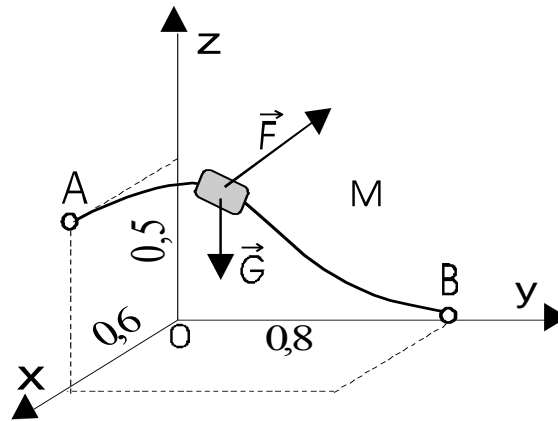


## RAD I VIRTUALNI RAD

### PRIMJERI:

1). Blok mase 2kg kreće se po glatkoj nepomičnoj vodilici pod djelovanjem sile teže  $G$  i konstantne sile  $\vec{F} = -15\vec{i} + 10\vec{j} + 15\vec{k}$  [kN], crtež 10.6. Ako klizač krene iz točke  $A(0,6; 0; 0,5)$  [m], odredite rad koji će izvršiti do točke  $B(0; 0,8; 0)$  [m].



Crtež 10.6.

RJEŠENJE:

$$\text{RAD SILE } \vec{F}: U_F = \int_A^B (-15\vec{i} + 10\vec{j} + 15\vec{k})(dx\vec{i} + dy\vec{j} + dz\vec{k}),$$

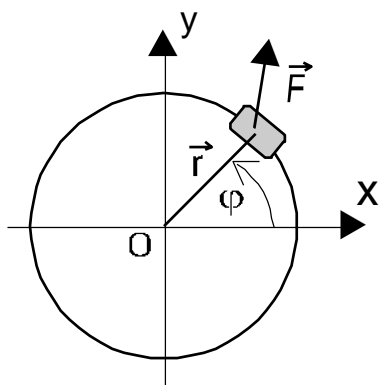
$$U_F = \int_A^B -15dx + \int_A^B 10dy + \int_A^B 15dz = -15x \Big|_{0,6}^0 + 10y \Big|_0^{0,8} + 15z \Big|_{0,5}^0 = 9,5J,$$

$$\text{RAD SILE } \vec{G}: U_G = \int_A^B (-2 \cdot 9,81\vec{k})(dx\vec{i} + dy\vec{j} + dz\vec{k}),$$

$$U_G = \int_A^B -2 \cdot 9,81dz = -19,62z \Big|_{0,5}^0 = 9,81J,$$

$$\text{UKUPNI RAD: } \underline{\underline{U_{uk} = 9,5 + 9,81 = 19,31J.}}$$

2). Blok zanemarive mase giba se u ravnini  $x$ - $y$ , po kružnici koja ima centar u ishodištu i polumjer 3cm, crtež 10.7. Odredite rad sile koja djeluje na blok, ako je ona zadana kao funkcija položaja,  $\vec{F} = (2x - y + z)\vec{i} + (x + y - z^2)\vec{j} + (3x - 2y + 4z)\vec{k}$ , na putu kada blok prođe jedan puni krug.



Crtež 10.7.

RJEŠENJE:

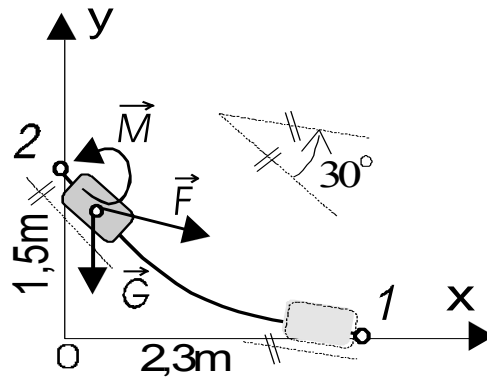
$$x = r \cdot \cos \varphi, \quad y = r \cdot \sin \varphi, \quad dx = -r \cdot \sin \varphi d\varphi, \quad dy = r \cdot \cos \varphi d\varphi,$$

$$A = -\int_0^{2\pi} F_x dx + \int_0^{2\pi} F_y dy = -\int_0^{2\pi} (2x - y + z) \cdot 3 \sin \varphi d\varphi + \int_0^{2\pi} (x + y - z^2) \cdot 3 \cos \varphi d\varphi,$$

$$A = -18 \int_0^{2\pi} \cos \varphi \sin \varphi d\varphi + 9 \int_0^{2\pi} \sin^2 \varphi d\varphi + 9 \int_0^{2\pi} \cos^2 \varphi d\varphi + 9 \int_0^{2\pi} \sin \varphi \cdot \cos \varphi d\varphi,$$

$$A = \frac{9}{4} \cos 2\varphi d\varphi \Big|_0^{2\pi} + 18\pi = \underline{\underline{18\pi}}.$$

3). Blok mase  $m=12\text{kg}$  giba se u ravlini  $x$ - $y$ . Na njegovo težište djeluje konstantna sila  $\vec{F} = 25\vec{i} - 18\vec{j}$  i konstantni moment  $\vec{M} = 78\vec{k}$ , crtež 10.8. Odredite rad svih sila i momenata koji djeluju na blok, na putu od položaja 1 do položaja 2, ako se blok putem zakrenuo za kut  $30^\circ$  u prikazanom smjeru.



Crtež 10.8.

RJEŠENJE:

$$\text{RAD SILE } \vec{F} : U_F = \int_1^2 (25\vec{i} - 18\vec{j})(dx\vec{i} + dy\vec{j}),$$

$$U_F = \int_1^2 25dx - \int_1^2 18dy = 25x \Big|_{2,3}^0 - 18y \Big|_0^{1,5} = -57,5 - 27 = -84,5J,$$

$$\text{RAD SILE } \vec{G} : U_G = \int_1^2 (-12 \cdot 9,81\vec{j})(dx\vec{i} + dy\vec{j}),$$

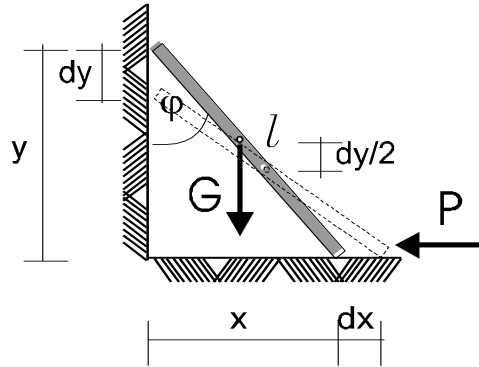
$$U_G = \int_1^2 -12 \cdot 9,81dy = -117,72y \Big|_0^{1,5} = -176,58J,$$

$$\text{RAD MOMENTA } \vec{M} : U_M = \int_0^{\pi/6} (78\vec{k})(d\phi\vec{k}),$$

$$U_M = \int_0^{\pi/6} 78d\phi = 78\phi \Big|_0^{\pi/6} = 40,82J,$$

$$\text{UKUPNI RAD : } \underline{\underline{U_{uk} = -84,5 - 176,58 + 40,82 = -220,26J.}}$$

4). Štap duljine  $l=1m$  i težine  $G=50N$  oslonjen je na glatki zid i glatku podlogu te je pridržan silom  $P=15N$  u ravnoteži, crtež 10.9. Odredite kut  $\varphi$  za slučaj ravnoteže, pomoću načela virtualnog rada.



Crtež 10.9.

RJEŠENJE:

$$x = l \cdot \sin \varphi = l \sin \varphi, \Rightarrow dx = \cos \varphi d\varphi,$$

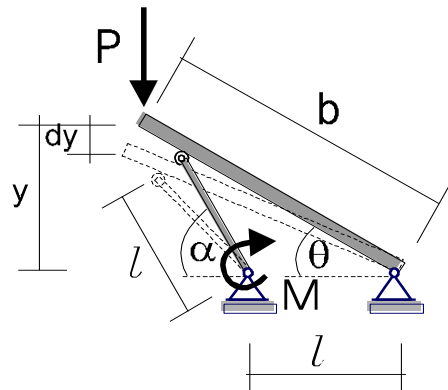
$$y = l \cos \varphi = l \cos \varphi, \Rightarrow dy = -\sin \varphi d\varphi,$$

$dA = 0$ , JEDNADŽBA VIRTUALNOG RADA,

$$-P \cdot dx + G \cdot \frac{dy}{2} = 0, \Rightarrow -15 \cdot \cos \varphi d\varphi + 50 \cdot (-\sin \varphi d\varphi) = 0,$$

$$\frac{\sin \varphi}{\cos \varphi} = -\frac{30}{50} = \operatorname{tg} \varphi, \quad \varphi = -30,96^\circ.$$

5). Štap duljine  $b$  zanemarive težine oslonjen je na manji štap duljine  $l$  i opterećen silom  $P$  kako pokazuje crtež 10.10. Odredite moment  $M$  koji će održavati stanje ravnoteže za ovaj sustav, pomoću načela virtualnog rada.



Crtež 10.10.

RJEŠENJE:

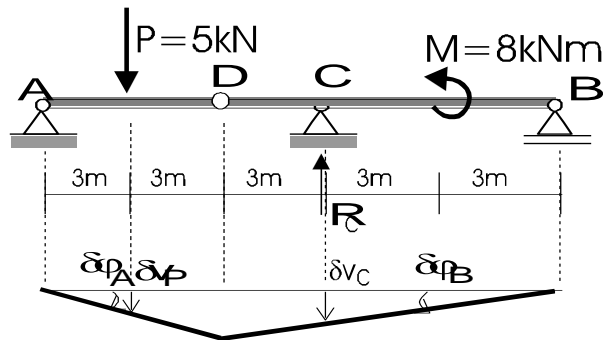
$$y = b \cdot \sin \theta, \Rightarrow dy = b \cos \theta d\theta, \quad \alpha = 2\theta, \quad d\alpha = 2d\theta,$$

$$\underline{dA = 0}, \text{ JEDNADŽBA VIRTUALNOG RADA,}$$

$$P \cdot dy - M \cdot d\alpha = 0, \Rightarrow P \cdot b \cdot \cos \theta \underline{d\theta} - M \cdot \underline{2d\theta} = 0,$$

$$M = \frac{P \cdot b \cdot \cos \theta}{2}.$$

6). Pomoću načela virtualnog rada odredite veličinu reakcije u osloncu C za opterećenje Gerber-ovog nosača prikazanog na crtežu 10.11.



Crtež 10.11.

RJEŠENJE:

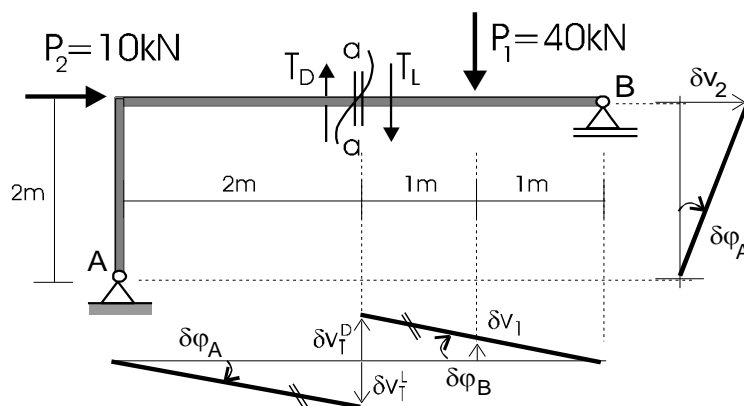
$$\delta v_P = 3 \cdot \delta \varphi_A, \quad \delta v_C = 6 \cdot \delta \varphi_B, \quad 6 \cdot \delta \varphi_A = 9 \cdot \delta \varphi_B, \quad \Rightarrow \quad \delta \varphi_A = \frac{3}{2} \delta \varphi_B,$$

$dA = 0$ , JEDNADŽBA VIRTUALNOG RADA,

$$P \cdot \delta v_P - R_C \cdot \delta v_C + M \cdot \delta \varphi_B = 0, \quad \Rightarrow \quad 5 \cdot \frac{3}{2} \cdot \delta \varphi_B \cdot 3 - R_C \cdot \delta \varphi_B \cdot 6 + 8 \cdot \delta \varphi_B = 0,$$

$$R_C = \frac{30,5}{6} = 5,08 \text{ kN}.$$

7). Pomoću načela virtualnog rada odredite veličinu poprečne sile u presjeku  $a-a$  za opterećenje okvirnog nosača prikazanog na crtežu 10.12.



Crtež 10.12.

RJEŠENJE:

$$\delta v_T^L = 2 \cdot \delta \varphi_A, \quad \delta v_T^D = 2 \cdot \delta \varphi_B, \quad \delta v_1 = 1 \cdot \delta \varphi_B, \quad \delta v_2 = 2 \cdot \delta \varphi_A,$$

$$\delta \varphi_A = \delta \varphi_B, \quad T_L = T_D = T,$$

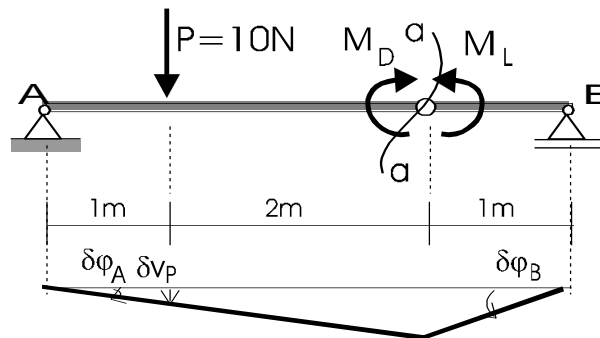
$dA = 0$  JEDNADŽBA VIRTUALNOG RADA,

$$T_L \cdot \delta v_T^L + T_D \cdot \delta v_T^D - F_1 \cdot \delta v_1 + F_2 \cdot \delta v_2 = 0,$$

$$T \cdot 2 \cdot \delta \varphi_A + T \cdot 2 \cdot \delta \varphi_A - 40 \cdot \delta \varphi_A + 20 \cdot \delta \varphi_A = 0,$$

$$4 \cdot T = 20, \quad \Rightarrow \quad \underline{\underline{T = 5 \text{ kN}}}.$$

8). Pomoću načela virtualnog rada odredite veličinu momenta u presjeku  $a-a$  za opterećenje jednostavne grede prikazane na crtežu 10.13.



Crtež 10.13.

RJEŠENJE:

$$\delta v_P = 1 \cdot \delta \varphi_A, \quad 3 \cdot \delta \varphi_A = 1 \cdot \delta \varphi_B, \quad M_L = M_D = M,$$

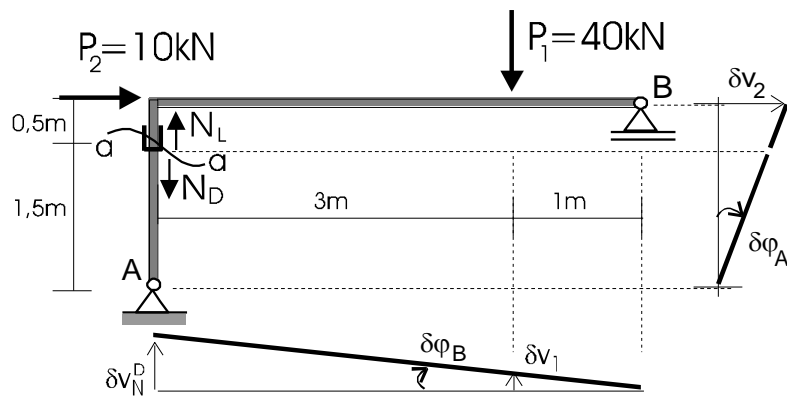
$$dA = 0 \quad \text{JEDNADŽBA VIRTUALNOGRADA},$$

$$P \cdot \delta v_P - M_L \cdot \delta \varphi_A - M_D \cdot \delta \varphi_B = 0,$$

$$P \cdot \delta \varphi_A - M \cdot \delta \varphi_A - M \cdot 3 \delta \varphi_A = 0,$$

$$10 = 4M, \quad M = 2,5 \text{ Nm}.$$

9). Pomoću načela virtualnog rada odredite veličinu uzdužne sile u presjeku  $a-a$  za opterećenje okvirnog nosača prikazanog na crtežu 10.14.



Crtež 10.14.

RJEŠENJE:

$$\delta v_1 = 1 \cdot \delta \varphi_B, \quad \delta v_2 = 2 \cdot \delta \varphi_A, \quad \delta v_N^D = 4 \cdot \delta \varphi_B,$$

$$\delta \varphi_A = \delta \varphi_B, \quad N_L = N_D = N,$$

$$dA = 0 \quad \text{JEDNADŽBA VIRTUALNOGRADA},$$

$$N_L \cdot 0 - N_D \cdot \delta v_N^D - F_1 \cdot \delta v_1 + F_2 \cdot \delta v_2 = 0,$$

$$N \cdot 0 - N \cdot 4 \cdot \delta \varphi_B - F_1 \cdot \delta \varphi_B + F_2 \cdot 2 \cdot \delta \varphi_B = 0,$$

$$4 \cdot N = -20, \quad \Rightarrow \underline{N = -5 \text{ kN}}, \quad \Rightarrow \text{TLAK}.$$