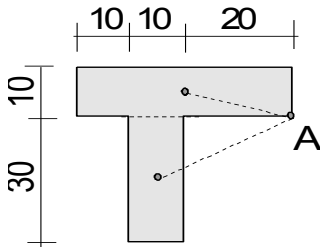
DINAMIKA – DINAMIKA KRUTOG TIJELA

PRIMJER:

Momenti tromosti mase tijela

1. Primjenom Steinerovog stavka odredite moment tromosti na točku *A*, za homogenu ploču mase 0,5 kg/m^2 prikazane na crtežu 6.4. Prikazane dimenzije su u metrima.



Crtež 6.4.

RJEŠENJE:

$$r_1 = \sqrt{20^2 + 5^2} = 20{,}62\,m,\ r_2 = \sqrt{25^2 + 15^2} = 29{,}15\,m,$$

$$m_1 = 10 \cdot 40 \cdot 0{,}5 = 200\,kg,\ m_2 = 10 \cdot 30 \cdot 0{,}5 = 150\,kg,$$

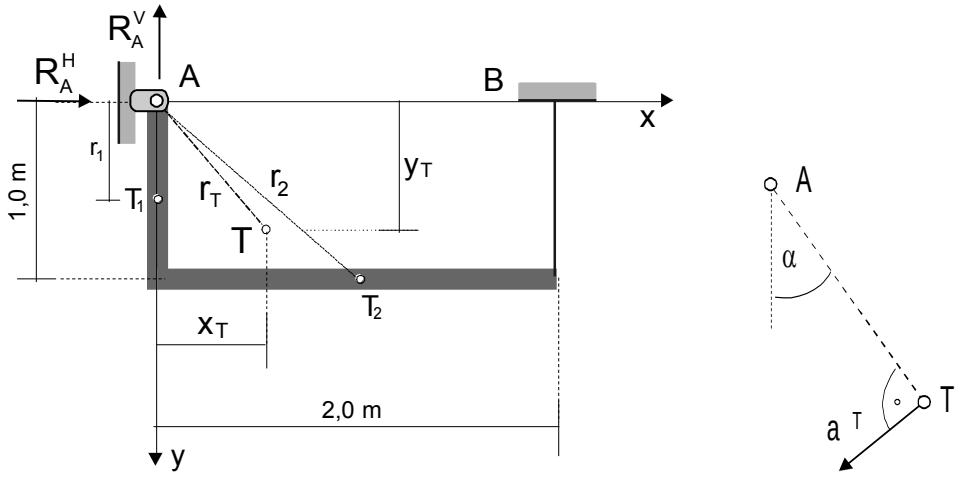
$$I_A = \sum I_{Ci} + m_i \cdot r_i^2,$$

$$I_A = 200 \cdot \frac{\left(10^2 + 40^2\right)}{12} + 200 \cdot 20{,}62^2 + 150 \cdot \frac{\left(10^2 + 30^2\right)}{12} + 150 \cdot 29{,}15^2,$$

$$I_A = 28333{,}33 + 85036{,}88 + 12500 + 127458{,}38 = 253328{,}59\,kgm^2.$$

Jednadžba gibanja i zakon momenta

1. Tijelo miruje oslonjeno u zglobu *A* i užetu *B*, crtež 6.6. Ako uže iznenada pukne, odredite dinamičke reakcije u osloncu *A* u tom trenutku. Homogeno tijelo ima jednoliko raspodijeljenu masu 5,0 *kg/m*.



RJEŠENJE:

$r_1 = 0{,}50\,m, \quad r_2 = \sqrt{1^2 + 1^2} = 1{,}41 m,$

$m_1 = 1{,}0 \cdot 5 = 5\,kg, \quad m_2 = 2{,}0 \cdot 5 = 10\,kg,$

$I_A = \frac{5 \cdot 1^2}{3} + \frac{10 \cdot 2^2}{12} + 10 \cdot 1{,}41^2 = 24{,}88\,kgm^2,$

$\sum M_A = I_A \cdot \varepsilon, \quad \Rightarrow \quad 10 \cdot 9{,}81 = 24{,}88 \cdot \varepsilon, \quad \Rightarrow \quad \varepsilon = 3{,}943\,rad/s^2,$

$x_T \cdot 15 = 10 \cdot 1 \Rightarrow x_T = 0{,}67\,m, \; y_T \cdot 15 = 5 \cdot 0{,}5 + 10 \cdot 1 \Rightarrow y_T = 0{,}83 m,$

$r_T = \sqrt{0{,}67^2 + 0{,}83^2} = 1{,}07 m, \quad atg\,\alpha = \frac{0{,}67}{0{,}83} \Rightarrow \alpha = 38{,}91^o,$

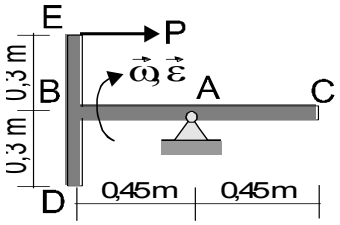
$a_T = r_C \cdot \varepsilon = 1{,}07 \cdot 3{,}943 = 4{,}22\,m/s^2,$

$a_T^x = -4{,}22 \cdot \cos\alpha = -3{,}28\,m/s^2, \; a_T^y = 4{,}22 \cdot \sin\alpha = 2{,}65\,m/s^2,$

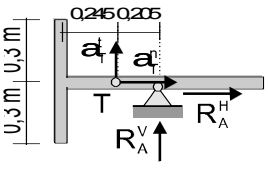
$\sum F_x = m \cdot a_C^x, \quad \Rightarrow \quad R_A^H = -15 \cdot 3{,}28 = -49{,}20\,N \leftarrow,$

$\sum F_y = m \cdot a_C^y, \quad \Rightarrow \quad -R_A^V + 98{,}1 + 49{,}20 = 15 \cdot 2{,}64, \quad \Rightarrow \quad R_A^V = 105{,}55\,N \uparrow.$

2. Dva jednolika štapa *BC* mase 6 *kg* i *DE* mase 5 *kg*, zavarena čine tijelo T-oblika, crtež 6.7. U prikazanom trenutku tijelo rotira u vertikalnoj ravnini kutnom brzinom 10 *rad/s* i kutnim ubrzanjem 22 *rad/s*^{2}, oko zgloba *A*, obje u smjeru kazaljke sata. Odredite silu *P* i ukupnu dinamičku reakciju u osloncu *A*.



RJEŠENJE:



$\sum M_A = I_A \cdot \varepsilon_A, \quad \Rightarrow \quad -P \cdot 0{,}3 + 5 \cdot 9{,}81 \cdot 0{,}45 = -I_A \cdot 22,$

$I_A = \frac{6 \cdot 0{,}9^2}{12} + \frac{5 \cdot 0{,}6^2}{12} + 5 \cdot 0{,}45^2 = 1{,}57\,kgm^2,$

$P = 188{,}53\,N, \qquad 11 \cdot x_T = 6 \cdot 0{,}45, \; \Rightarrow \; x_T = 0{,}245 m,$

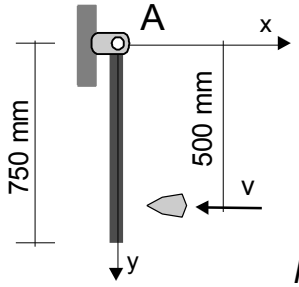
$a_T^n = 0{,}205 \cdot 10^2 = 20{,}5\,m/s^2, \quad a_T^t = 0{,}205 \cdot 22 = 4{,}51 m/s^2,$

$\sum F_x = m \cdot a_x, \quad \Rightarrow \quad R_A^H + 188{,}53 = 20{,}5 \cdot 11, \quad \Rightarrow \quad R_A^H = 36{,}97\,N,$

$\sum F_y = m \cdot a_y, \quad \Rightarrow \quad R_A^V - 11 \cdot 9{,}81 = 4{,}51 \cdot 11, \quad \Rightarrow \quad R_A^V = 157{,}52\,N.$

Količina gibanja, moment količine gibanja, impuls i moment impulsa

1. Puščano zrno mase 20 *g* zabije se u vertikalno obješen homogeni štap mase 10 *kg* brzinom 500 *m/s*, crtež 6.9. Odredite brzinu centra mase štapa neposredno nakon sudara.



RJEŠENJE:

$$I_2 \cdot \omega_2 - I_1 \cdot \omega_1 = m \cdot v \cdot h,$$

$$I_1 = \frac{m \cdot l^2}{3} = \frac{10 \cdot 0{,}75^2}{3} = 1{,}875\, kgm^2,$$

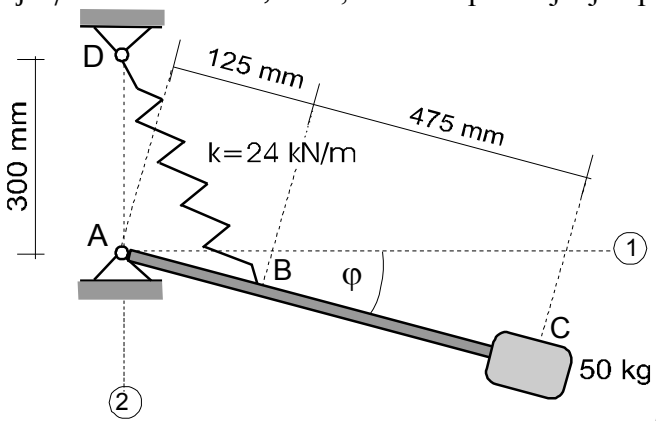
$$I_2 = 1{,}875 + 0{,}02 \cdot 0{,}5^2 = 1{,}880\, kgm^2,$$

$$1{,}880 \cdot \omega_2 - I_1 \cdot 0 = 0{,}02 \cdot 500 \cdot 0{,}5, \quad \Rightarrow \quad \omega_2 = 2{,}660\, rad/s,$$

$$v_C = r_C \cdot \omega_2 = 0{,}375 \cdot 2{,}660 = 1{,}00\, m/s.$$

Zakon održanja mehaničke energije - opće načelo rada i energije

1. Blok mase 50 *kg* otpušten je iz stanja mirovanja kada je kut φ=0^o, crtež 6.11. Ako je brzina u trenutku kada je φ=90^o iznosila 2,5 *m/s*, odredite produljenje opruge u početnom položaju.



RJEŠENJE:

$$E_{K1} + E_{P1} = E_{K2} + E_{P2}, \; E_{K1} = 0, \; E_{P1} = \frac{k \cdot \Delta l_1^2}{2},$$

$$l_1 = \sqrt{0{,}3^2 + 0{,}125^2} = 0{,}325\, m, \qquad l_2 = 0{,}425\, m,$$

$$\Delta l_1 = 0{,}325 - l, \qquad \Delta l_2 = 0{,}425 - l,$$

$$E_{K2} = \frac{I \cdot \omega^2}{2}, \; I = m \cdot 0{,}6^2 = 18\, kgm^2, \; \omega = \frac{v}{r} = \frac{2{,}5}{0{,}6} = 4{,}17\, rad/s,$$

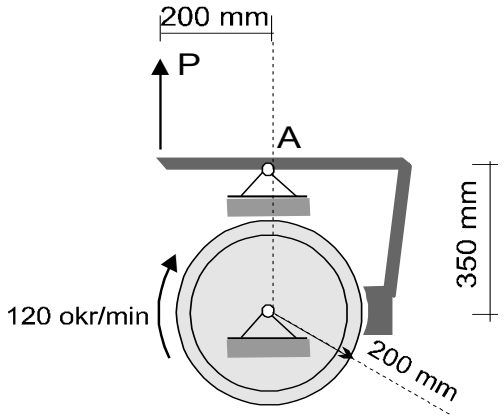
$$E_{K2} = \frac{18 \cdot 4{,}17^2}{2} = 156{,}25\, J, \qquad E_{P2} = \frac{k \cdot \Delta l_2^2}{2} + \left(-50 \cdot 9{,}81 \cdot 0{,}6\right),$$

$$0 + \frac{24000 \cdot \left(0{,}325 - l\right)^2}{2} = 156{,}25 + \frac{24000 \cdot \left(0{,}425 - l\right)^2}{2} - 50 \cdot 9{,}81 \cdot 0{,}6,$$

$$l = 0{,}317\, m, \quad \Rightarrow \quad \Delta l_1 = 0{,}325 - l = 0{,}325 - 0{,}317 = 0{,}008\, m,$$

$$\Delta l_2 = 0{,}425 - l = 0{,}425 - 0{,}317 = 0{,}108\, m.$$

2. Poluga kočnice tlači valjak čiji je moment tromosti 8 kgm^2, crtež 6.12. Ako je u trenutku početka kočenja brzina valjka bila 120 okr/min, odredite silu potrebnu da se valjak zaustavi nakon 8 okretaja. Koeficijenti statičkog i dinamičkog trenja su: μ_{st}=0,50 i μ_{din}=0,40.



Crtež 6.12.

RJEŠENJE:

$$\omega_1 = \frac{120 \cdot 2\pi}{60} = 12{,}56\, rad/s,$$

$$E_{K1} + E_{P1} + U_{1-2} = E_{K2} + E_{P2},$$

$$E_{K1} = \frac{8 \cdot 12{,}56^2}{2} = 631{,}01\, J, \qquad E_{P1} = 0,$$

$$E_{K2} = 0, \qquad E_{P2} = 0, \qquad U_{1-2} = T \cdot 8 \cdot 2 \cdot 0{,}2 \cdot \pi,$$

$$631{,}01 - T \cdot 8 \cdot 2 \cdot 0{,}2 \cdot \pi = 0, \;\Rightarrow\; T = 62{,}8\, N,$$

$$\sum M_A = 0, \;\Rightarrow\; P \cdot 0{,}2 = N \cdot 0{,}35, \;\Rightarrow\; N = \frac{0{,}2P}{0{,}35},$$

$$T = 0{,}4 \cdot N, \quad 62{,}8 = 0{,}4 \cdot \frac{0{,}2P}{0{,}35}, \;\Rightarrow\; P = 274{,}75\, N.$$