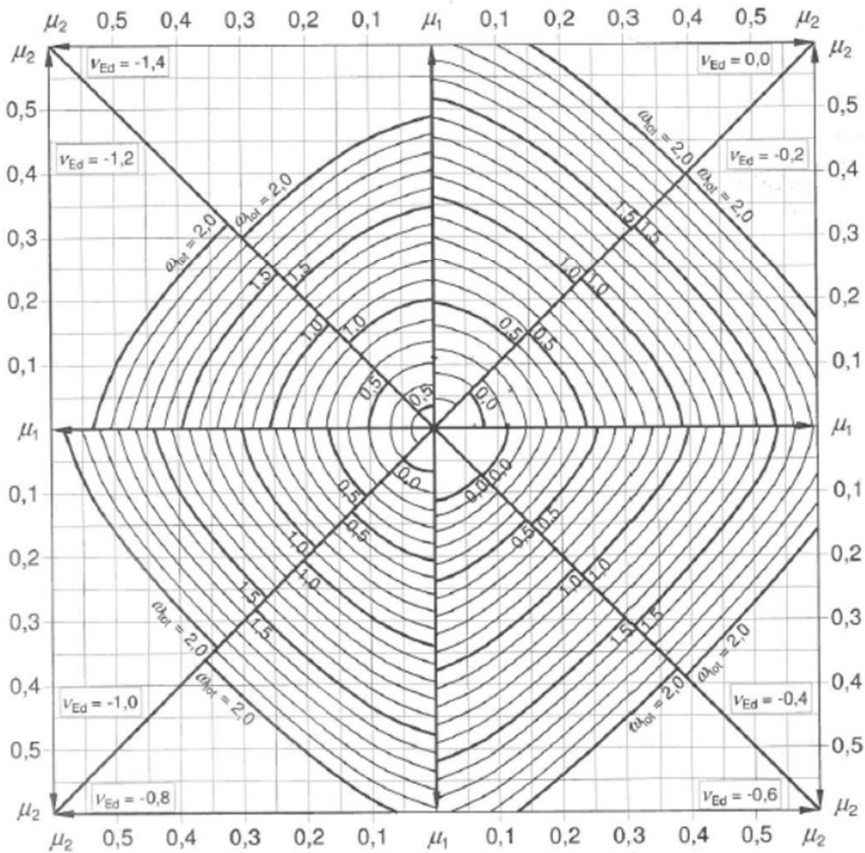
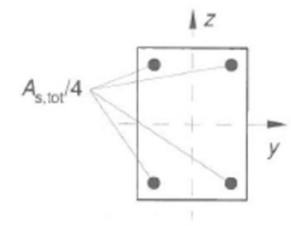
$$\begin{aligned} M_{Rdz} &= N_{Rd} \cdot e_y \\ &= F_{cd} \cdot \left(\frac{b}{2} - \bar{e}_y\right) + (F_{sd1} - F_{sd3}) \cdot \left(\frac{b}{2} - b_1\right) + (F_{sd4} - F_{sd2}) \cdot \left(\frac{b}{2} - b_1\right) \\ &= 0.95 \cdot f_{cd} \cdot \bar{x} \cdot b \cdot \left(\frac{b}{2} - \bar{e}_y\right) + (A_{s1} \cdot \sigma_{s1} - A_{s3} \cdot \sigma_{s3}) \cdot \left(\frac{b}{2} - b_1\right) + (A_{s4} \cdot \sigma_{s4} - A_{s2} \cdot \sigma_{s2}) \cdot \left(\frac{b}{2} - b_1\right) \end{aligned}$$



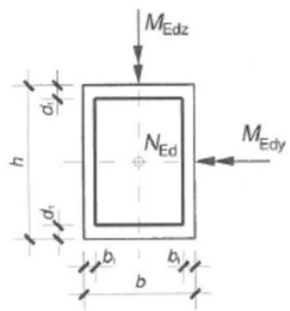
Beton: C12/15 do C50/60  
 Čelik: B500B  
 $d/h = b'/b = 0,1$   
 $\gamma_s = 1,15; \gamma_c = 1,5$



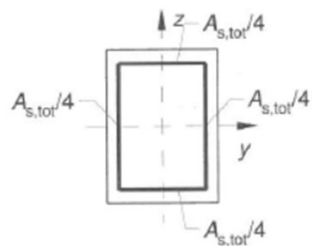
Beton: C12/15 do C50/60  
 Čelik: B500B  
 $d/h = b'/b = 0,1$   
 $\gamma_s = 1,15; \gamma_c = 1,5$



etonske konstrukcije 2



Beton: C12/15 do C50/60  
 Čelik: B500B  
 $d_1/h = b_1/b = 0,1$   
 $\gamma_s = 1,15; \gamma_c = 1,5$



$$\mu_{Edy} = \frac{|M_{Edy}|}{b \cdot h^2 \cdot f_{cd}}$$

$$\mu_{Edz} = \frac{|M_{Edz}|}{b^2 \cdot h \cdot f_{cd}}$$

$\mu_1$  je uvijek veći od ova dva

$$v_{Ed} = \frac{N_{Ed}}{b \cdot h \cdot f_{cd}}$$

Primjer:

$$\mu_{Edy} = 0,112 = \mu_1$$

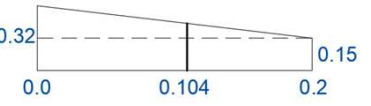
$$\mu_{Edz} = 0,066 = \mu_2$$

$$v_{Ed} = -0,104$$

$$\omega_{tot} = 0,236$$

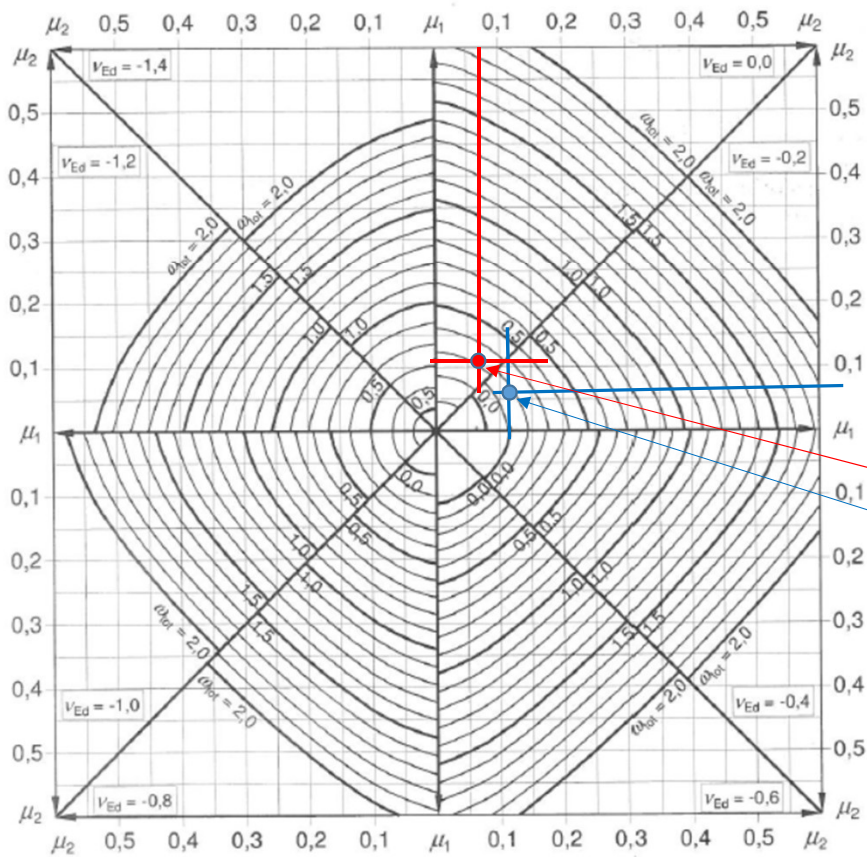
U području  $v_{Ed} = 0,0 \Rightarrow \omega_{tot} = 0,32$

U području  $v_{Ed} = 0,2 \Rightarrow \omega_{tot} = 0,15$



$$A_{s,tot} = \omega_{tot} \cdot b \cdot h \frac{f_{cd}}{f_{yd}}$$

Ukupna armatura za sve četiri stranice!



## Odvojena provjera oko svake osi posebno

Odvojene provjere oko svake osi (kao jednoosno savijanje) smiju se provesti samo ako su ispunjena oba uvjeta:

$$\frac{\lambda_y}{\lambda_z} \leq 2 \quad \text{i} \quad \frac{\lambda_z}{\lambda_y} \leq 2$$

i jedan od sljedeća dva uvjeta:

$$\frac{\frac{e_y}{b_{eq}}}{\frac{e_z}{h_{eq}}} \leq 2 \quad \text{ili} \quad \frac{\frac{e_z}{h_{eq}}}{\frac{e_y}{b_{eq}}} \leq 2$$

gdje je:

$$\lambda_y, \lambda_z = \frac{l_0}{i} \quad \text{duljine izvijanja}$$

$$e_z = \frac{M_{Edy}}{N_{Ed}}$$

$$e_y = \frac{M_{Edz}}{N_{Ed}}$$

ekscentriciteti

$$b_{eq} = i_z \cdot \sqrt{12}$$

$$h_{eq} = i_y \cdot \sqrt{12}$$

Istovrijedna širina i visina poprečnog presjeka ( $b_{eq}=b$  i  $h_{eq} = h$  za pravokutne presjeke)

## Odvojena provjera oko svake osi posebno

