REŠETKASTI NOSAČI

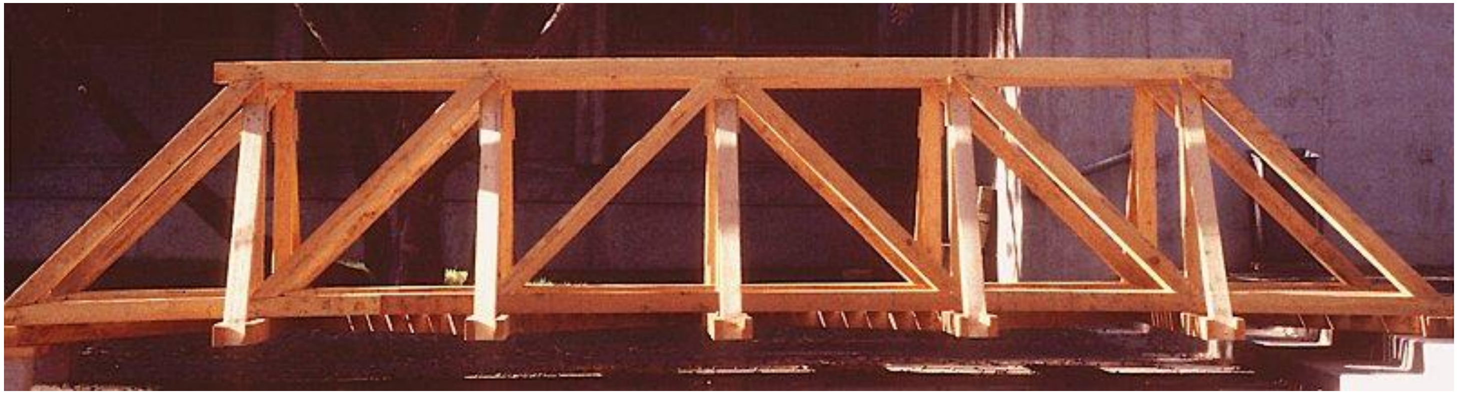
REŠETKASTI NOSAČI

PRIMJENA NOSAČA:

U VISOKOGRADNJI: KROVIŠTA; UKRUTE(SPREGOVI); HALE; DALEKOVODI

MOSTOVI: ŽELJEZNIČKI;

PJEŠAČKI; CESTOVNI

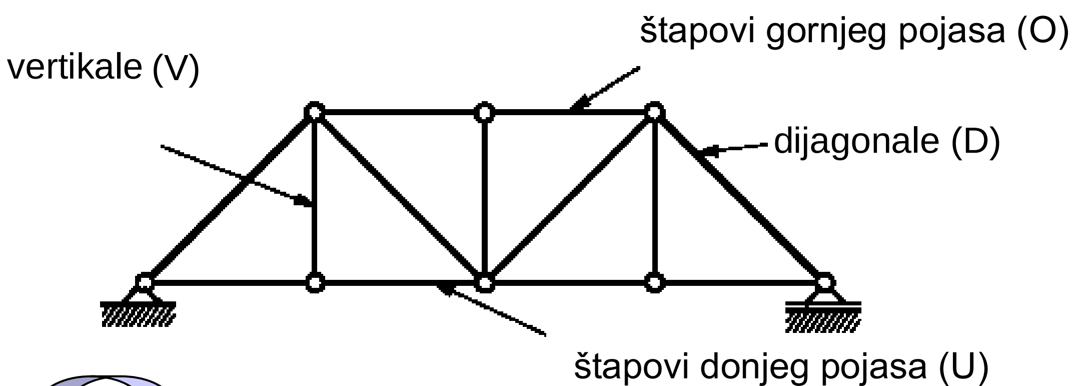


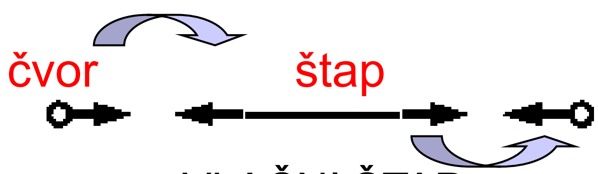
REŠETKASTI NOSAČI

PRIMJENA NOSAČA:

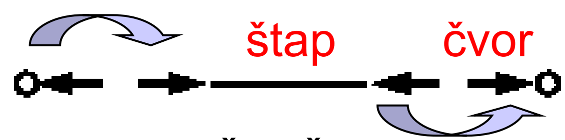
RAVNINSKE REŠETKE

REŠETKASTI NOSAČI: sustavi sastavljeni od u čvorovima zglobno spojenih štapova.





VLAČNI ŠTAP

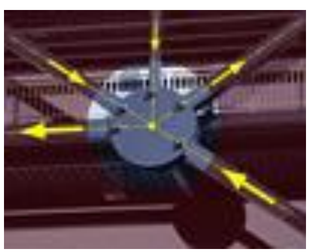


TLAČNI ŠTAP

REŠETKASTI NOSAČI

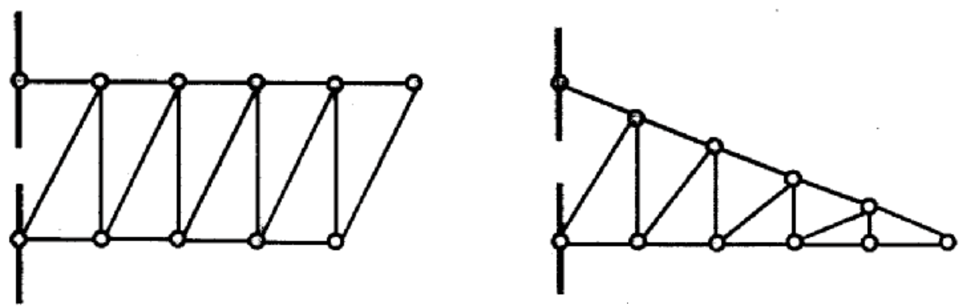
PRETPOSTAVKE:

- štapovi spojeni u čvorovima idealnim zglobovima (bez trenja).
- optrećenja-koncentrirane sile djeluju u čvoru - M=0→samo uzdužne sile u elementima rešetke (samo N dijagrami un. sila)

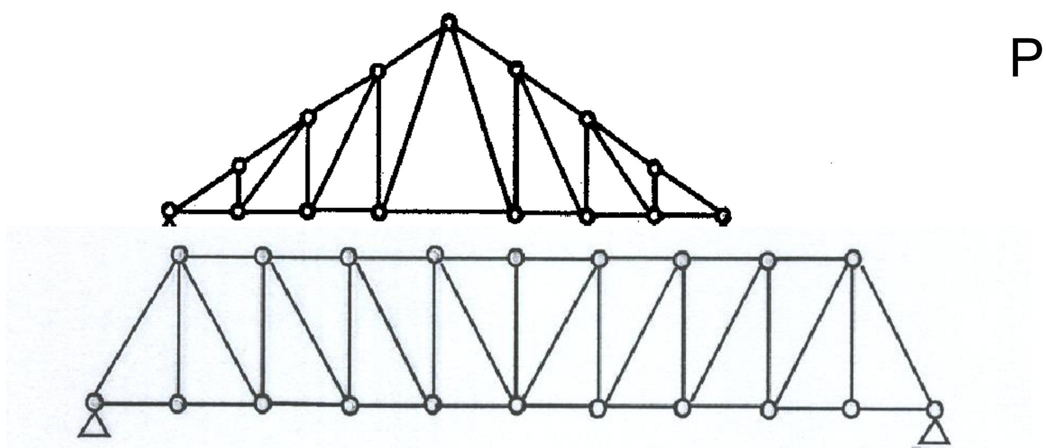


VRSTE RAVNINSKIH REŠETKI

Rešetkaste nosače nazivamo prema štapovima ispune,
 prema obliku pojasa (II,trokutni,parabolični), prema kreatorima istih i prema ležajnim uvjetima-odnosno statičkom sustavu.

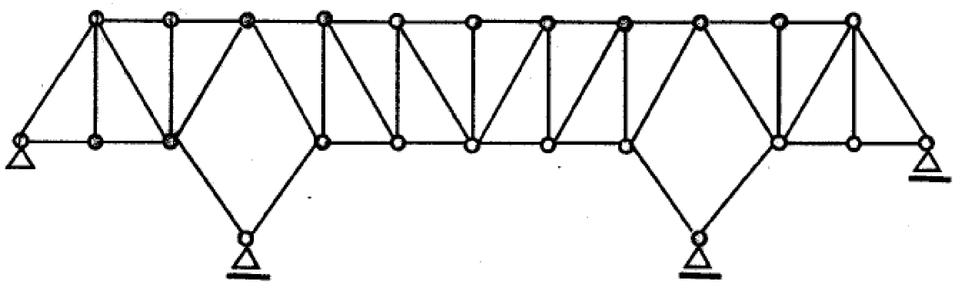


Konzolne N rešetka

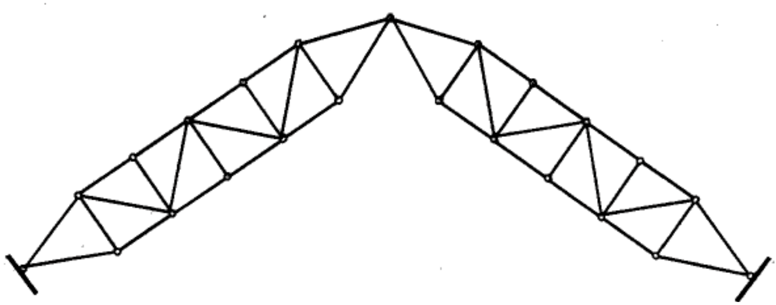


Prattove rešetke

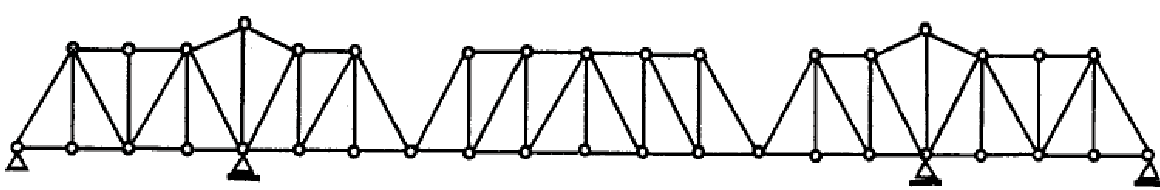
VRSTE RAVNINSKIH REŠETKI



Složena rešetka

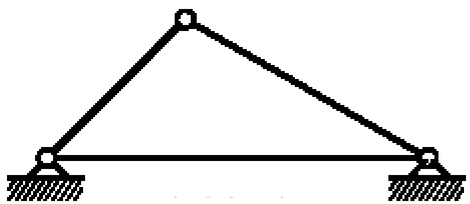


Trozglobna rešetka



Gerberova rešetka

GEOM. NEPROMJENJIVOST REŠETKI

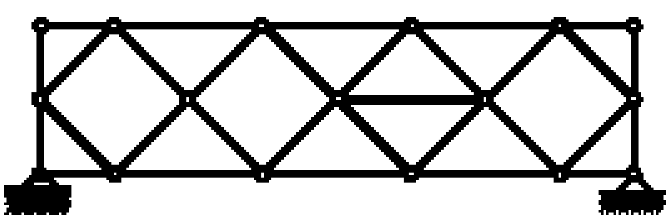


Trokut-osnovni geometrijski nepromjenjiv oblik. Rešetka iz trokuta je stabilna.

$\boxed{Š \geq 2 * Č - L}$

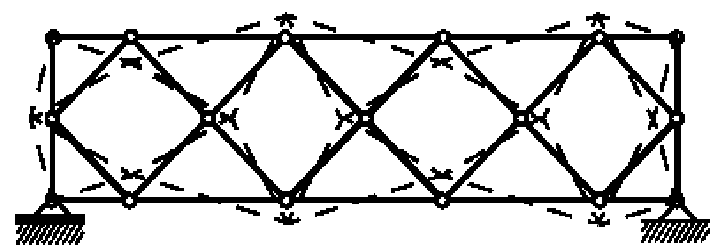
Nužan uvjet geometrijske nepromjenjivosti.

Dovoljan: ispravan rapored štapova rešetke.



Geom.nepromjenjiv sustav

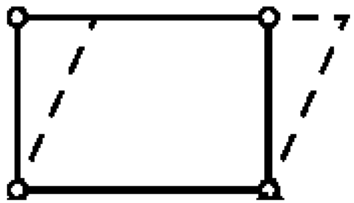
31 (Š) = 2x17 (Č) – 3 (L)



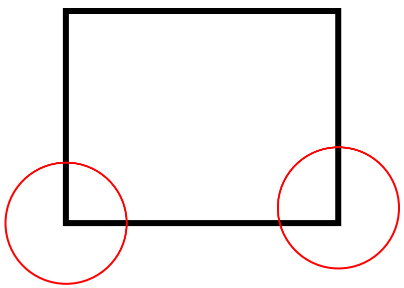
Geom.promjenjiv sustav-nestabilan

30 (Š) < 2x17 (Č) – 3 (L)

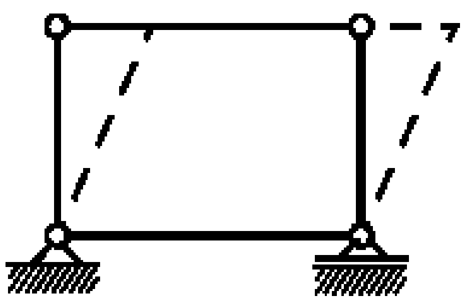
GEOM. NEPROMJENJIVOST REŠETKI



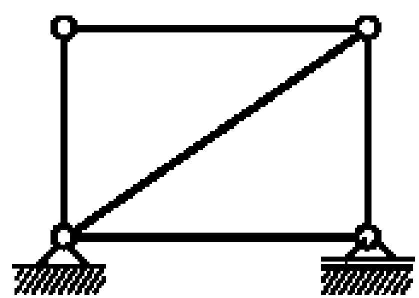
Četverokut-osnovni geometrijski promjenjiv oblik.



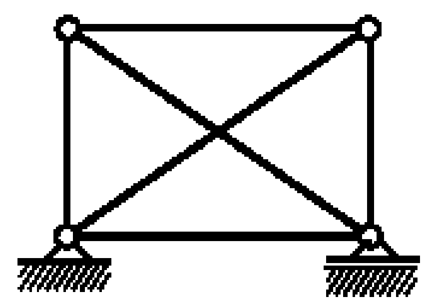
Ovaj četverokut je stabilan.



pomjerljiv sustav
nestabilan



statički određen



statički neodređen

METODE PRORAČUNA REŠETKASTIH NOSAČA

RAVNOTEŽA	čvora	presjeka	
analitički	metoda isjecanja čvorova	metoda momentnih točaka (Ritterova metoda) + metoda projekcija	metoda zamjene štapova
grafički	Cremonin plan sila	Culmanova metoda	
numerički	MKE-MP-programi		

METODA ČVOROVA

Konstrukcija je u ravnoteži ako svaki čvor u ravnoteži.

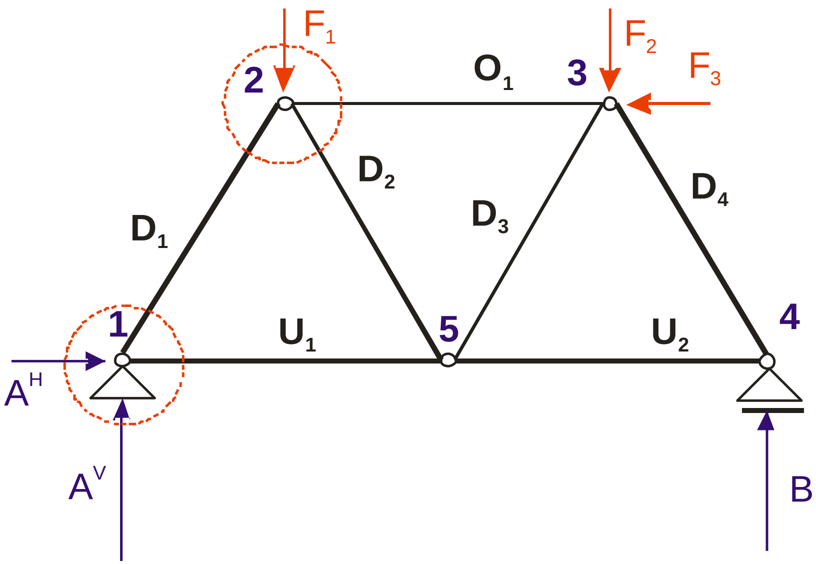
METODA ČVOROVA

- **uravnoteženje čvor po čvor**
- **uravnoteženje svih čvorova**

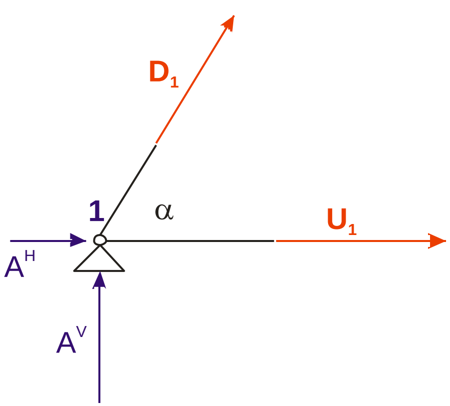
- Ova metoda postavlja po dvije jednadžbe ravnoteže za svaki čvor.
- Primjenjiva je ako možemo krenuti od čvora gdje su nepoznate dvije sile i u svakom slijedećem čvoru da su nepoznate dvije sile.
- Prvo se odrede reakcije sustava, a onda ide na određivanje sila u štapovima.

METODA ČVOROVA

uravnoteženje čvor po čvor



1. čvor



$$\boxed{\sum F_y = 0}$$

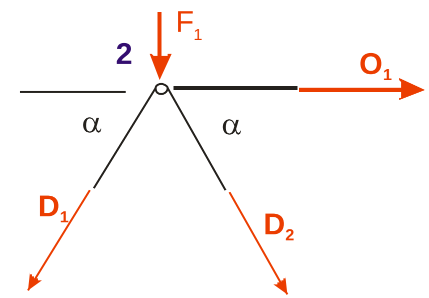
$$-D_1 \text{ x } \sin\alpha + A^V = 0$$

$$\boxed{\sum F_x = 0}$$

$$U_1 + D_1 \text{ x } \cos\alpha + A^H = 0$$

$$\Rightarrow \quad D_1; \quad U_1$$

2. čvor



$$\boxed{\sum F_y = 0}$$

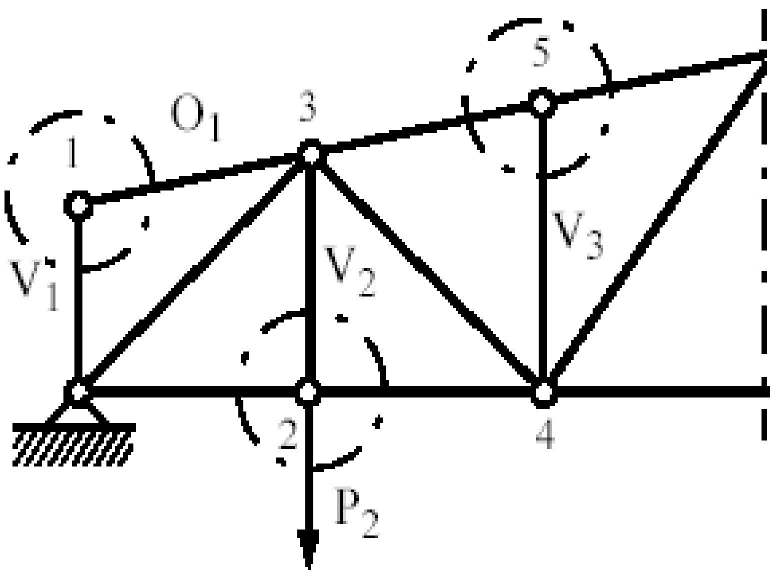
$$-F_1 - D_1 \text{ x } \sin\alpha - D_2 \text{x } \sin\alpha = 0$$

$$\boxed{\sum F_x = 0}$$

$$-D_1 \text{ x } \cos\alpha + D_2 \text{x } \cos\alpha + O_1 = 0$$

$$\Rightarrow \quad D_2; \quad O_1$$

Prepoznavanje nul štapova:



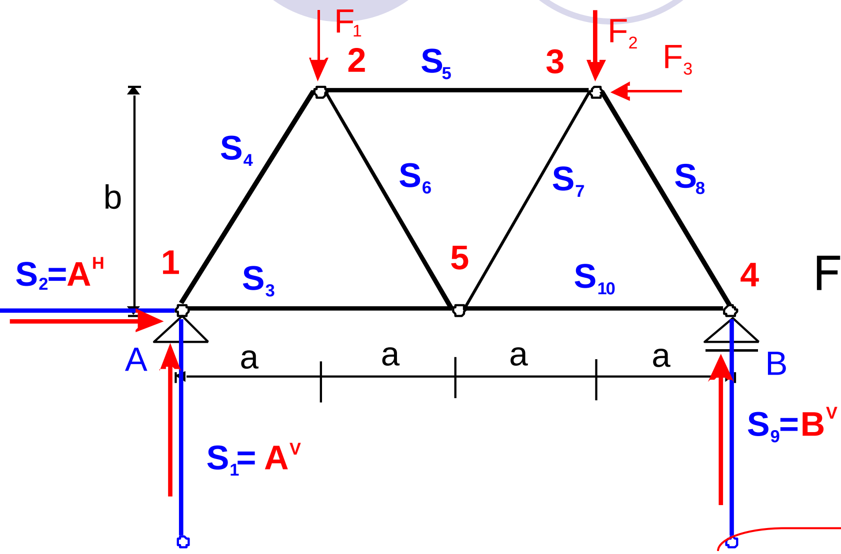
čvor 1
$V_1=0$
$O_1=0$

čvor 5
$V_3=0$

čvor 2
$V_2=P_2$

METODA ČVOROVA

uravnoteženje svih čvorova



Formiranje sustava jednadžbi za cijeli nosač:

$$
\begin{array}{l}
\Sigma Fx=0 \\
\Sigma Fy=0 \quad \text{čvor 1} \\
\\ \\ \\ \\ \\ \\
\text{čvor 5} \\
\\
\end{array}
\overbrace{\begin{bmatrix}
0 & 1 & 1 & \cos\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sin\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos\alpha & 1 & \cos\alpha & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sin\alpha & 0 & \sin\alpha & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & \cos\alpha & \cos\alpha & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sin\alpha & \sin\alpha & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos\alpha & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin\alpha & 1 & 0 \\
0 & 0 & -1 & 0 & 0 & \cos\alpha & \cos\alpha & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & \sin\alpha & \sin\alpha & 0 & 0 & 0
\end{bmatrix}}^{\mathbf{D}}
\overset{\mathbf{s}}{\begin{Bmatrix} 0 \\ 0 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \\ 0 \\ S_{10} \end{Bmatrix}}
=
\overset{\mathbf{f}}{\begin{Bmatrix} A^H \\ A^V \\ 0 \\ F_1 \\ F_3 \\ F_2 \\ 0 \\ 0 \\ B^V \\ 0 \end{Bmatrix}}
$$

METODA ČVOROVA

U matričnom obliku:

$$D\ s = f$$

rješenje: $D\ f^{-1} = s$

-matrica **D** koeficijenti f-je geometrijskog položaja štapova rešetke

-vektor s **-** nepoznate sile

- vektor **f** opterećenje u čvorovima

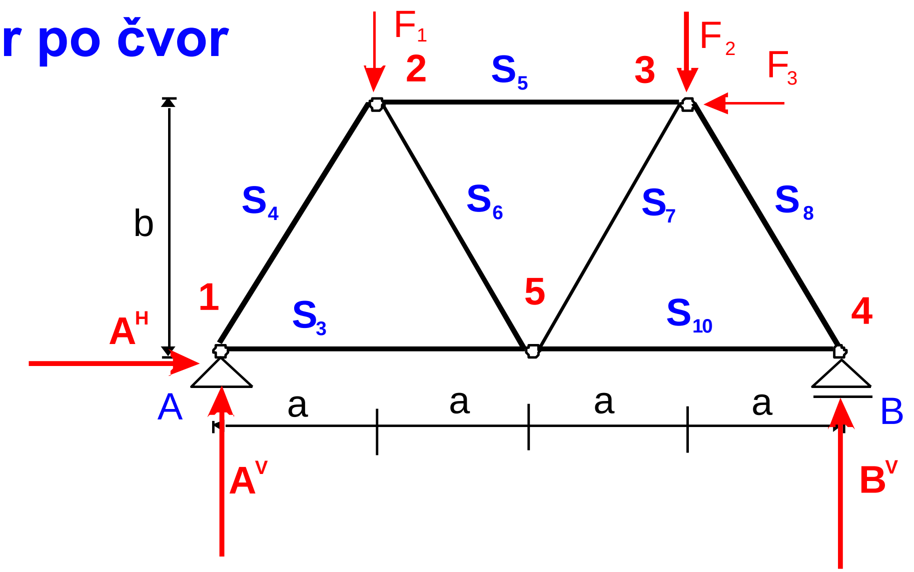
Nužan uvjet geometrijske nepromjenjivosti:

det D=0

METODA ČVOROVA

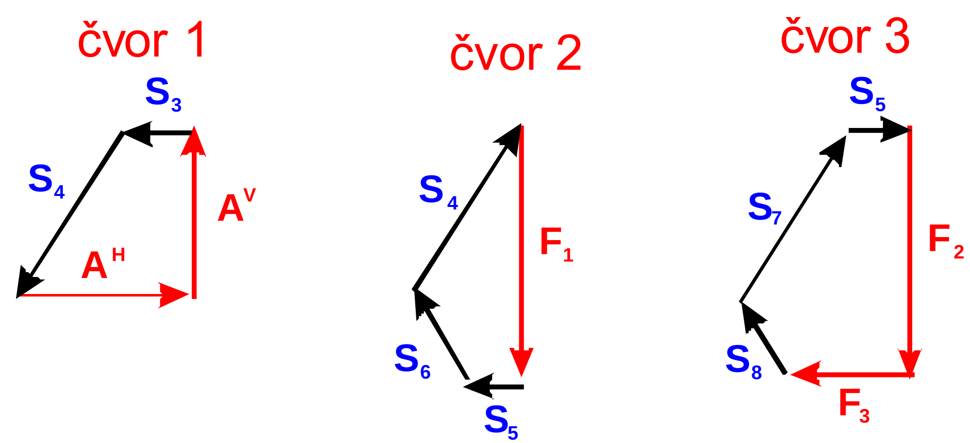
GRAFIČKI NAČIN

uravnoteženje čvor po čvor



Zatvoren poligon

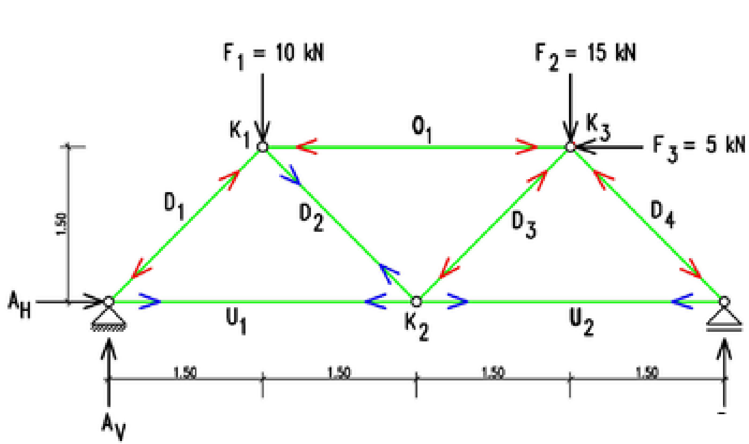
sila za svaki čvor.



METODA ČVOROVA

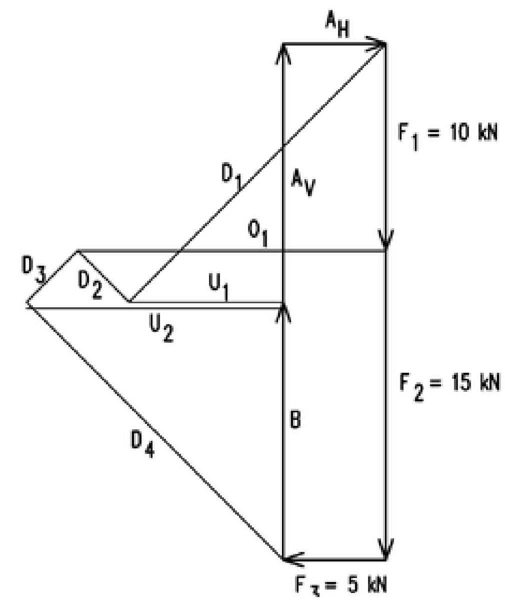
GRAFIČKI NAČIN

uravnoteženje svih čvorova → Cremonin plan sila



tlak

vlak



Jedinstven poligon sila za sve čvorove.

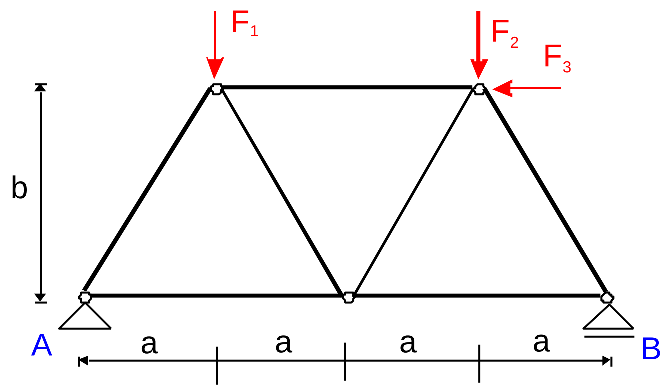
Svaka sila se jednom pojavljuje u poligonu.

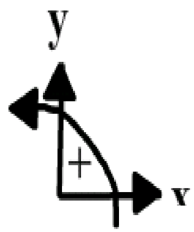
METODA PRESJEKA

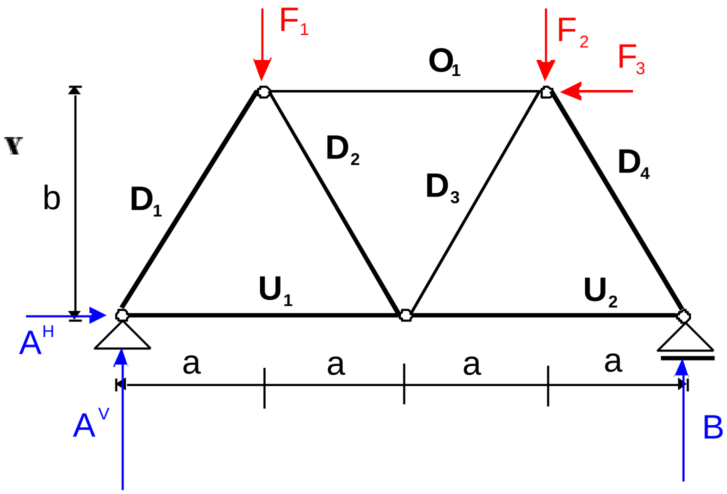
Ritterova metoda-momentnih točaka

Nosač u ravnoteži ako svaki njegov dio u ravnoteži

t.j. rezultanta vanjskih sila = rezultanti unutarnjih sila.







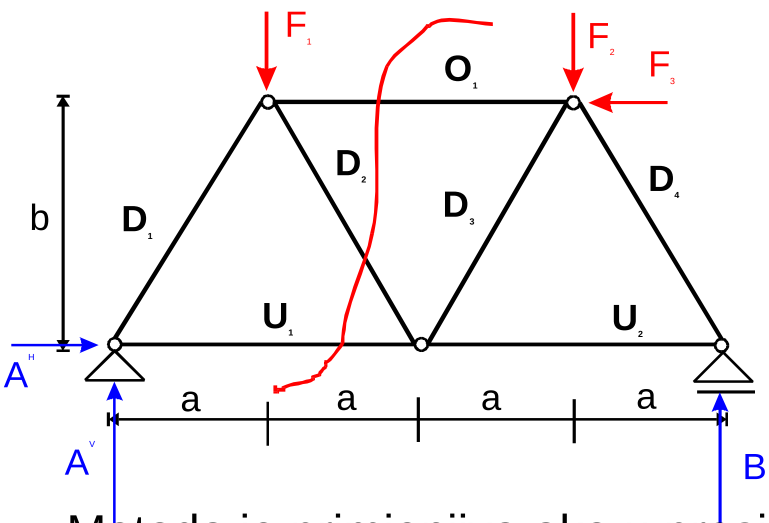
1. se odrede reakcije iz uvjeta ravnoteže.

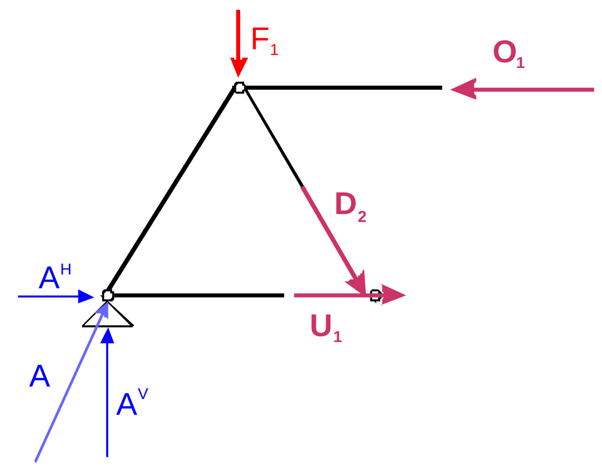
$\Sigma X=0 \Rightarrow R_{Ax}$

$\Sigma Y=0;\ \Sigma M_B=0 \Rightarrow R_{AY};\ R_B$

METODA PRESJEKA

2. se pravi presjek onih štapova u kojima tražimo silu.



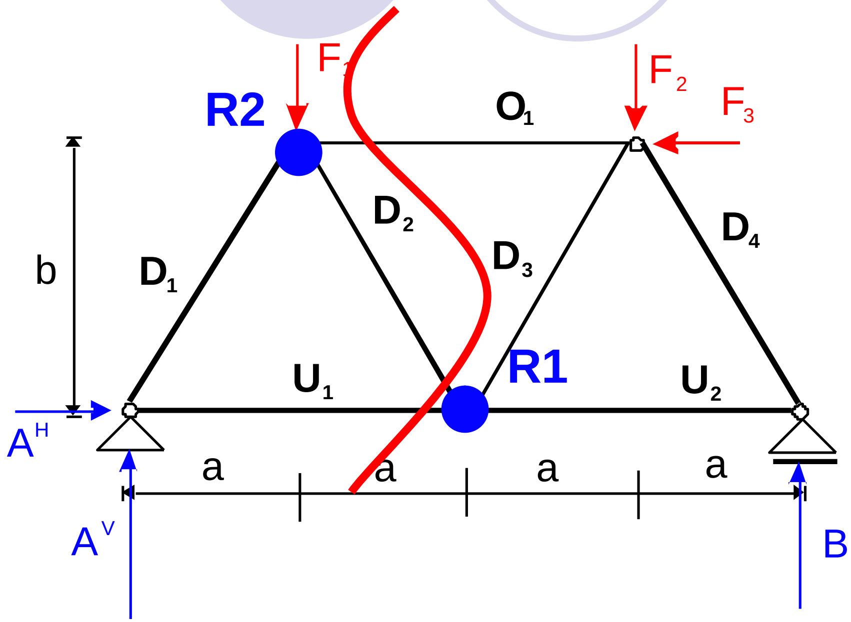


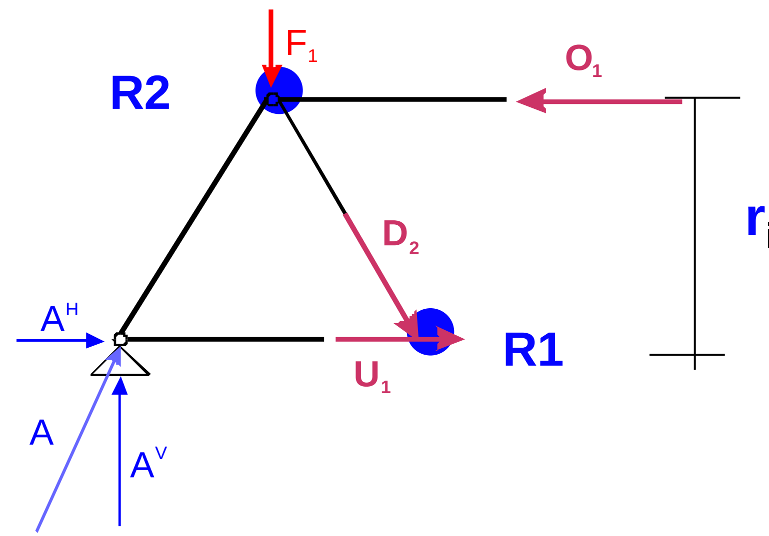
Metoda je primjenjiva ako u presjeku 3 nepoznate sile.

U algebarskoj formulaciji postavljamo 3 jednadžbe ravnoteže, a u geometrijskoj imamo 3 pravca koja uravnotežujemo.

Traže se točke u kojima se sijeku pravci dvije presječene sile-riterove točke, u odnosu na njih postavljamo uvjet da je suma momenata svih sila(vanjskih i unutarnjih) lijevo ili desno od presjeka nula [1]// ili suma vertikalnih projekcija svih sila(vanjskih i unutarnjih) lijevo ili desno od presjeka nula (kada nema riterove točke-II štapovi) [2]

METODA PRESJEKA





$R_{vanjskih} = R_A + F_1$

$R_{unutarnjih} = O_1 + U_1 + D_2$

$R_{vanjskih} + R_{unutarnjih} = 0 \}$ ravnoteža

$\Sigma M_R = 0 \Rightarrow Si = M_R / r_i$

1

$\Sigma M_{R1} = 0$

$R_A^V x 2a - F_1 x a + O_1 x b = 0 \Rightarrow O_1$

$\Sigma M_{R2} = 0$

$R_A^V x a + U_1 x b = 0 \Rightarrow U_1$

2

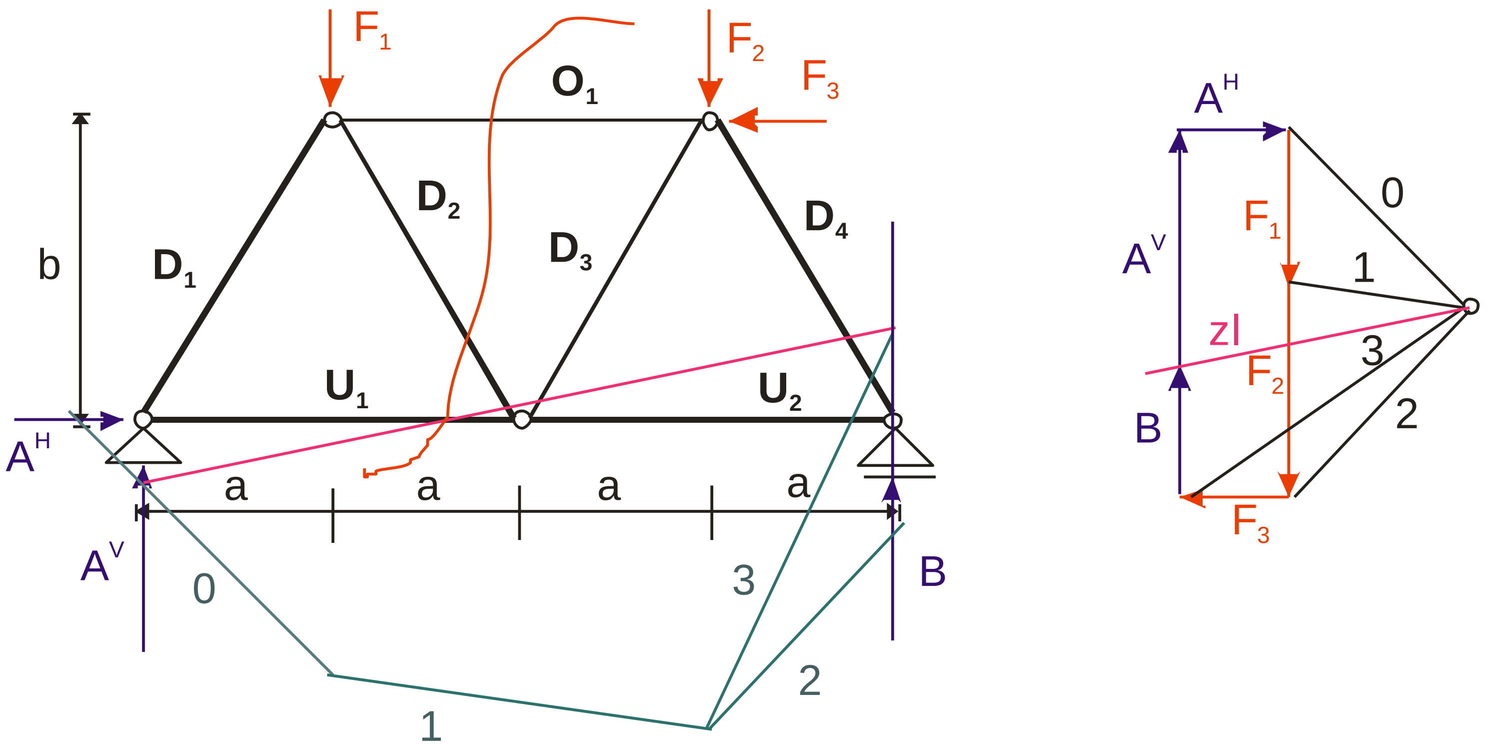
$\Sigma F_y = 0$

$RA^V - F1 + D_2 x \sin\alpha = 0 \Rightarrow D_2$

METODA PRESJEKA

GRAFIČKI NAČIN

Culmanova metoda

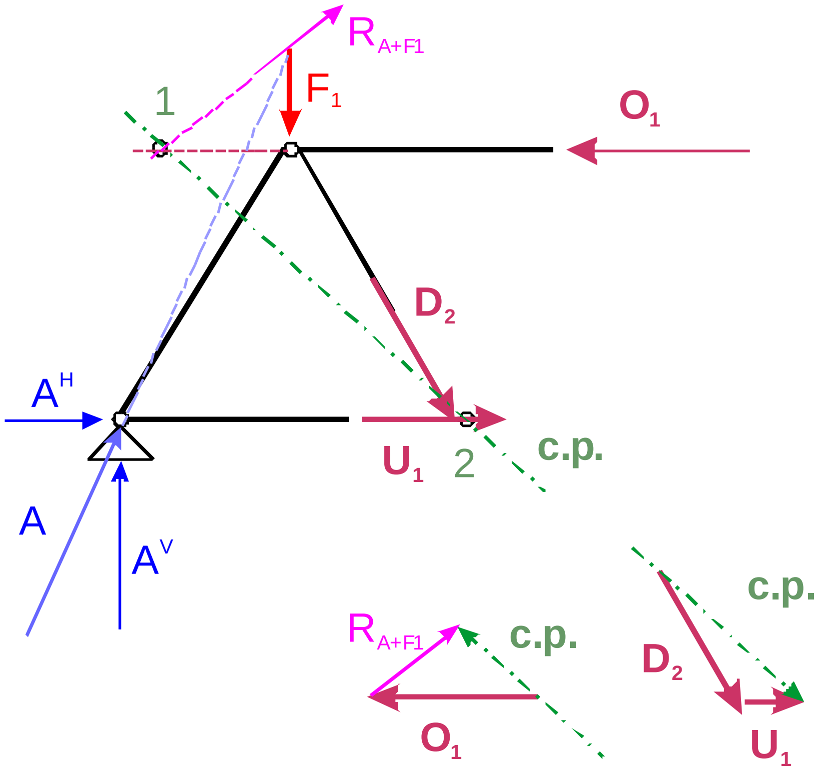


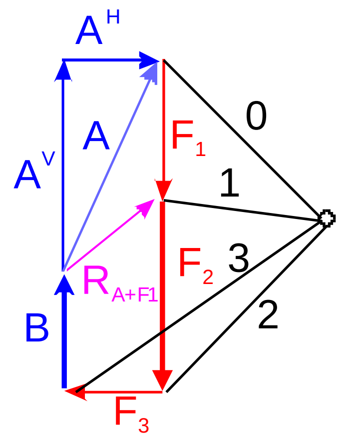
1. se odrede reakcije, te se pravi presjek za koji treba odrediti sile u štapovima.

METODA PRESJEKA

GRAFIČKI NAČIN

Culmanova metoda





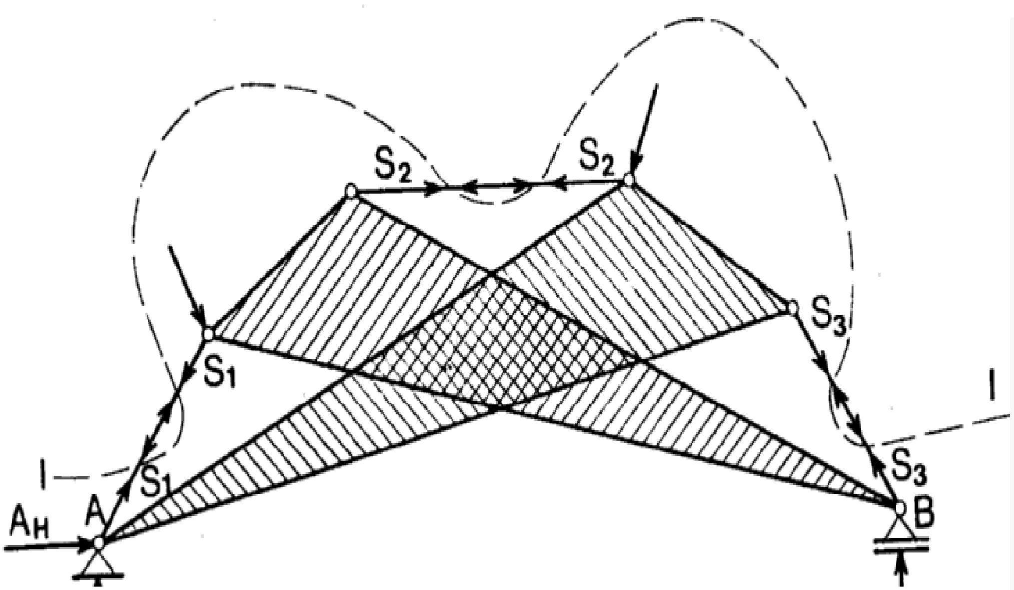
$R_{vanjskih}=R_A + F_1$

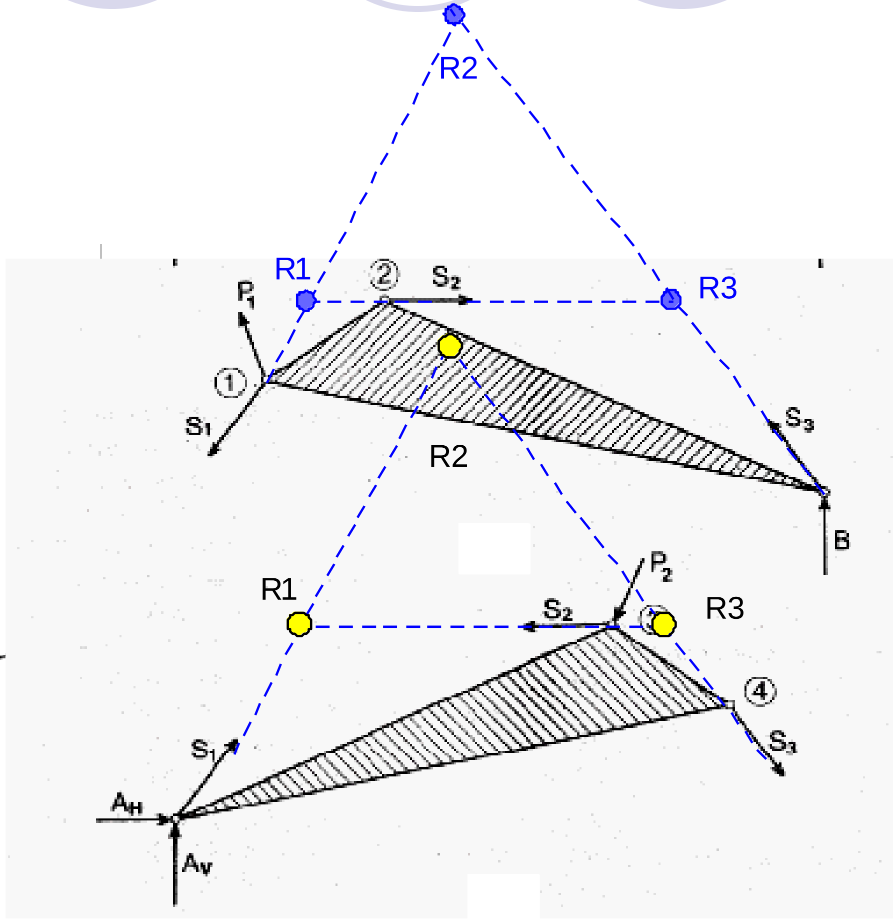
$R_{unutarnjih}=O_1+U_1+D_2$

$R_{vanjskih} + R_{unutarnjih} = 0$

C.P.-culmanov pravac prolazi kroz 2 točke presjeka po 2 sile ($R_{A,F1}$; O_1 i D_2;U_1)

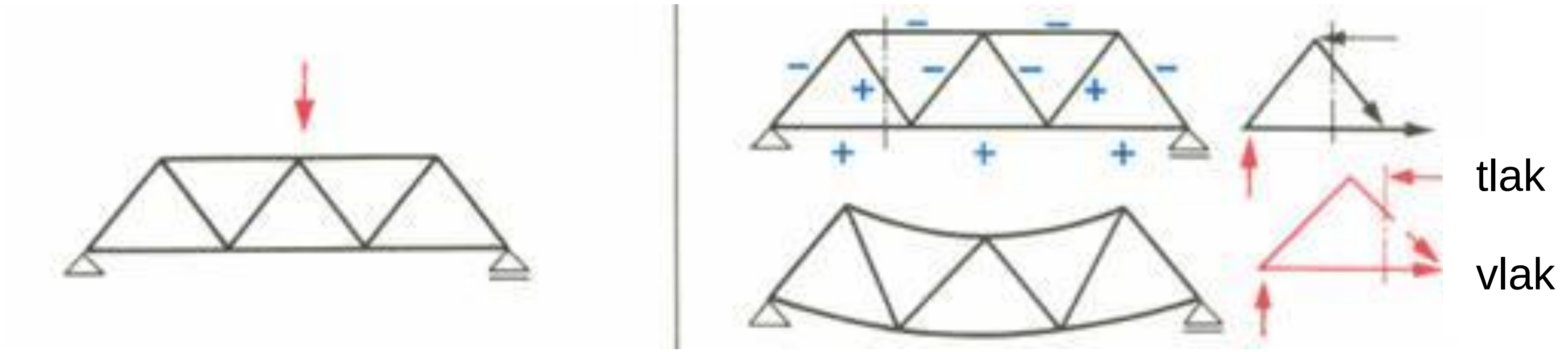
METODA PRESJEKA

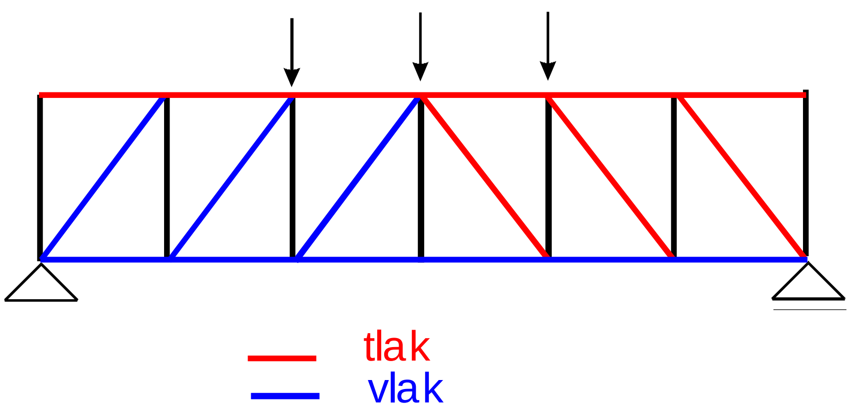




Presjek ne mora ići ravno. Rastavimo rešetku na 2 dijela, te radimo riterovom metodom.

SMJER SILA U ŠTAPOVIMA



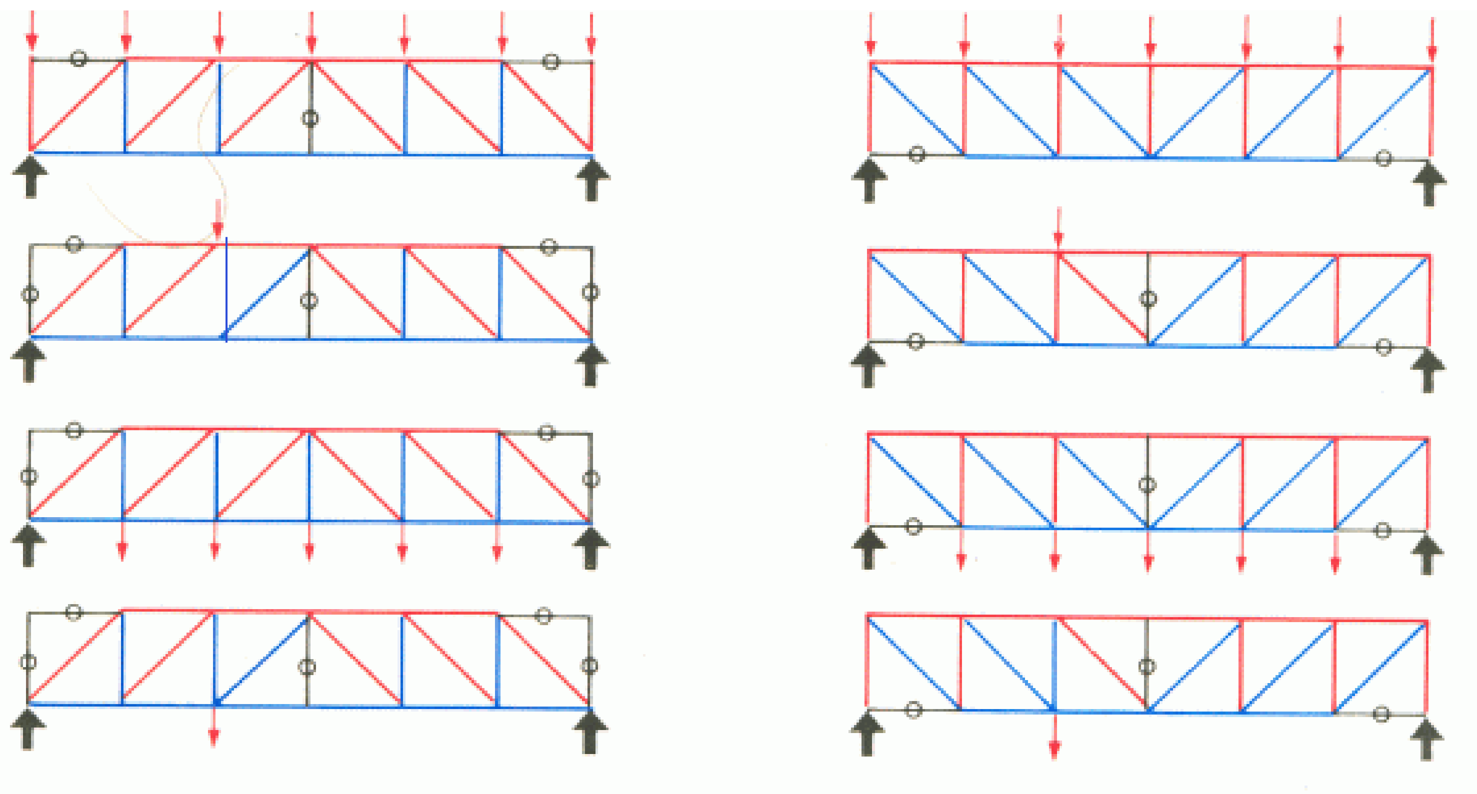


NOSAČI SA II POJASEVIMA

SMJER SILA U ŠTAPOVIMA

Howe rešetka

Pratt rešetka

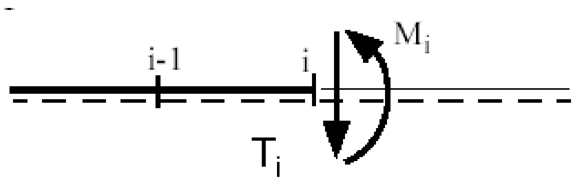


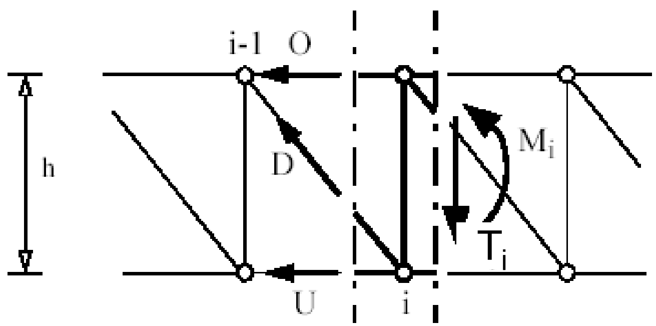
tlak

vlak

0 nul štap

NOSAČI SA II POJASEVIMA



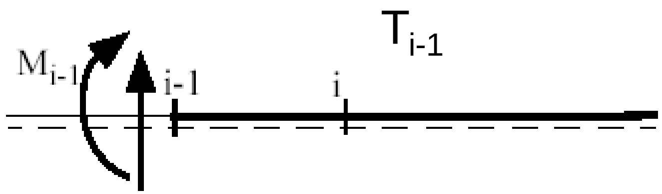


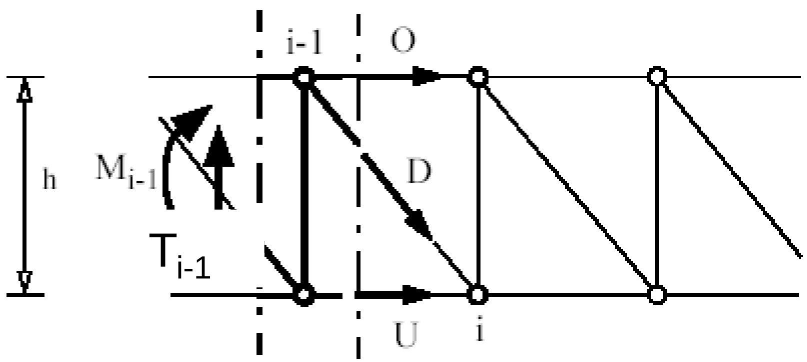
$$O = -\frac{M_i}{h}$$

$$D = \frac{T_i}{\sin\alpha}$$

Sile u štapovima rešetke mogu se odrediti pomoću sila u "zamjenskoj prostoj gredi".

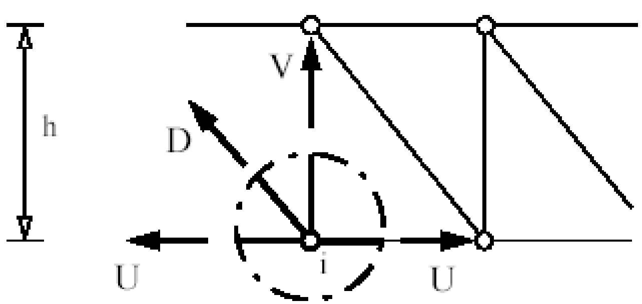
NOSAČI SA II POJASEVIMA





$$U = \frac{M_{i-1}}{h}$$

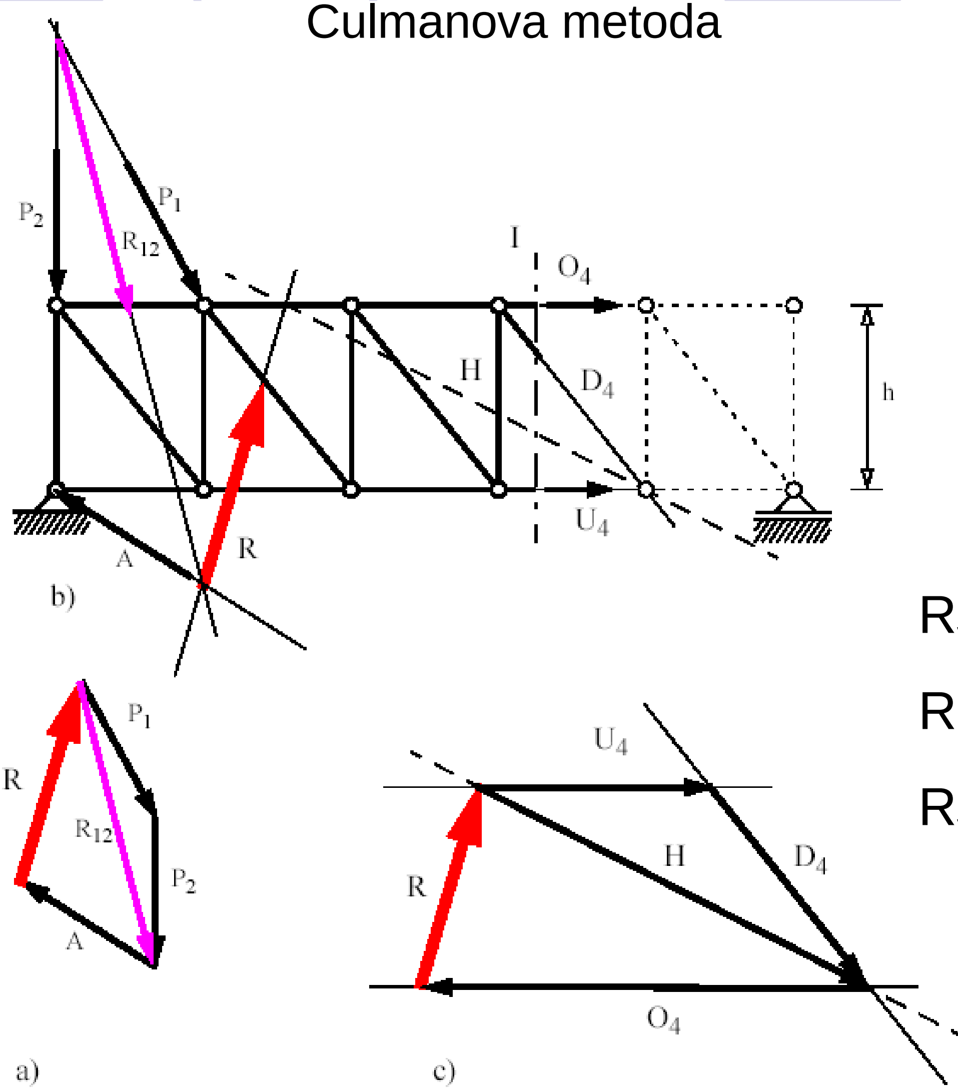
Isjecanje čvora



$$V = -D x \sin\alpha = -T_i$$

NOSAČI SA II POJASEVIMA

Culmanova metoda



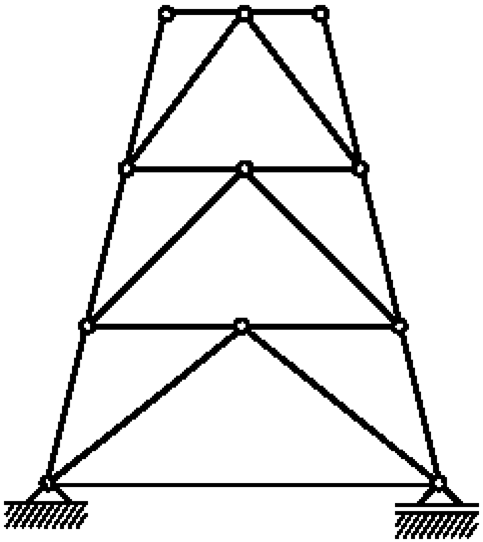
$R_{vanjskih} = R_A + P_1 + P_2$

$R_{unutarnjih} = O_4 + U_4 + D_4$

$R_{vanjskih} + R_{unutarnjih} = 0$

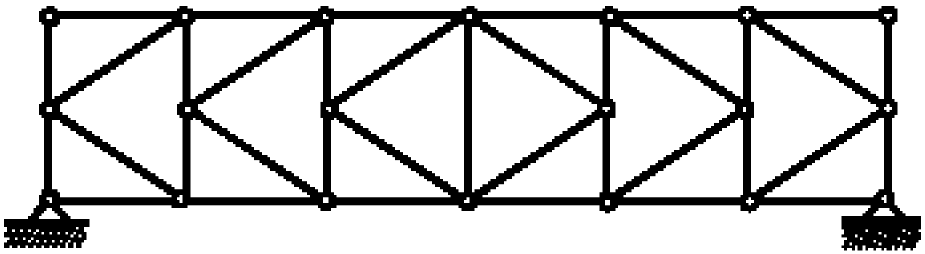
K REŠETKE

Ispuna u obliku slova k. Time se smanjuju duljine štapova ispune.

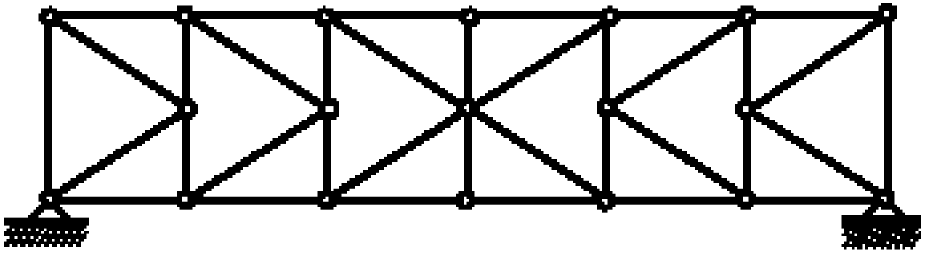


stabilna

19=2*11-3

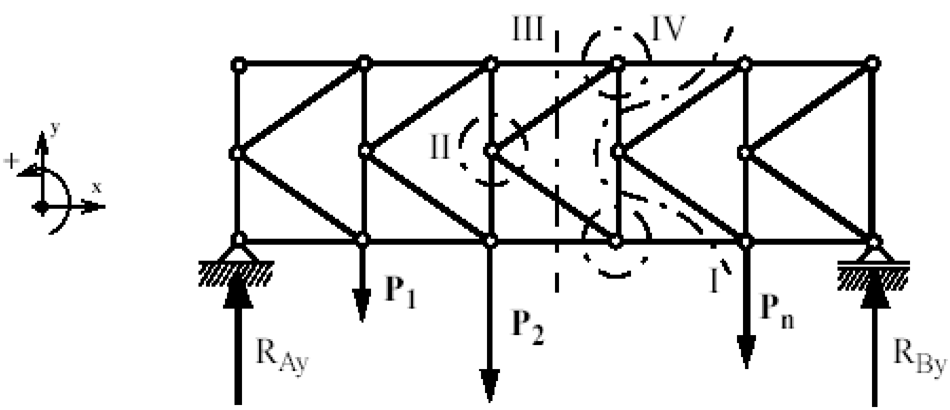


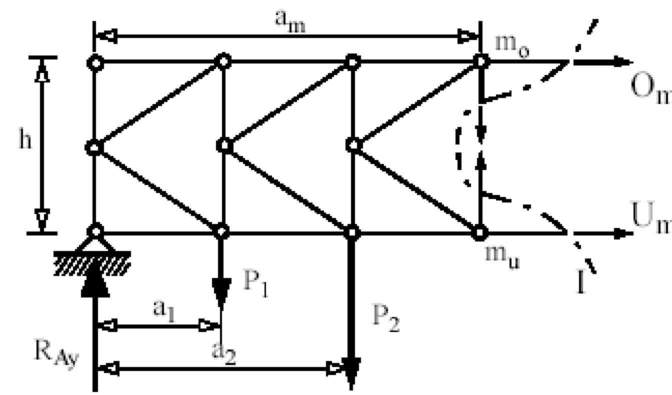
stabilna 37=2x20-3



nestabilna 36≠2x19-3

K REŠETKE





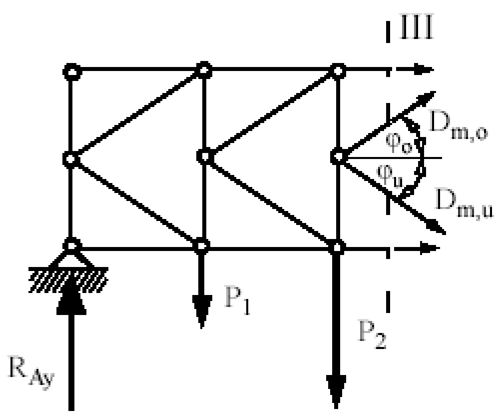
Za drugačiji presjek:

$$\sum M_{m_u} = 0$$

$$\underbrace{R_{AY}\ x\ a_m - \sum_{i=1}^{2} P_i\ x\ (a_m - a_i)}_{M^V_{m_u}} + O_m\ x\ h = 0$$

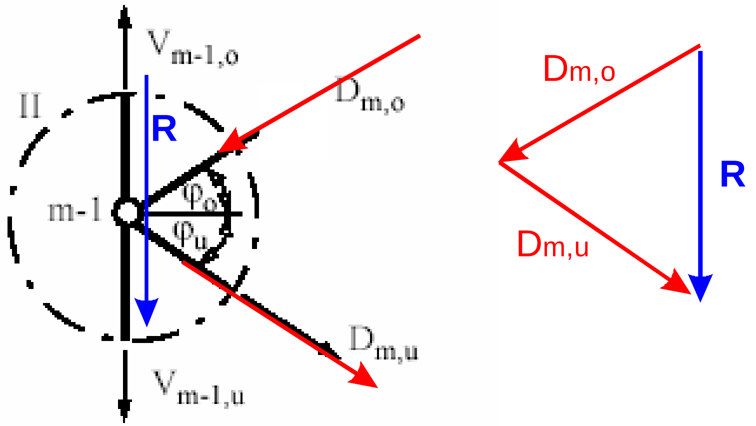
$$M^V_{m_u} + O_m\ x\ h = 0 \quad \Rightarrow \quad O_m = -\frac{M^V_{m_u}}{h}$$

$$\sum M_{m_o} = 0 \quad \Rightarrow \quad U_m = \frac{M^V_{m_o}}{h}$$



Presjekom sječemo 4 štapa.

Isječemo li čvor:



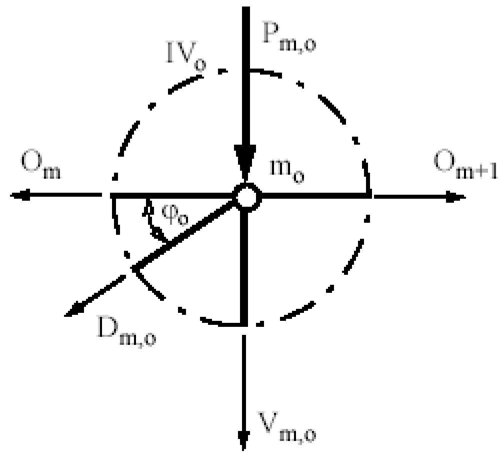
$\sum H=0$

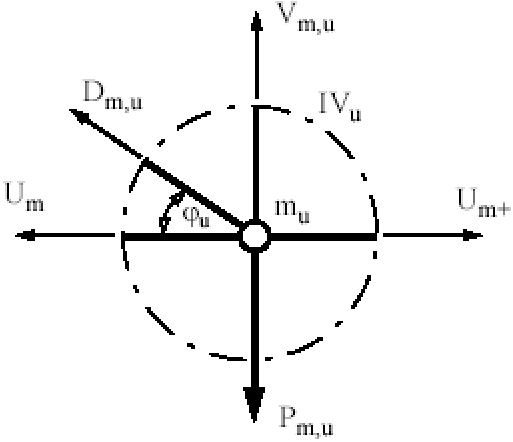
$$D_{m,o} x \cos\varphi_o + D_{m,u} x \cos\varphi_u = 0 \quad \Rightarrow \quad D_{m,u} = -D_{m,o} x \frac{\cos\varphi_0}{\cos\varphi_u}$$

$$\varphi_u = \varphi_o = \varphi$$

$$D_{m,u} = -D_{m,o}$$

K REŠETKE

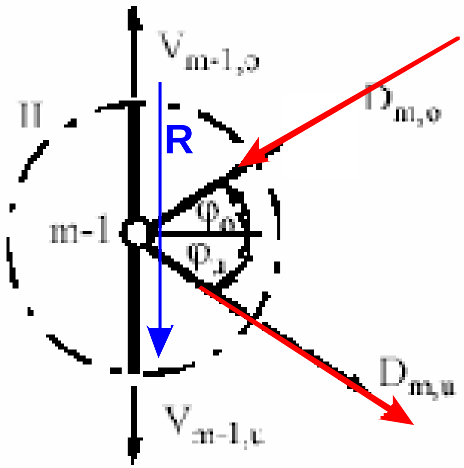


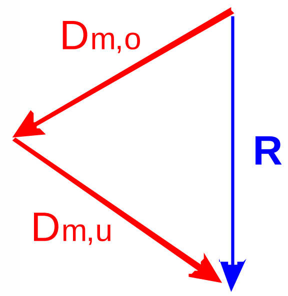


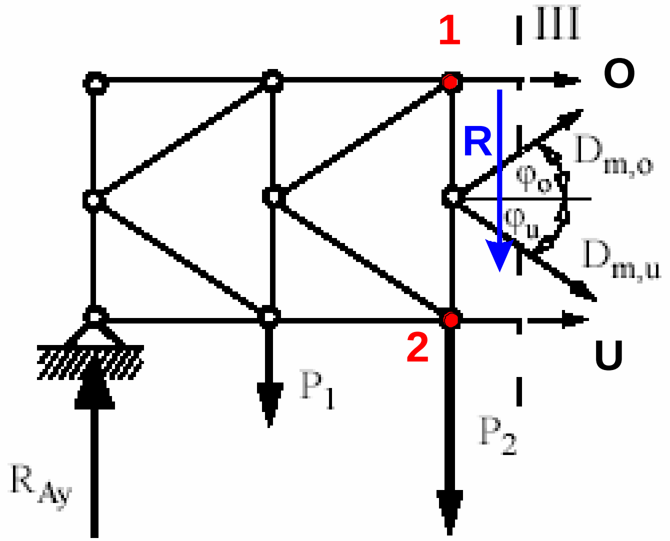
$\Sigma V=0$

$$V_{m,o} = -P_{m,o} - D_{m,o} \times \sin\varphi_0$$

K REŠETKE







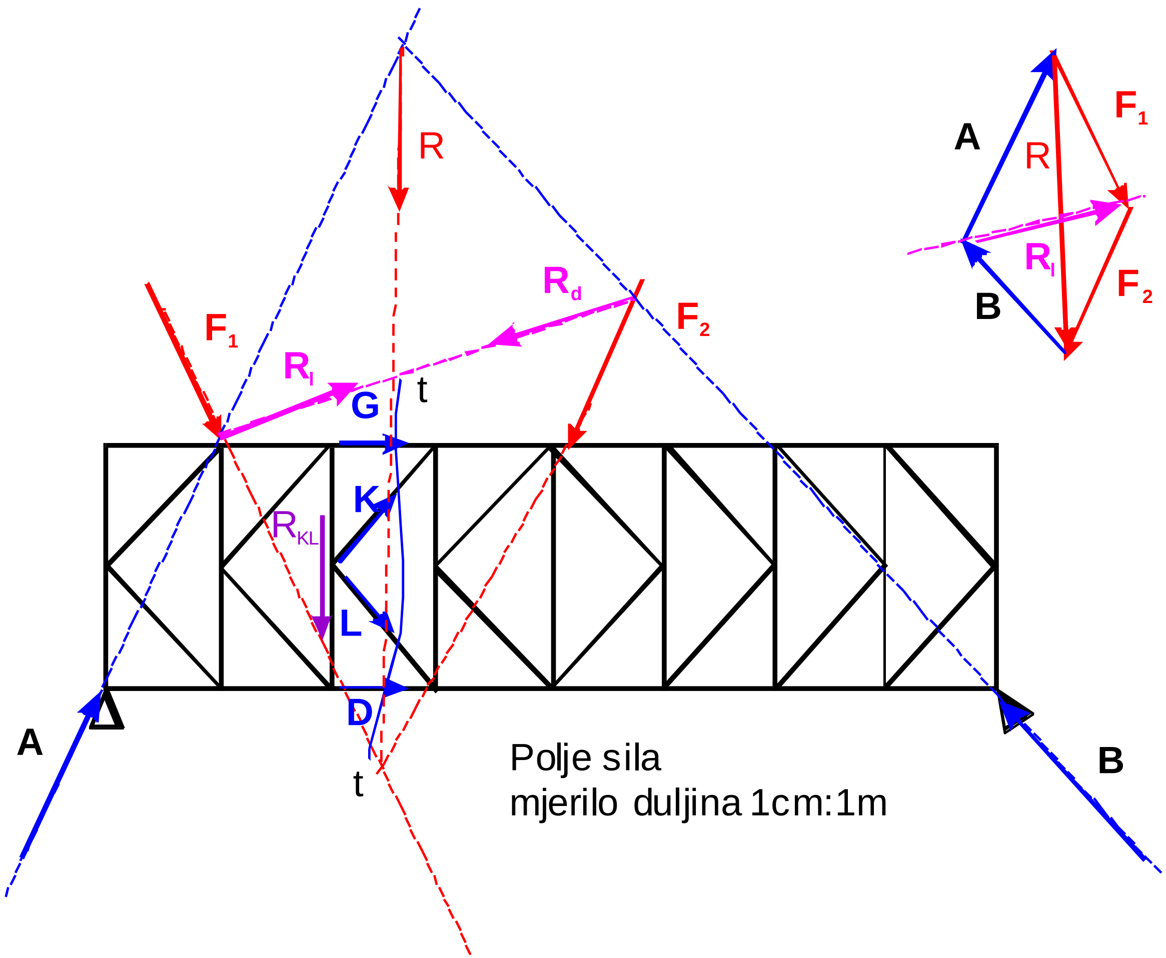
Koristimo metodu presjeka,

jer su nam nepoznate ipak 3 sile.

$\sum M_1=0 \Rightarrow U;$ $\sum M_2=0 \Rightarrow O\ ;$

$\sum V=0 \Rightarrow R$ $R=2*D*\sin\varphi \Rightarrow D$

Grafičko rješenje metodom presjeka



Polje sila
mjerilo duljina 1cm:1m

Poligon sila
mjerilo sila 1cm:1kN

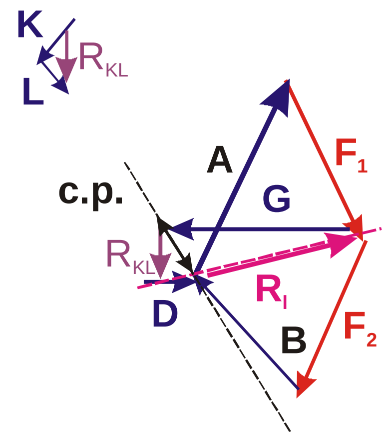
$$R_{vanjskih} = R_A + F_1$$

$$R_{unutarnjih} = G + R_{KL} + D$$

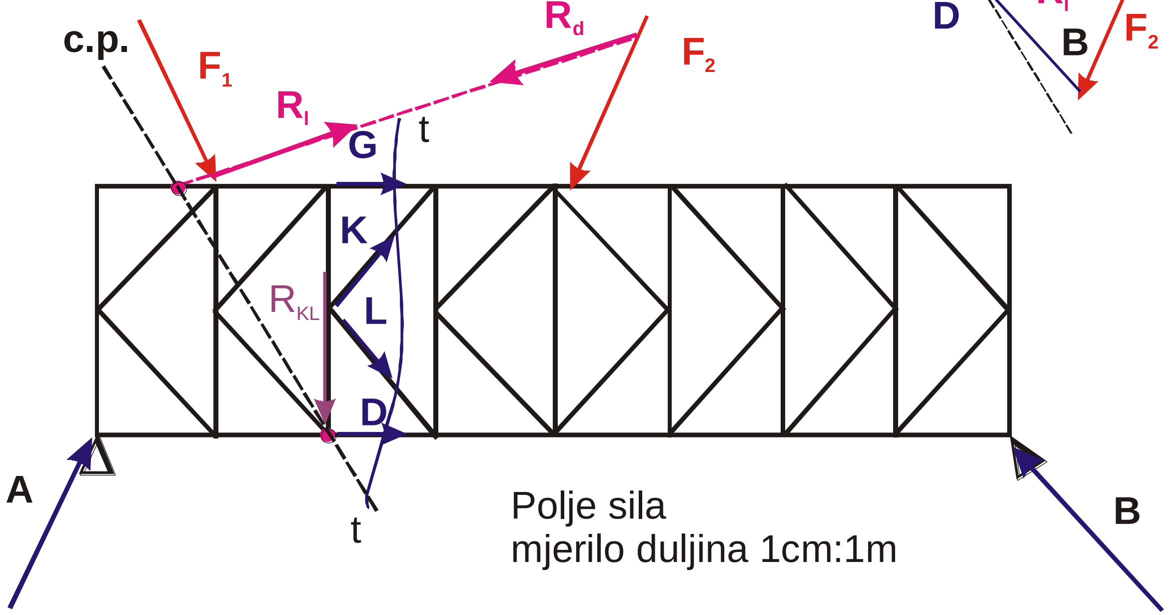
$$R_{vanjskih} + R_{unutarnjih} = 0$$

K REŠETKE

Grafičko rješenje metodom presjeka   Culmanova metoda

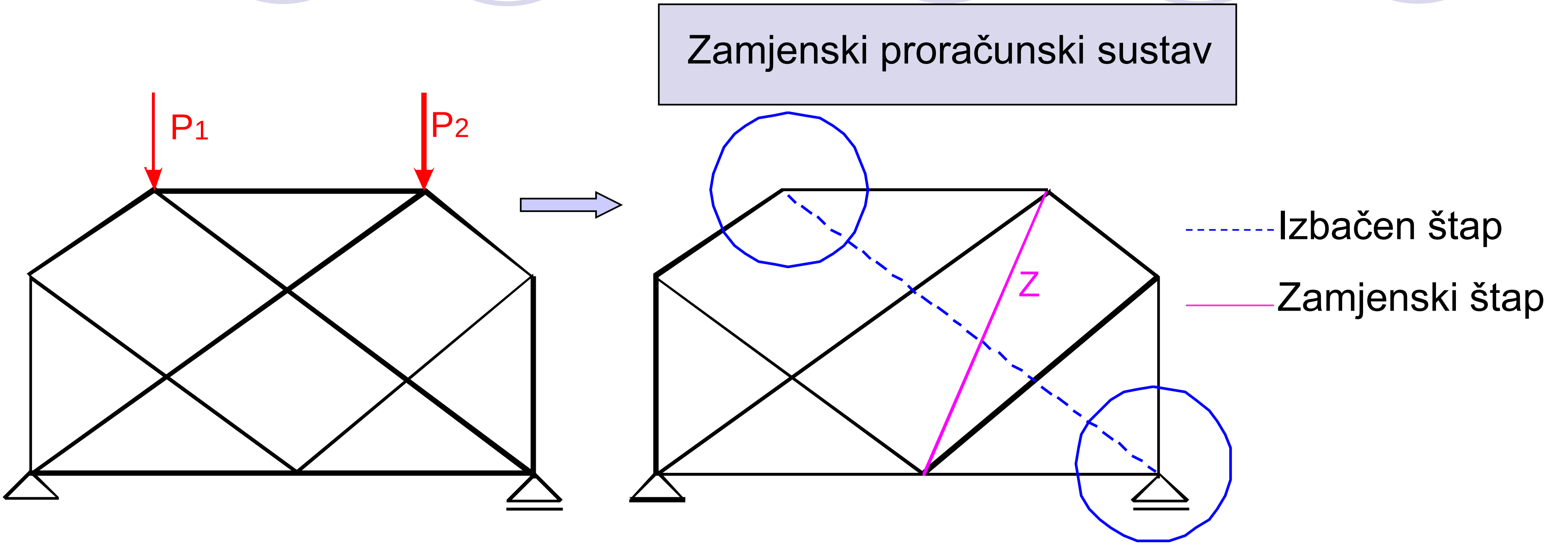


Poligon sila
mjerilo sila 1cm:1kN



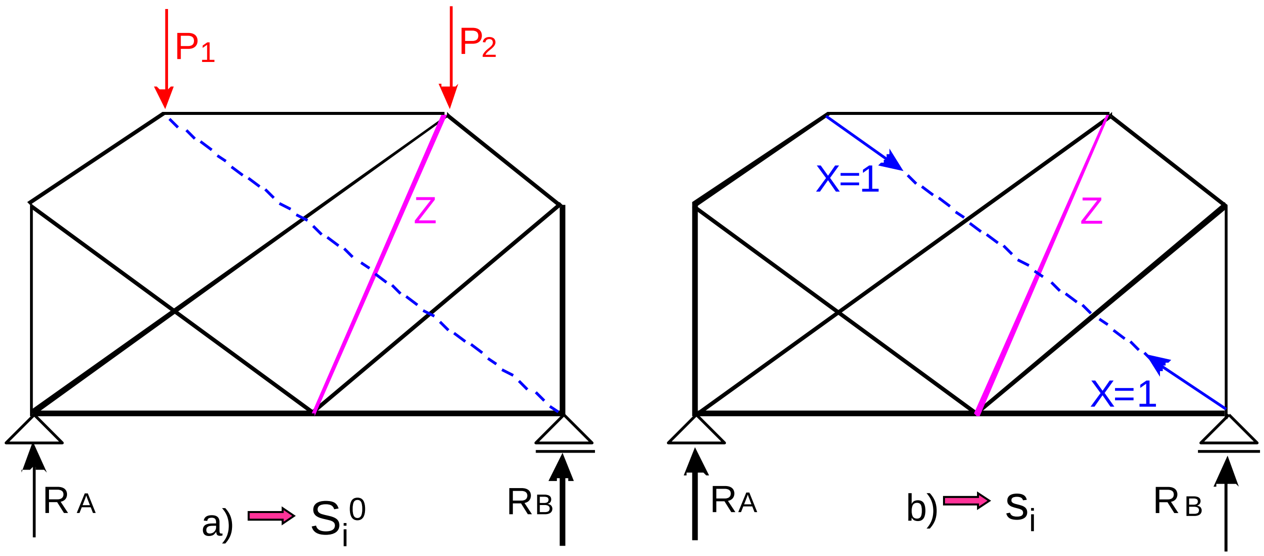
Polje sila
mjerilo duljina 1cm:1m

METODA ZAMJENE ŠTAPOVA



Koristi se kada veći broj nepoznanica od 3-kada je neprimjenjiva i metoda presjeka i metoda čvorova.

METODA ZAMJENE ŠTAPOVA



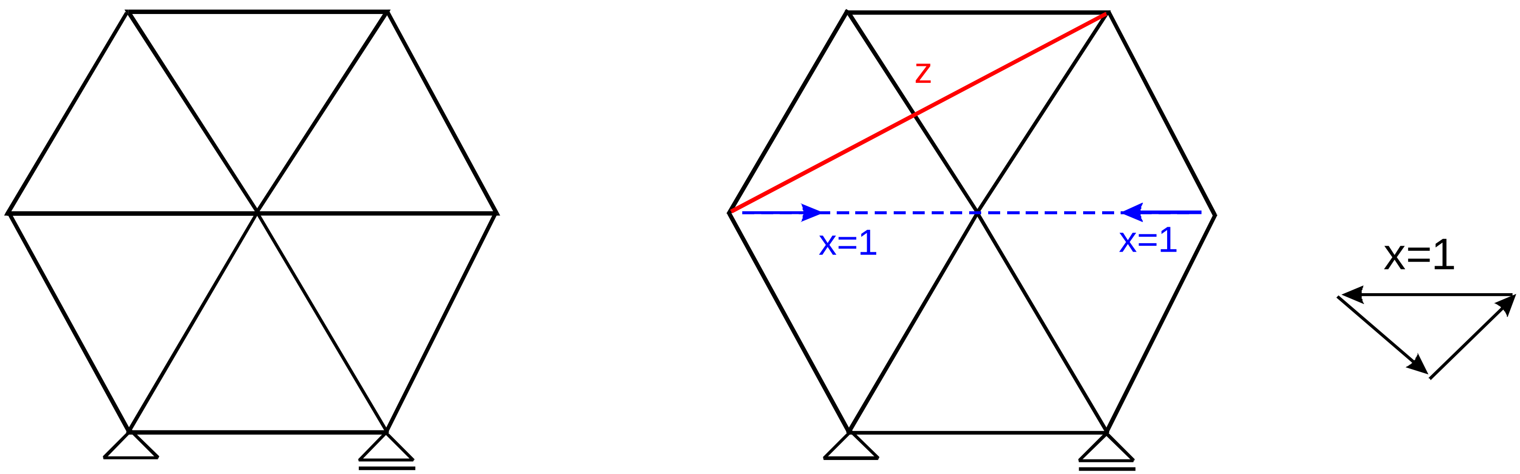
Sile u štapovima zamjenskog sustava su superpozicija rješenja kroz 2 proračunska koraka:

$$Si = S_i^0 + X_i{*}s_i$$

Iz poznatog rješenje: $0 = S_z^0 + X_i{*}s_z \;\Rightarrow\; \mathbf{X_i = -\,S_z^0 / s_z}$

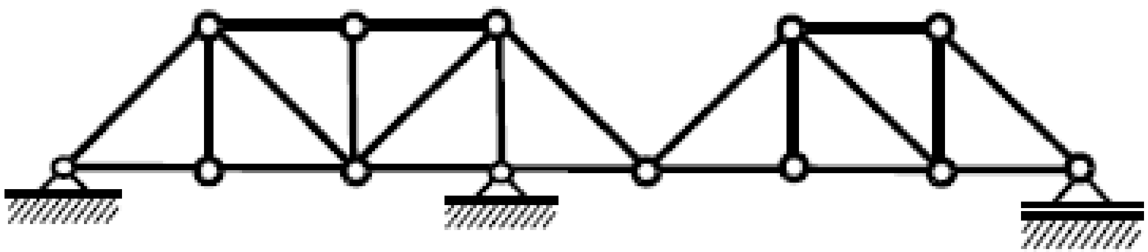
METODA ZAMJENE ŠTAPOVA

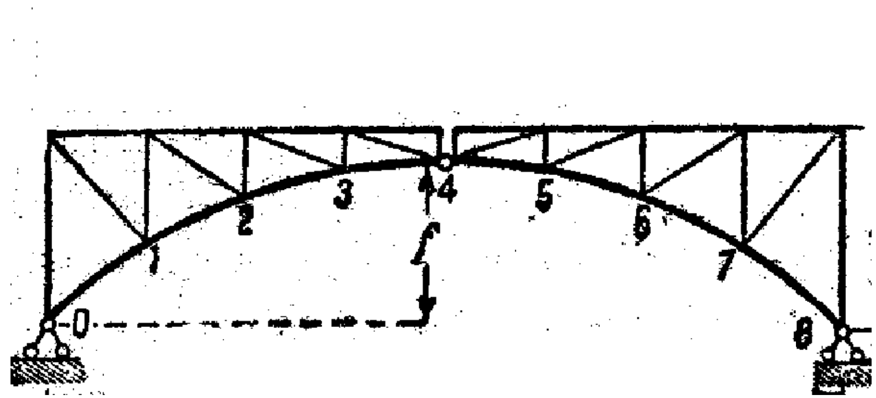
Primjer primjene metode za određivanje geom.nepromjenjivosti:



$s_z=0$ ⇨ $X_i \rightarrow \infty$ ⇨ **geom. promjenjiv sist.**

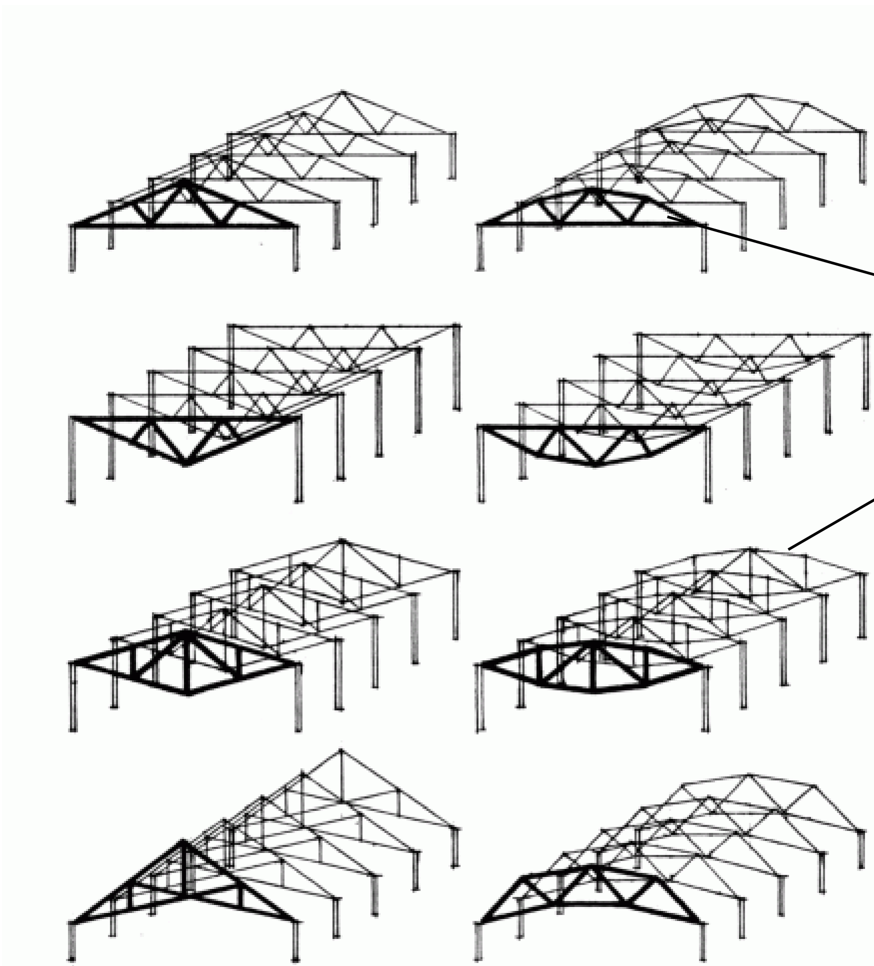
PRORAČUN REŠETKASTIH NOSAČA





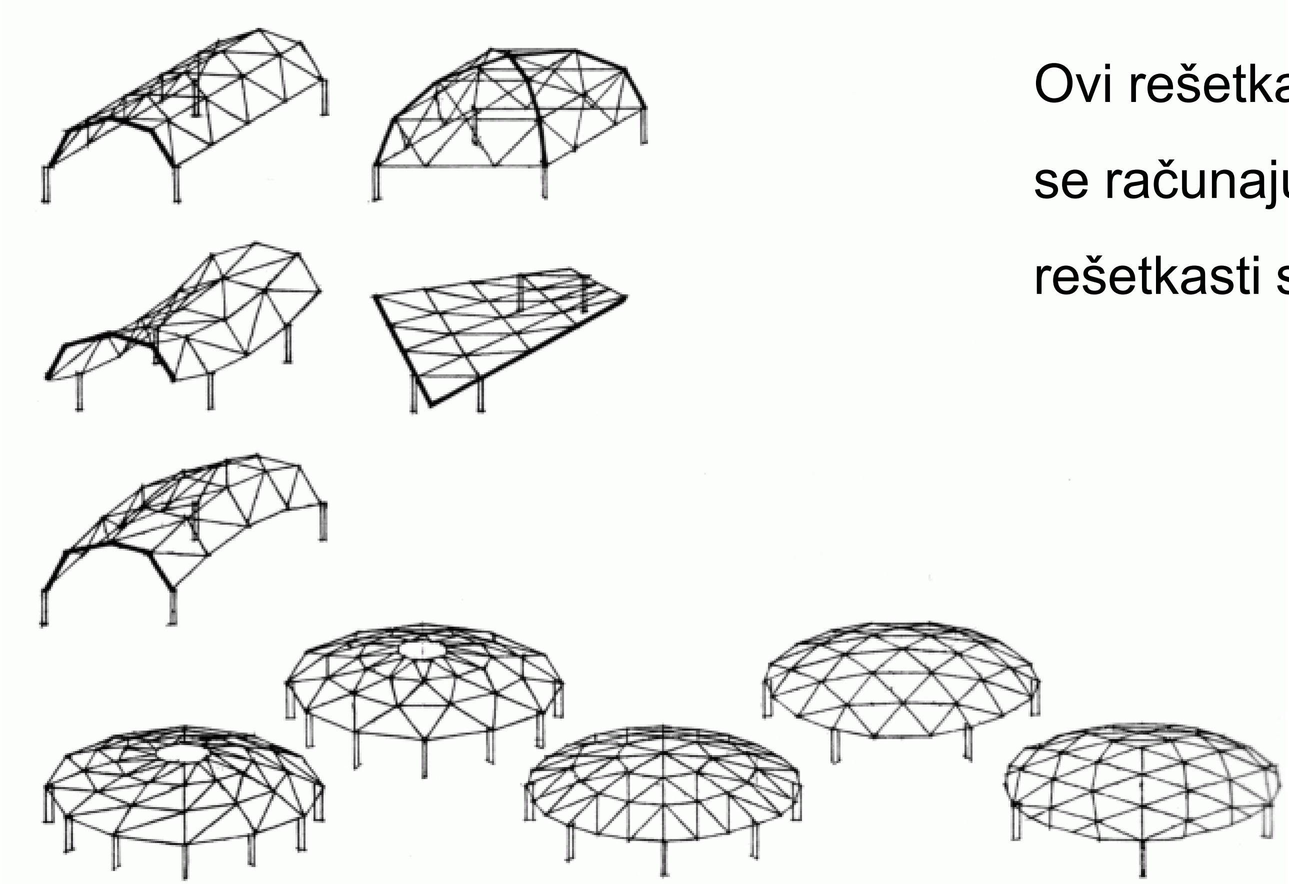
Reš. nosači mogu biti bilo kojeg statičkog sustava, računaju se kombinacijom postupaka za st. sustav - (reakcije) i rešetku (štapovi).

PRORAČUN REŠETKASTIH NOSAČA



Ovi rešetkasti nosači koji su dio prostornog nosivog sustava računaju se kao ravninski rešetkasti sustavi.

PRORAČUN REŠETKASTIH NOSAČA

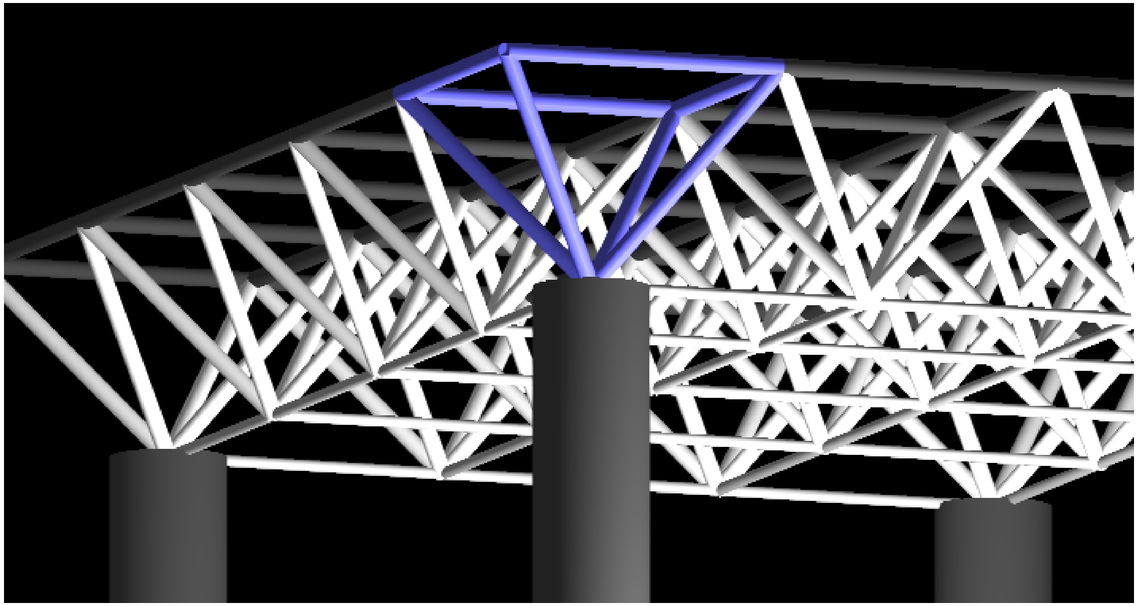


Ovi rešetkasti nosači

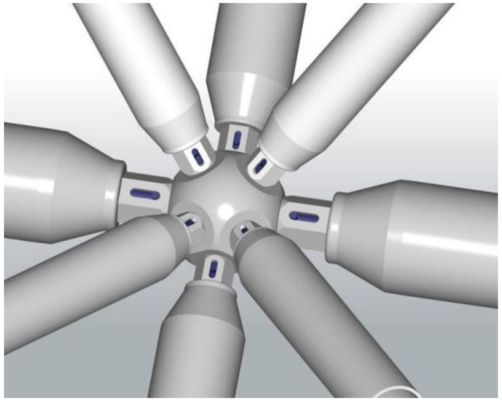
se računaju kao prostorni

rešetkasti sustavi.

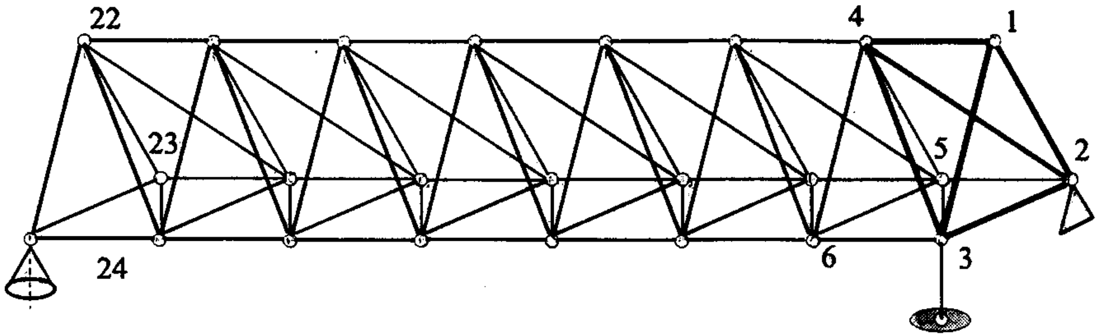
PROSTORNI REŠETKASTI NOSAČI



Najpoznatiji prostorni sustavi rešetki su mero sustavi.



PROSTORNI REŠETKASTI NOSAČI



Dokaz geometrijske nepromjenjivosti prostornih sustava:

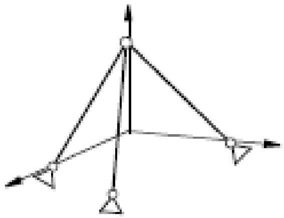
$\boxed{S \geq 3*n-š}$ nužan uvjet; dovoljan uvjet pravilan raspored štapova

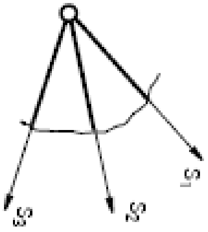
Osnovni geometrijski nepromjenjiv prostorni oblik je tetraedar.
Prostorna rešetka iz tetraedara je stabilna.

PROSTORNI REŠETKASTI NOSAČI

Za čvor u prostoru 3 uvjeta ravnoteže:

$\Sigma X=0$; $\Sigma Y=0$; $\Sigma Z=0$





Iste su metode rješavanja prostornih rešetkastih nosača kao i ravninskih, povećan broj jednadžbi iz kojih se određuju sile u štapovima rešetki.