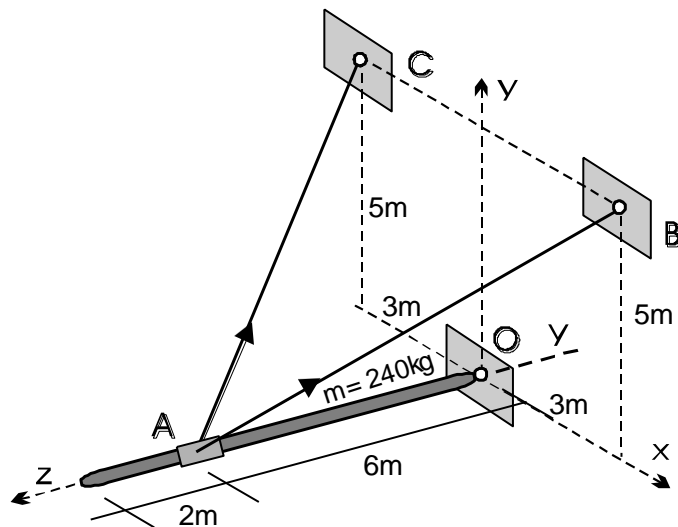


## RAVNOTEŽA

### PRIMJERI:

I). Jednoliki kruti štap duljine 8 m i mase 240 kg pridržan je s dva užeta AB i AC, kako pokazuje crtež 2.26. Odredite silu u užetu AC.



Crtež 2.26.

RJEŠENJE:

$$\vec{S}_C = S_C \cdot \vec{e}_C = S_C \cdot \frac{-3\vec{i} + 5\vec{j} - 6\vec{k}}{\sqrt{9 + 25 + 36}} = -0,359S_C\vec{i} + 0,598S_C\vec{j} - 0,717S_C\vec{k},$$

$$\vec{S}_B = S_B \cdot \vec{e}_B = \frac{3\vec{i} + 5\vec{j} - 6\vec{k}}{\sqrt{9 + 25 + 36}} = 0,359S_B\vec{i} + 0,598S_B\vec{j} - 0,717S_B\vec{k},$$

$$\vec{G} = G \cdot \vec{e}_G = 240 \cdot 9,81 \cdot (-\vec{j}) = -2354,4\vec{j} [N],$$

$$\sum M_O = 0, \Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 6 \\ -0,359S_C & 0,598S_C & -0,717S_C \end{vmatrix} +$$
$$+ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 6 \\ 0,359S_B & 0,598S_B & -0,717S_B \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 4 \\ 0 & -2354,4 & 0 \end{vmatrix} = 0$$

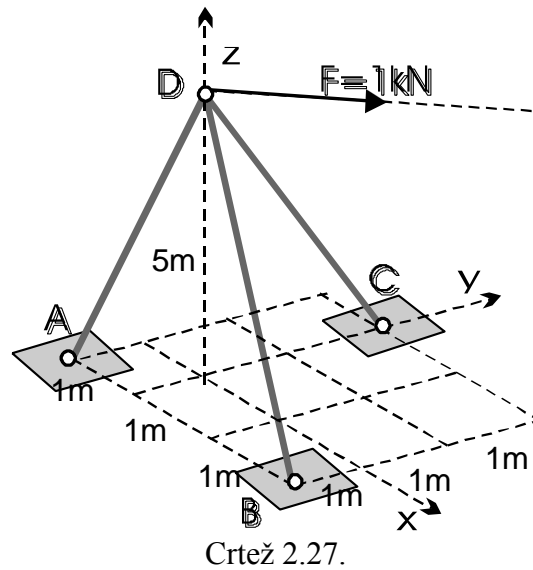
$$(-3,588S_C)\vec{i} - (2,154S_C)\vec{j} + (-3,588S_B)\vec{i} - (-2,154S_B)\vec{j} + 9417,6\vec{i} = 0,$$

$$\sum M_x = 0 \Rightarrow -3,588S_C - 3,588S_B + 9417,6 = 0,$$

$$\sum M_y = 0 \Rightarrow -2,154S_C + 2,154S_B = 0, \Rightarrow S_B = S_C,$$

$$-3,588S_C - 3,588S_C + 9417,6 = 0, \Rightarrow \underline{S_C = 1312N}.$$

- 2). Sila  $F=1\text{kN}$  djeluje na sastavu tri kruta štapa zanemarive mase, obostrano zglobno vezana, crtež 2.27. Odredite sile u štapovima  $S_A$ ,  $S_B$  i  $S_C$ .



RJEŠENJE:

*Pretpostavka – sve vlačne sile*

$$\vec{S}_A = S_A \cdot \vec{e}_A = S_A \cdot \frac{-1\vec{i} - 1\vec{j} - 5\vec{k}}{\sqrt{1+1+25}} = -0,192S_A\vec{i} - 0,192S_A\vec{j} - 0,962S_A\vec{k},$$

$$\vec{S}_B = S_B \cdot \vec{e}_B = \frac{2\vec{i} - 1\vec{j} - 5\vec{k}}{\sqrt{4+1+25}} = 0,365S_B\vec{i} - 0,182S_B\vec{j} - 0,912S_B\vec{k},$$

$$\vec{S}_C = S_C \cdot \vec{e}_C = S_C \cdot \frac{0\vec{i} + 2\vec{j} - 5\vec{k}}{\sqrt{4+25}} = 0,371S_C\vec{j} - 0,928S_C\vec{k},$$

$$\vec{F} = F \cdot \vec{e}_F = 1 \cdot \frac{2\vec{i} + 2\vec{j}}{\sqrt{4+4}} = 0,707\vec{i} - 0,707\vec{j},$$

$$\sum F_x = 0 \Rightarrow -0,192S_A + 0,365S_B + 0,707 = 0, \quad (1)$$

$$\sum F_y = 0 \Rightarrow -0,192S_A - 0,182S_B + 0,371S_C - 0,707 = 0, \quad (2)$$

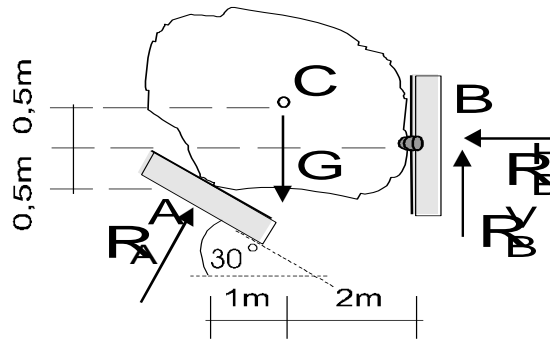
$$\sum F_z = 0 \Rightarrow -0,962S_A - 0,912S_B - 0,928S_C = 0, \quad (3)$$

$$S_A = 2,04\text{kN} \Rightarrow (\text{pretpostavkavlak}) \Rightarrow \text{VLAK},$$

$$S_B = -0,86\text{kN} \Rightarrow (\text{pretpostavkavlak}) \Rightarrow \text{TLAK},$$

$$S_C = -1,27\text{kN} \Rightarrow (\text{pretpostavkavlak}) \Rightarrow \text{TLAK}.$$

3). Kamen težine  $G=28\text{kN}$  oslonjen je u točki A u glatkoj plohi, a u točki B je zglobno povezan žicom, crtež 2.28. Odredite sile u osloncima A i B, ako težina G djeluje u težištu kamena C.



Crtež 2.28.

RJEŠENJE:

$$\sum F_x = 0, \Rightarrow R_A \cdot \sin 30^\circ - R_B^H = 0, \quad (1)$$

$$\sum F_y = 0, \Rightarrow -G + R_A \cdot \cos 30^\circ + R_B^V = 0, \quad (2)$$

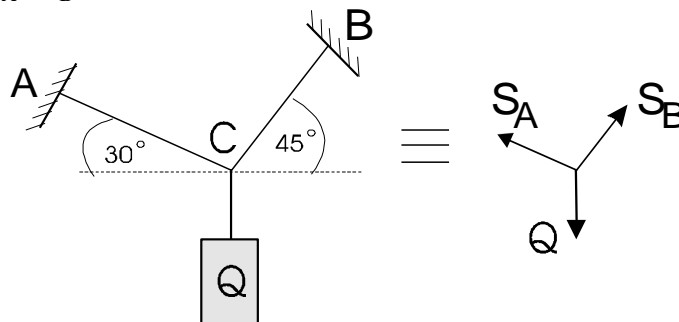
$$\sum M_B = 0. \Rightarrow R_A \cdot \sin 30^\circ \cdot 0,5 - R_A \cdot \cos 30^\circ \cdot 3 + G \cdot 2 = 0, \quad (3)$$

$$\text{iz (3)} \Rightarrow R_A = \frac{2 \cdot 28}{(-\sin 30^\circ \cdot 0,5 + \cos 30^\circ \cdot 3)} = \underline{\underline{23,85\text{kN}}},$$

$$\text{iz (2)} \Rightarrow R_B^V = G - R_A \cdot \cos 30^\circ = \underline{\underline{7,35\text{kN}}},$$

$$\text{iz (1)} \Rightarrow R_B^H = R_A \cdot \sin 30^\circ = \underline{\underline{11,93\text{kN}}}.$$

4). Teret težine  $Q=200\text{N}$  obješen je u točki C užetom za točke B i A, crtež 2.29. Odredite sile u dijelovima užeta  $S_A$  i  $S_B$ .



Crtež 2.29.

RJEŠENJE:

$$\sum F_x = 0, \Rightarrow -S_A \cdot \cos 30^\circ + S_B \cdot \cos 45^\circ = 0, \quad (1)$$

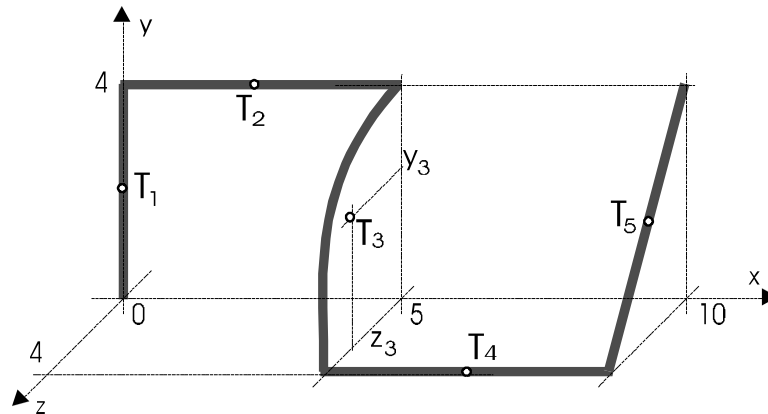
$$\sum F_y = 0, \Rightarrow S_A \cdot \sin 30^\circ + S_B \cdot \sin 45^\circ - Q = 0, \quad (2)$$

$$\text{iz (1)} \Rightarrow S_A = \frac{S_B \cdot \cos 45^\circ}{\cos 30^\circ}, \quad \text{u (2)} \Rightarrow \frac{S_B \cdot \cos 45^\circ}{\cos 30^\circ} \cdot \sin 30^\circ + S_B \cdot \sin 45^\circ - Q = 0$$

$$S_B = 179,32\text{N} \quad S_A = 146,41\text{N}.$$

## TEŽIŠTE

1). Odredite položaj težišta složene crte, odnosno isavijane homogene žice prikazane na crtežu 4.11.



Crtež 4.11.

RJEŠENJE:

$$L_1 = 4m, L_2 = 5m, L_3 = \frac{2 \cdot 4\pi}{4} = 6,28m, L_4 = 5m, L_5 = \sqrt{4^2 + 4^2} = 5,66m,$$

$$x_1 = 0m, x_2 = 2,5m, x_3 = 5m, x_4 = 7,5m, x_5 = 10m,$$

$$y_1 = 2m, y_2 = 4m, y_3 = \frac{2 \cdot 4}{\pi} = 2,55m, y_4 = 0m, y_5 = 2m,$$

$$z_1 = 0m, z_2 = 0m, z_3 = \frac{2 \cdot 4}{\pi} = 2,55m, z_4 = 4m, z_5 = 2m,$$

$$x_T \cdot \mathbf{L} = \sum_{i=1}^n x_i \cdot L_i \quad \text{gdje je} \quad \mathbf{L} = \sum_{i=1}^n L_i,$$

$$x_T \cdot (4 + 5 + 6,28 + 5 + 5,66) = 4 \cdot 0 + 5 \cdot 2,5 + 6,28 \cdot 5 + 5 \cdot 7,5 + 5,66 \cdot 10 \Rightarrow \underline{x_T = 5,32m},$$

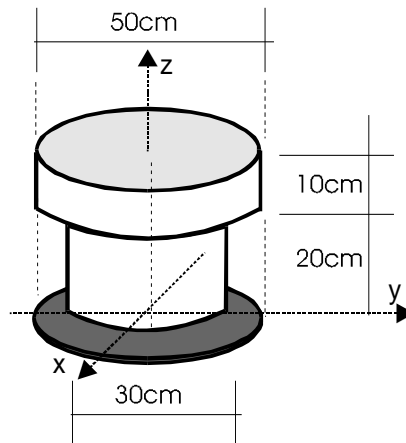
$$y_T \cdot \mathbf{L} = \sum_{i=1}^n y_i \cdot L_i \quad \text{gdje je} \quad \mathbf{L} = \sum_{i=1}^n L_i,$$

$$y_T \cdot (4 + 5 + 6,28 + 5 + 5,66) = 4 \cdot 2 + 5 \cdot 4 + 6,28 \cdot 2,55 + 5 \cdot 0 + 5,66 \cdot 2 \Rightarrow \underline{y_T = 2,13m},$$

$$z_T \cdot \mathbf{L} = \sum_{i=1}^n z_i \cdot L_i \quad \text{gdje je} \quad \mathbf{L} = \sum_{i=1}^n L_i,$$

$$z_T \cdot (4 + 5 + 6,28 + 5 + 5,66) = 4 \cdot 0 + 5 \cdot 0 + 6,28 \cdot 2,55 + 5 \cdot 4 + 5,66 \cdot 2 \Rightarrow \underline{z_T = 1,82m}.$$

2). Odredite z položaj težišta složenog tijela, sastavljenog iz dva puna valjka specifične težine  $\gamma=4\text{N/dm}^3$  i ploče specifične težine  $\gamma=8\text{N/dm}^2$ , prikazanog na crtežu 4.12.



Crtež 4.12.

RJEŠENJE:

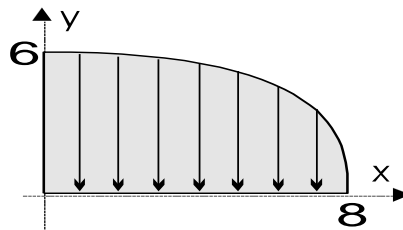
$$G_1 = 2,5^2 \cdot \pi \cdot 1 \cdot 4 = 78,5\text{N}, \quad G_2 = 1,5^2 \cdot \pi \cdot 2 \cdot 4 = 56,52\text{N}, \quad G_3 = 2,5^2 \cdot \pi \cdot 8 = 157\text{N},$$

$$z_1 = 2,5\text{dm}, \quad z_2 = 1\text{dm}, \quad z_3 = 0\text{dm},$$

$$z_T \cdot L = \sum_{i=1}^n z_i \cdot L_i, \quad \text{gdje je} \quad L = \sum_{i=1}^n L_i,$$

$$z_T \cdot (78,5 + 56,52 + 157) = 2,5 \cdot 78,5 + 1 \cdot 56,52 + 0 \cdot 157 \Rightarrow \underline{z_T = 0,866\text{dm} = 8,66\text{cm}}.$$

3). Odredite položaj težišta raspodijeljenog opterećenja prikazanog na crtežu 4.13., čija je gornja stranica parabola<sup>2</sup>.



Crtež 4.13.

RJEŠENJE:

$$y = ax^2 + bx + c, \quad y = ax^2 + c,$$

$$\text{za } x = 0 \Rightarrow y = 6 \Rightarrow c = 6,$$

$$\text{za } x = 8 \Rightarrow y = 0 \Rightarrow 0 = a \cdot 8^2 + 6, \Rightarrow a = -\frac{3}{32}, \Rightarrow \underline{y = -\frac{3}{32}x^2 + 6},$$

$$x_T \cdot \int_0^8 \left(-\frac{3}{32}x^2 + 6\right) dx = \int_0^8 \left(-\frac{3}{32}x^2 + 6\right) \cdot x dx,$$

$$x_T \cdot \left(-\frac{3}{32} \cdot \frac{x^3}{3} + 6x\right)_0^8 = \left(-\frac{3}{32} \cdot \frac{x^4}{4} + 6 \cdot \frac{x^2}{2}\right)_0^8, \Rightarrow x_T = \frac{96}{32} = 3\text{cm}.$$