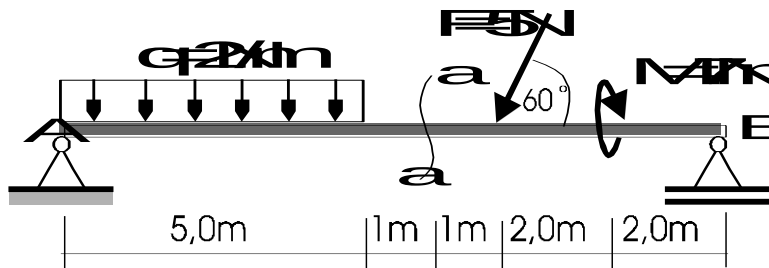


UNUTRAŠNJE SILE

PRIMJERI:

1. Odredite unutrašnje sile u presjeku a-a za slučaj opterećene jednostavne grede kao na crtežu 7.4.



Crtež 7.4.

RJEŠENJE:

Reakcije :

$$\sum M_A = 0 \Rightarrow -2 \cdot 5 \cdot 2,5 - 5 \cdot \sin 60^\circ \cdot 7 - 7 + R_B \cdot 11 = 0, \Rightarrow \underline{R_B = 5,66 N},$$

$$\sum M_B = 0 \Rightarrow 2 \cdot 5 \cdot 8,5 + 5 \cdot \sin 60^\circ \cdot 4 - 7 - R_A^V \cdot 11 = 0, \Rightarrow \underline{R_A^V = 8,67 N},$$

$$\sum F_x = 0 \Rightarrow -5 \cdot \cos 60^\circ + R_A^H = 0, \Rightarrow \underline{R_A^H = 2,5 N},$$

$$\text{Kontrola reakcija} \Rightarrow \sum F_y = 0 \Rightarrow -2 \cdot 5 - 5 \cdot \sin 60^\circ + 5,66 + 8,67 = 0.$$

Unutrašnje sile u presjeku:

Slijeva:

$$\sum T_{a-a} = 0 = 8,67 - 2 \cdot 5 - T_{a-a},$$

$$T_{a-a} = -1,33 N,$$

$$\sum M_{a-a} = 0 = -8,67 \cdot 6 + 2 \cdot 5 \cdot 3,5 + M_{a-a},$$

$$M_{a-a} = 17,02 Nm.$$

$$\sum N_{a-a} = 0 = 2,5 + N_{a-a},$$

$$N_{a-a} = -2,5 N \Rightarrow \text{Tlak}.$$

Kontrola zdesna:

$$\sum T_{a-a} = 0 = 5,66 - 5 \cdot \sin 60^\circ + T_{a-a},$$

$$T_{a-a} = -1,33 N,$$

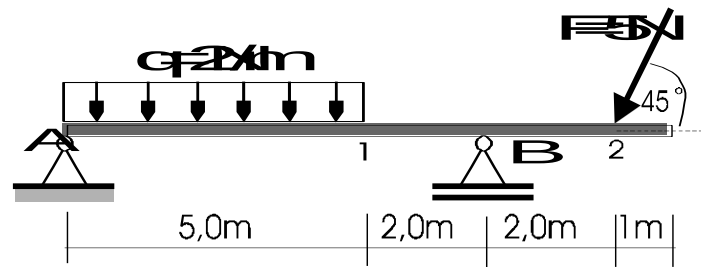
$$\sum M_{a-a} = 0 = 5,66 \cdot 5 - 7 - 5 \cdot \sin 60^\circ \cdot 1 - M_{a-a}, \Rightarrow M_{a-a} = 17,02 Nm.$$

$$\sum N_{a-a} = 0 = -5 \cdot \cos 60^\circ - N_{a-a},$$

$$N_{a-a} = -2,5 N \Rightarrow \text{Tlak}.$$

Izračun unutrašnjih sila u nekom presjeku postupcima slijeva ili zdesna ostaje po slobodnom izboru. Uobičajeno je izabrati postupak koji je kraći, odnosno, postupak koji u izračunima ima manji broj vanjskih sila, dok preostali postupak može služiti kao kontrola.

2. Nacrtajte dijagrame unutrašnjih sila za slučaj opterećene grede s prijepustom kao na crtežu 7.8.



Crtež 7.8.

RJEŠENJE:

Reakcije :

$$\sum M_A = 0 \Rightarrow -2 \cdot 5 \cdot 2,5 - 5 \cdot \sin 45^\circ \cdot 9 + R_B \cdot 7 = 0, \Rightarrow \underline{R_B = 8,12 N},$$

$$\sum M_B = 0 \Rightarrow 2 \cdot 5 \cdot 4,5 - 5 \cdot \sin 45^\circ \cdot 2 - R_A^V \cdot 7 = 0, \Rightarrow \underline{R_A^V = 5,42 N},$$

$$\sum F_x = 0 \Rightarrow -5 \cdot \cos 45^\circ + R_A^H = 0, \Rightarrow \underline{R_A^H = 3,54 N},$$

$$\text{Kontrola reakcija} \Rightarrow \sum F_y = 0 \Rightarrow -2 \cdot 5 - 5 \cdot \sin 45^\circ + 8,12 + 5,42 = 0.$$

Unutarnje sile slijeva :

$$\sum T_A = 0 \Rightarrow 5,42 - T_A = 0, \Rightarrow T_A = 5,42 N,$$

$$\sum N_A = 0 \Rightarrow 3,54 + N_A = 0, \Rightarrow N_A = -3,54 N,$$

$$\sum T_1 = 0 \Rightarrow 5,42 - 2 \cdot 5 - T_1 = 0, \Rightarrow T_1 = -4,58 N,$$

$$\sum M_1 = 0 \Rightarrow -5,42 \cdot 5 + 2 \cdot 5 \cdot 2,5 + M_1 = 0, \Rightarrow M_1 = 2,10 Nm,$$

$$\sum T_B^L = 0 \Rightarrow 5,42 - 2 \cdot 5 - T_B^L = 0, \Rightarrow T_B^L = T_1 = -4,58 N,$$

$$\sum T_B^D = 0 \Rightarrow 5,42 - 2 \cdot 5 + 8,12 - T_B^D = 0, \Rightarrow T_B^D = 3,54 N,$$

Zdesna :

$$\sum M_B = 0 \Rightarrow -5 \cdot \sin 45^\circ \cdot 2 - M_B = 0, \Rightarrow M_B = -7,07 Nm,$$

$$\sum T_2^L = 0 \Rightarrow -5 \cdot \sin 45^\circ + T_2^L = 0, \Rightarrow T_2^L = 3,54 N,$$

$$\sum N_2^L = 0 \Rightarrow -5 \cdot \cos 45^\circ - N_2^L = 0, \Rightarrow N_2^L = -3,54 N,$$

$$\sum T_2^D = 0 \Rightarrow T_2^D = 0,$$

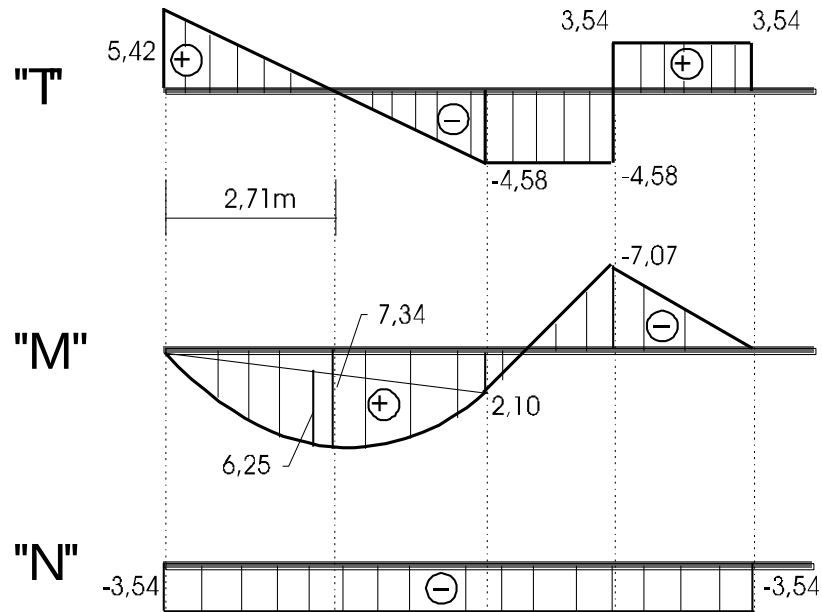
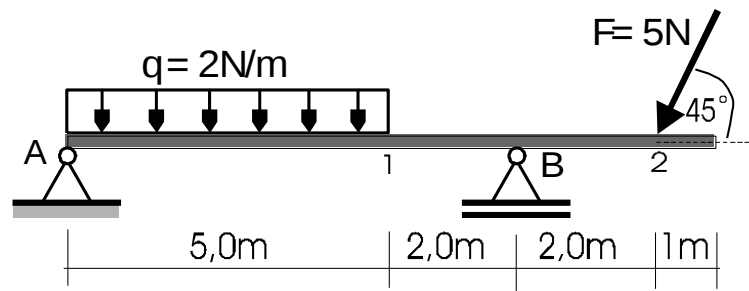
$$\sum N_2^D = 0 \Rightarrow N_2^D = 0,$$

$$\text{Ekstremnimoment} : \frac{10}{5} = \frac{5,42}{x} \Rightarrow x_E = 2,71 m,$$

$$\sum M_E = 0 \Rightarrow -5,42 \cdot 2,71 + 2 \cdot 2,71 \cdot 1,355 + M_E = 0, \Rightarrow M_E = 7,34 Nm,$$

$$\text{Konstrukcija momentnog dijagrama} : M_{\max} = \frac{q \cdot l^2}{8} = 6,25 Nm.$$

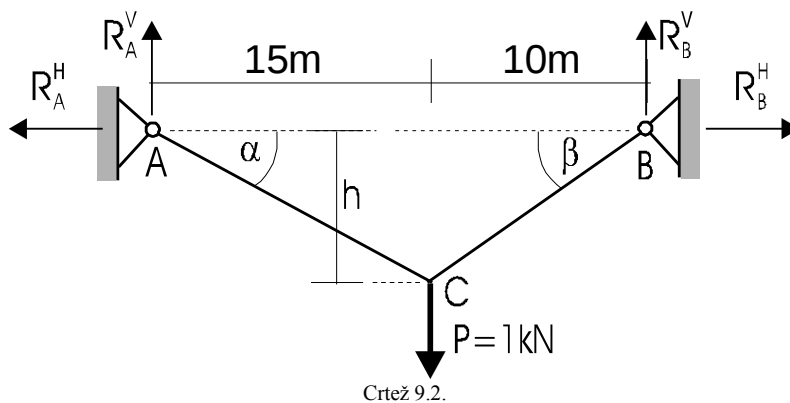
PRIKAZ DIJAGRAMA UNUTRAŠNJIH SILA



LANČANICE

PRIMJERI:

1). Uže duljine $L=30\text{m}$ opterećeno je koncentriranom silom $P=1\text{kN}$, crtež 9.2. Odredite sile u osloncima (reakcije) A i B.



RJEŠENJE:

$$\sum M_A = 0, \Rightarrow R_B^V \cdot 25 - 1 \cdot 15 = 0, \Rightarrow \underline{R_B^V = 0,6\text{kN}},$$

$$\sum F_Y = 0, \Rightarrow \underline{R_A^V = 0,4\text{kN}},$$

$$\text{s lijeva: } \sum M_C = 0, \Rightarrow -0,4 \cdot 15 + R_A^H \cdot h = 0, \quad (1)$$

$$\text{s desna: } \sum M_C = 0, \Rightarrow -0,6 \cdot 10 - R_B^H \cdot h = 0, \quad (2)$$

$$15^2 + h^2 = l_1^2 \Rightarrow h^2 = l_1^2 - 225, \quad (3)$$

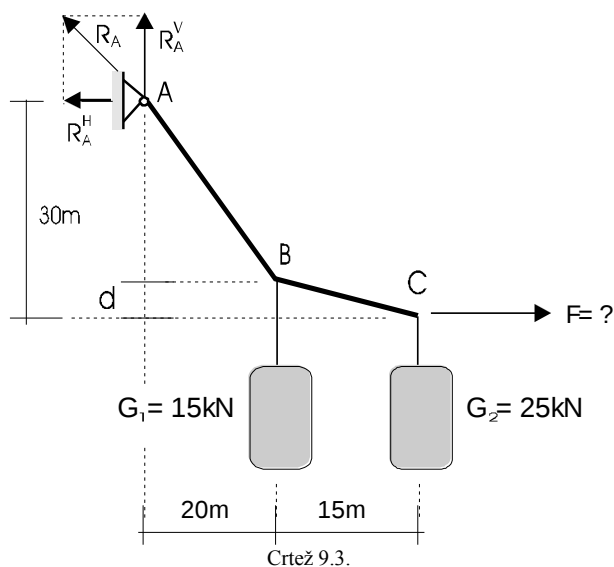
$$10^2 + h^2 = l_2^2 \Rightarrow h^2 = l_2^2 - 100, \quad (4)$$

$$l_1 + l_2 = 30, \Rightarrow l_2 = 30 - l_1, \quad (5)$$

$$\underline{l_1 = 17,08\text{m}}, \underline{l_2 = 12,92\text{m}}, \underline{h = 8,17\text{m}},$$

$$\underline{R_A^H = 0,735\text{kN}}, \underline{R_B^H = 0,735\text{kN}}.$$

2). Uže je s jedne strane vezano u točki A, a s druge se vuče horizontalnom silom F , crtež 9.3. Odredite veličinu horizontalne sile F , visinu d , sile u pojedinim dijelovima užeta te ukupnu duljinu užeta, ako je uže u stanju ravnoteže.



$$\sum M_A = 0, \Rightarrow -G_1 \cdot 20 - G_2 \cdot 35 + F \cdot 30 = 0, \Rightarrow \underline{\underline{F = 39,17kN}},$$

$$\sum M_B^D = 0, \Rightarrow F \cdot d - G_2 \cdot 15 = 0, \Rightarrow \underline{\underline{d = 9,57m}},$$

$$\sum F_Y = 0, \Rightarrow -G_1 - G_2 + R_A^V = 0, \Rightarrow \underline{\underline{R_A^V = 35,00kN}},$$

$$\sum F_X = 0, \Rightarrow F - R_A^H = 0, \Rightarrow \underline{\underline{R_A^H = 39,17kN}},$$

$$S_{AB} = \sqrt{(R_A^H)^2 + (R_A^V)^2} = 52,53kN,$$

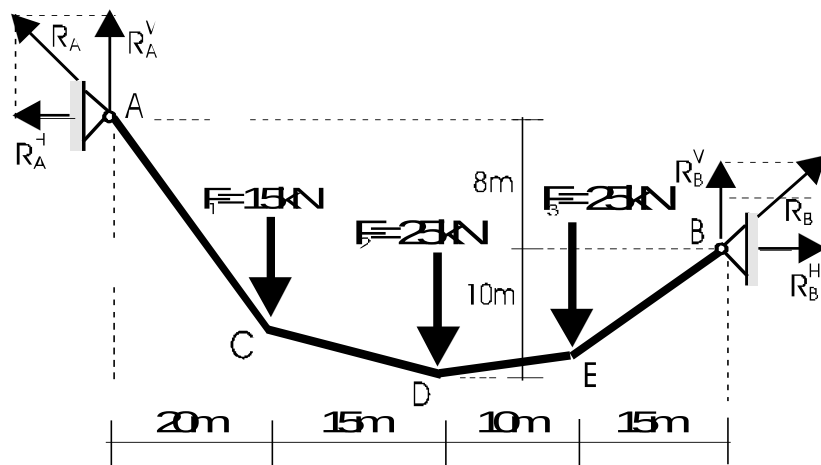
$$S_{BC} = \sqrt{(F)^2 + (G_2)^2} = 46,47kN,$$

$$l_{AB} = \sqrt{(30 - 9,57)^2 + 20^2} = 28,59m,$$

$$l_{BC} = \sqrt{9,57^2 + 15^2} = 17,79m,$$

$$L = 28,59 + 17,79 = 46,38m.$$

3). Uže vezano u točkama A i B opterećeno je koncentriranim silama u točkama C, D i E, crtež 9.4. Odredite sile u pojedinim dijelovima užeta i duljinu užeta.



Crtež 9.4.

RJEŠENJE:

$$\sum M_A = 0, \Rightarrow R_B^H \cdot 8 + R_B^V \cdot 60 - F_3 \cdot 45 - F_2 \cdot 35 - F_1 \cdot 20 = 0,$$

$$\sum M_D^D = 0, \Rightarrow -R_B^H \cdot 10 + R_B^V \cdot 25 - F_3 \cdot 10 = 0, \Rightarrow \underline{\underline{R_B^H = 43,28kN}}, \underline{\underline{R_B^V = 21,31kN}},$$

$$\sum F_X = 0, \Rightarrow \underline{\underline{R_A^H = 43,28kN}},$$

$$\sum F_Y = 0, \Rightarrow \underline{\underline{R_A^V = 28,69kN}},$$

$$S_{AC} = \sqrt{(R_A^H)^2 + (R_A^V)^2} = 51,93kN,$$

$$S_{EB} = \sqrt{(R_B^H)^2 + (R_B^V)^2} = 48,24kN,$$

$$S_{CD} = \sqrt{(R_A^H)^2 + (R_A^V - F_1)^2} = 45,39kN,$$

$$S_{DE} = \sqrt{(R_B^H)^2 + (R_B^V - F_3)^2} = 44,73kN,$$

$$\text{s lijeva: } \sum M_C^L = 0, \Rightarrow -R_A^V \cdot 20 + R_A^H \cdot y_C = 0, \Rightarrow y_C = 13,26m,$$

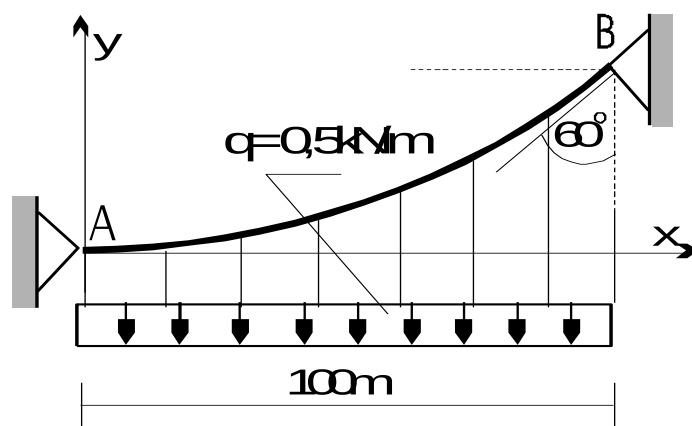
$$\text{s desna: } \sum M_E^D = 0, \Rightarrow R_B^V \cdot 15 - R_B^H \cdot y_E = 0, \Rightarrow y_E = 7,39m,$$

$$l_{AC} = \sqrt{20^2 + 13,26^2} = 24m, l_{CD} = \sqrt{15^2 + (18 - 13,26)^2} = 15,73m,$$

$$l_{DE} = \sqrt{10^2 + (10 - 7,39)^2} = 10,33m, l_{EB} = \sqrt{15^2 + 7,39^2} = 16,72m,$$

$$L = 24 + 15,73 + 10,33 + 16,72 = 66,78m.$$

1). Uže vezano u točkama A i B opterećeno je kontinuiranim opterećenjem po horizontali, crtež 9.12. Odredite najveću silu u užetu.



Crtež 9.12.

RJEŠENJE:

$$y = ax^2 + bx + c, \text{ ishodište u tjemenu!} \Rightarrow y = ax^2,$$

$$\text{za } x = 100 \text{ m} \Rightarrow y' = 2ax = \tan 30^\circ, \quad 2a \cdot 100 = 0.577, \Rightarrow a = 0.0029,$$

$$y = 0.0029x^2, \Rightarrow \text{za } x = 100 \text{ m} \Rightarrow y = 29 \text{ m},$$

$$\dot{y} = 2 \cdot 0.0029x,$$

$$\dot{y} = 0.0058x,$$

$$\ddot{y} = 0.0058,$$

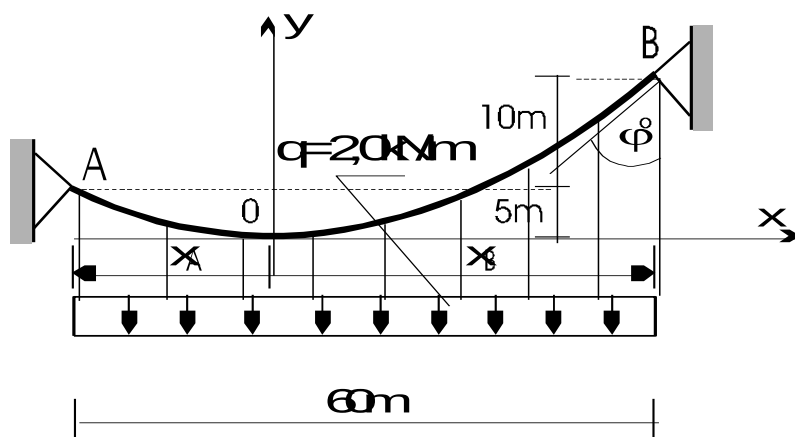
$$\ddot{y} = \frac{q}{H} \Rightarrow 0.0058 = \frac{q}{H}, \Rightarrow H = \frac{0.5}{0.00058} = 86.21 \text{ kN},$$

$$S = \frac{H}{\cos \varphi},$$

$$\varphi_{\text{MAX}} \Rightarrow S_{\text{MAX}}, \quad \varphi_{\text{MAX}} = 30^\circ,$$

$$S_{\text{MAX}} = \frac{86.21}{\cos 30^\circ} = 99.55 \text{ kN}.$$

2). Uže opterećeno kontinuiranim opterećenjem po horizontali vezano je tako da su oslonci u točkama A i B na različitim visinama, crtež 9.13. Ako je ishodište koordinatnog sustava postavljeno u tjemenu, odredite najveću silu u užetu.



Crtež 9.13.

RJEŠENJE:

$$y = ax^2 + bx + c, \text{ ishodište u tjemenu!} \Rightarrow y = ax^2,$$

$$-x_A + x_B = 60, \Rightarrow x_B = 60 + x_A,$$

$$\text{za } x = x_A \Rightarrow y = 5m \Rightarrow 5 = a \cdot x_A^2, \Rightarrow a = \frac{5}{x_A^2},$$

$$\text{za } x = x_B \Rightarrow y = 15m \Rightarrow 15 = a \cdot x_B^2, \Rightarrow 15 = a \cdot (60 + x_A)^2,$$

$$15 = 3600 \cdot \frac{5}{x_A^2} + 120 \cdot \frac{5}{x_A^2} \cdot x_A + \frac{5}{x_A^2} \cdot x_A^2, \Rightarrow -10x_A^2 + 600x_A + 18000 = 0,$$

$$Rj.^2 \quad x_A = -21,96m, \Rightarrow a = \frac{5}{(-21,96)^2} = 0,0104, \Rightarrow \underline{\underline{y = 0,0104x^2}},$$

$$y = 0,0104x^2,$$

$$\dot{y} = 2 \cdot 0,0104x,$$

$$\dot{y} = 0,0208x, \quad \dot{y} = \tan \varphi, \text{ za } x = x_B = 38,04m, \Rightarrow \varphi_{MAX} = 38,35^\circ,$$

$$\ddot{y} = 0,0208, \quad \ddot{y} = \frac{q}{H}, \Rightarrow H = \frac{2}{0,0208} = 96,15kN,$$

$$S_{MAX} = \frac{H}{\cos \varphi_{MAX}}, \Rightarrow S_{MAX} = \frac{96,15}{\cos 38,35^\circ} = 123,46kN.$$