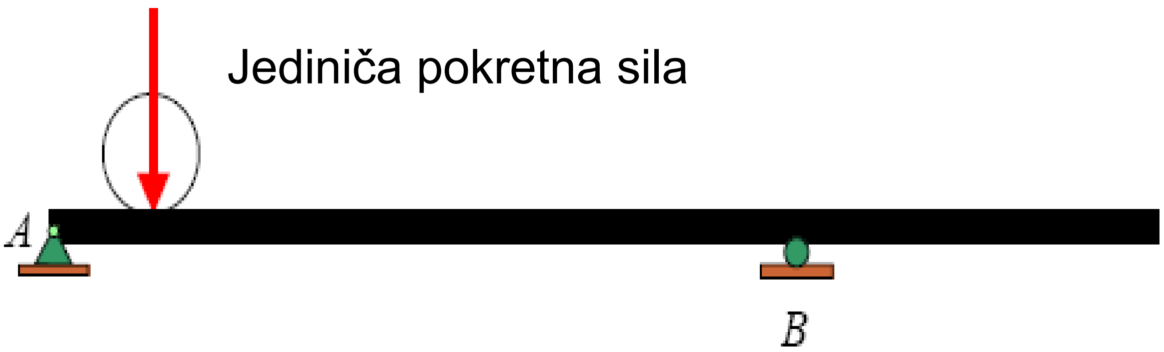
Utjecajne linije

- Važne u proračunu konstrukcija koje su dugo opterećene pokretnim opterećenjem.

-U dosadašnjem radu do ove točke, proučavali smo tehnike analize konstrukcija opterećenim stalnim ili nepomičnim ili mrtvim teretom.

-Ako je konstrukcija opterećena pokretnim opterećenjem promjenu posmičnih sila i momenata najbolje opisujemo utjecajnim linijama.

Utjecajne linije



Definicija utj. linija
Utjecajna linija prikazuje promjenu reakcija, posmičnih sila, momenata ili progiba u određenoj točki nosača kao posljedicu kretanja koncentrirane jedinične sile po štapu.

Određivanje utj. linija-statički i kinematski način.

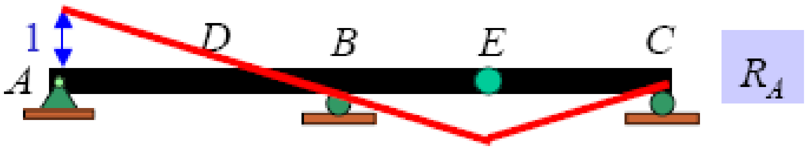
Utjecajne linije

U **statičkom** se **postupku** iz uvjeta ravnoteže (dijela) nosača opterećenoga jediničnom silom u po volji odabranoj točki *x* izvodi se izraz za iznos tražene veličine u obliku funkcije točke *x*.

Kinematski se **postupak** temelji na teoremu o virtualnim pomacima, odnosno, na Bettijevu teoremu uzajamnosti: *utjecajna linija je jednaka progibnoj liniji zamišljenog sistema, nastalog raskidanjem veze koja u izvornom nosaču prenosi dotičnu veličinu, ako se na mjestu i u smjeru te veličine zada jedinični pomak* (tzv. teorem Mueller-Breslaua).

Utjecajne linije

U.L.-pravci



Statički određen sistem

U.L.-krivulje

Statički neodređen sistem

Utjecajne linije

- Kada je nacrtana utjecajna linija vrlo brzo se može naći položaj pokretnog opterećenja koje će uzrokovati najveći utjecaj na konstrukciji.

- Utjecajne linije prikazuju učinak pokretnog opterećenja samo u određenoj točki štapa, dok dijagrama momenata i posmičnih sila prikazuju učinak nepomičnog opterećenja u svim točkama duž štapa.

Utjecajne linije

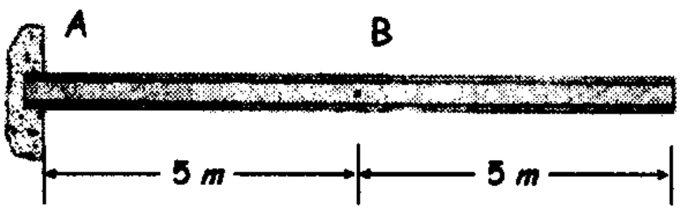
Statički postupak određivanja utjecajnih linija

u točki P za neku funkciju (reakciju, posmičnu silu ili momenat).

1. Postavim jedinično opterećenje (opterećenje čija je veličina =1) u točki x, od dužine štapa.
2. Koristim jednadžbe ravnoteže za pronaći veličinu tražene funkcije (reakcije, T ili M) u određenoj točki P uslijed koncentrirane sile u X.
3. Ponavljam korak 1 i 2 za različite vrijednosti x cijelog raspona grede.
4. Crtam veličine R, T ili M za posmatrani element konstrukcije.

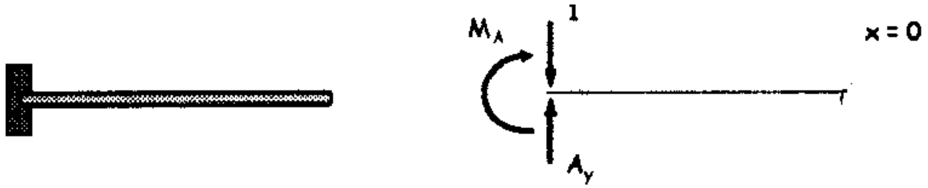
U.L. reakcije A

Konstruiranje u.l. za reakcije, posmične sile i momente
u dvije točke –A i B, na desnoj konzoli.



U.L. za vertikalnu reakciju A

1.



$$\sum F_y = 0 = A_y - 1$$

$$\boxed{A_y = 1}$$

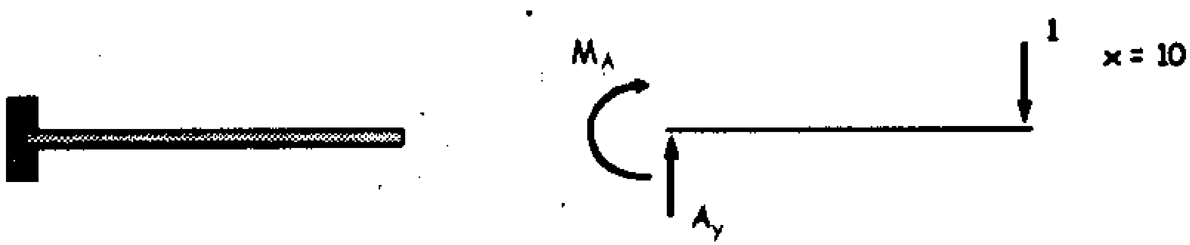
U.L. reakcije A

2.

$$\sum F_y = 0 = A_y - 1$$

$$\boxed{A_y = 1}$$

3.

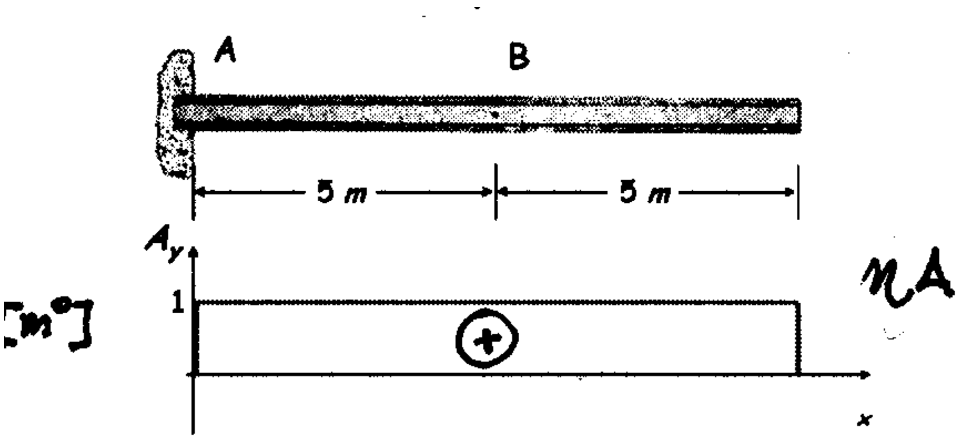


$$\sum F_y = 0 = A_y - 1$$

$$\boxed{A_y = 1}$$

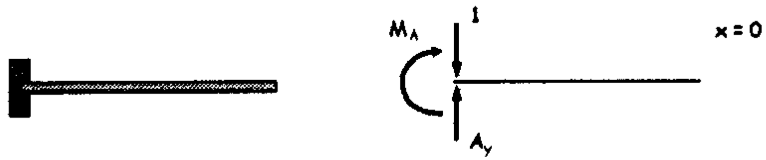
U.L. reakcije A

4.



U.L. REAKTIVNOG MOMENTA U A

1.



$$\sum M_A = 0 = -M_A - 1(0\ m) \qquad \boxed{M_A = 0}$$

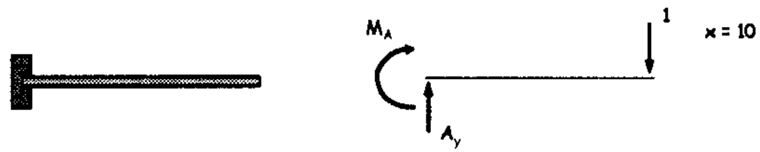
U.L. REAKTIVNOG MOMENTA U A

2.

$$\sum M_A = 0 = -M_A - 1(3\ m)$$

$$\boxed{M_A = -3\ m}$$

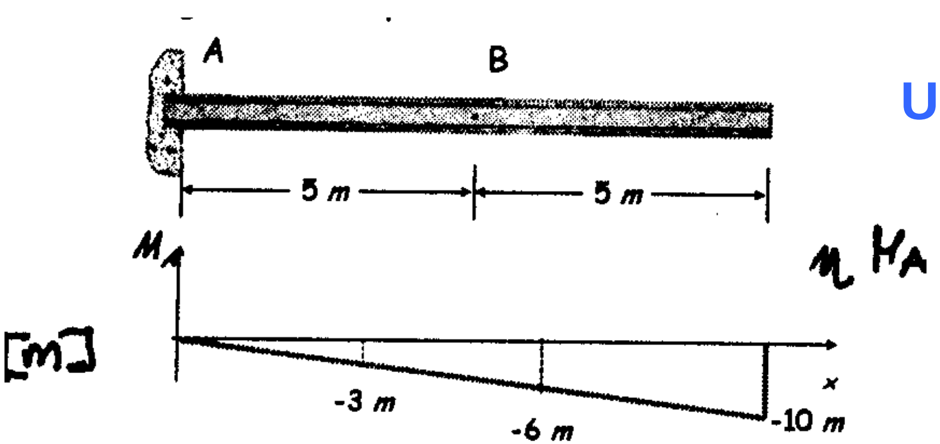
3.



$$\sum M_A = 0 = -M_A - 1(10\ m)$$

$$\boxed{M_A = -10\ m}$$

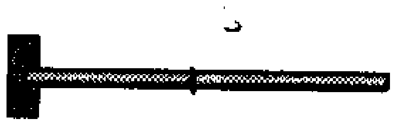
4.

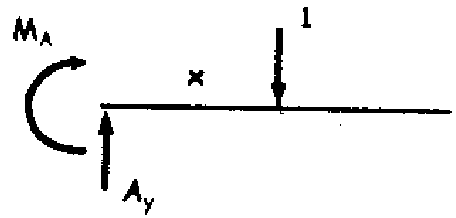


U.L. MA

U.L. ZA T U B

1.

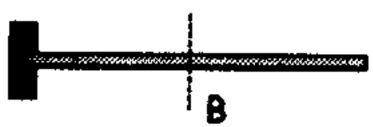


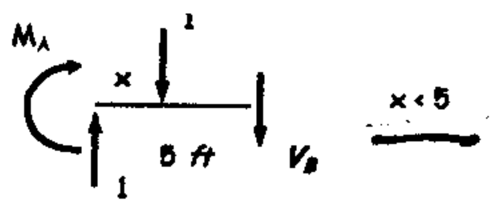


$$\sum F_y = 0 = A_y - 1$$

$$\boxed{A_y = 1}$$

2.



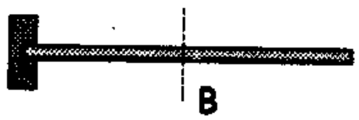


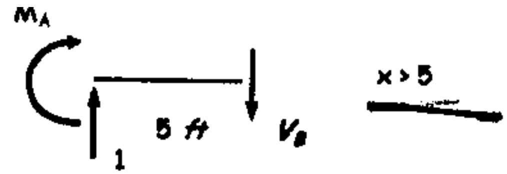
$$\sum F_y = 0 = V_B - 1 + 1$$

$$\boxed{V_B = 0}$$

U.L. ZA T U B

3.



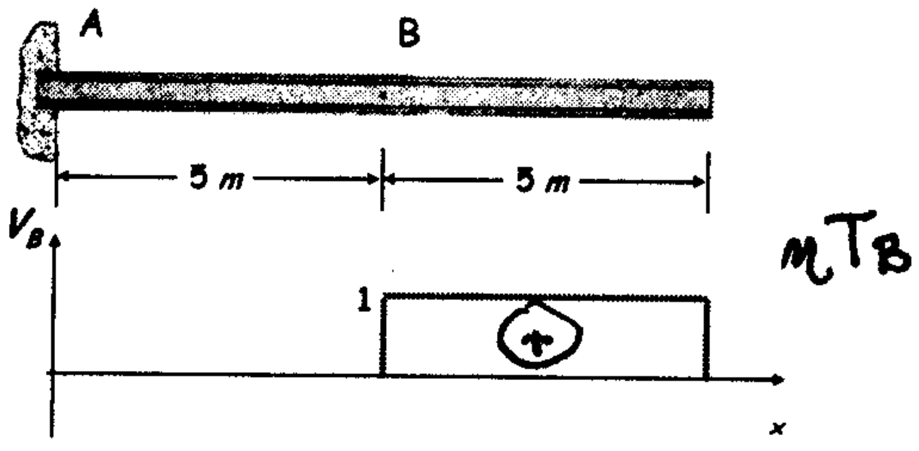


$$\sum F_y = 0 = -V_B + 1$$

$$\boxed{V_B = 1}$$

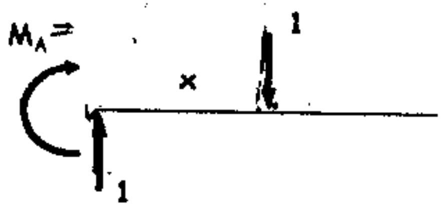
U.L. ZA T_B

4.



U.L. ZA M U B

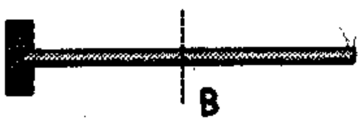
1.

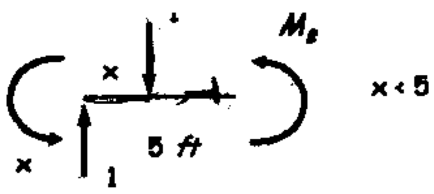


$$\sum F_y = 0 = -M_A - 1x$$

$$\boxed{M_A = -x}$$

2.



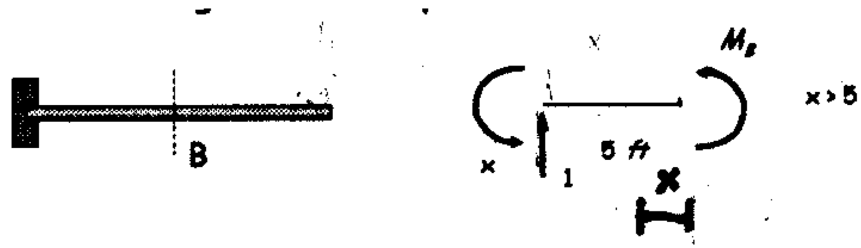


$$\sum M_{cut} = 0 = M_B + x - 5 + (5 - x)$$

$$\boxed{M_B = 0}$$

U.L. ZA M U B

3.

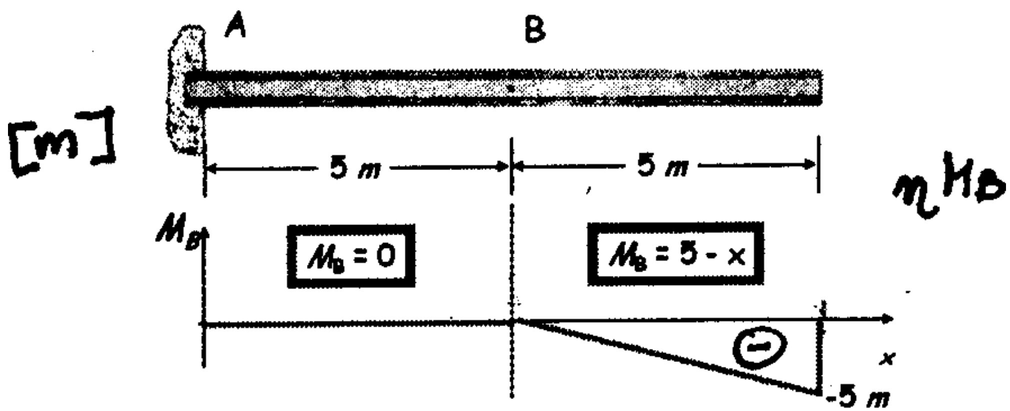


$$\sum M_{cut} = 0 = M_B + x - 5$$

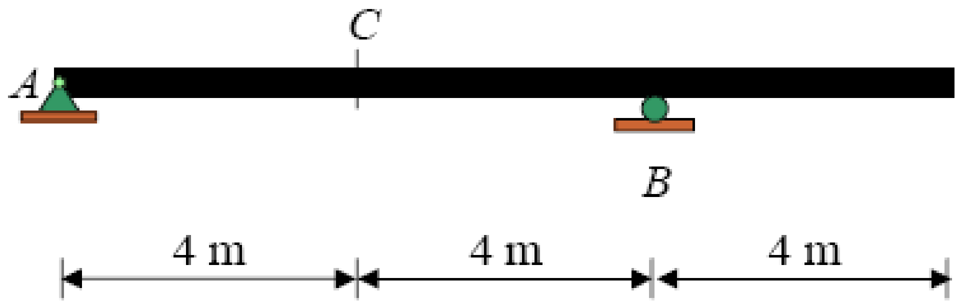
$$\boxed{M_B = 5 - x}$$

4.

U.L. ZA M_B



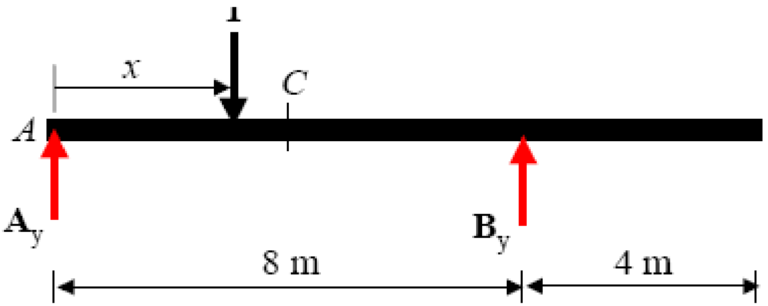
GREDA S PREPUSTOM



Primjenit ćemo tkzv. tablični pristup

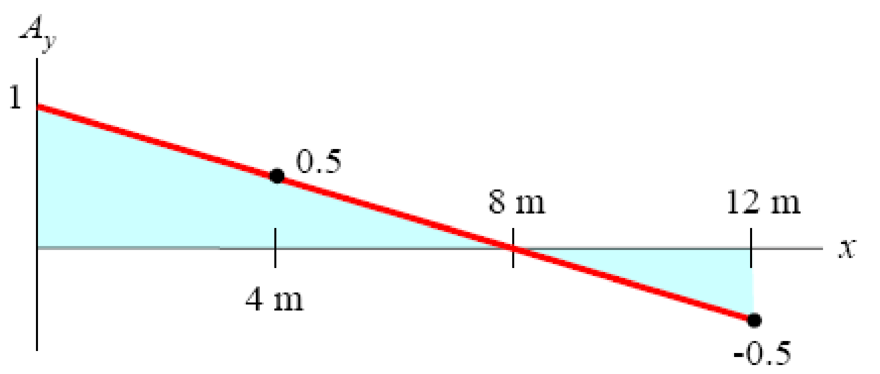
GREDA S PREPUSTOM

REAKCIJA U A



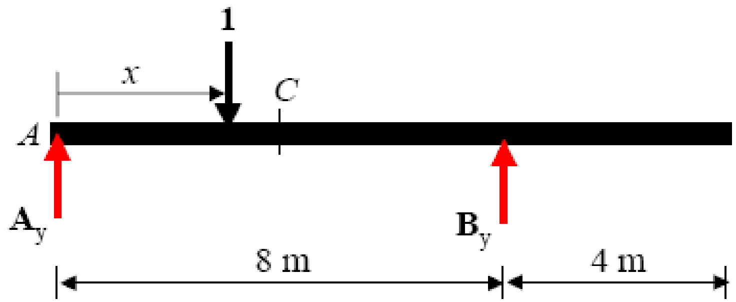
x	A_y
0	1
4	0.5
8	0
12	-0.5

$$+\circlearrowright \Sigma M_B = 0: \qquad -A_y(8) + 1(8-x) = 0, \qquad A_y = 1 - \frac{1}{8}x$$



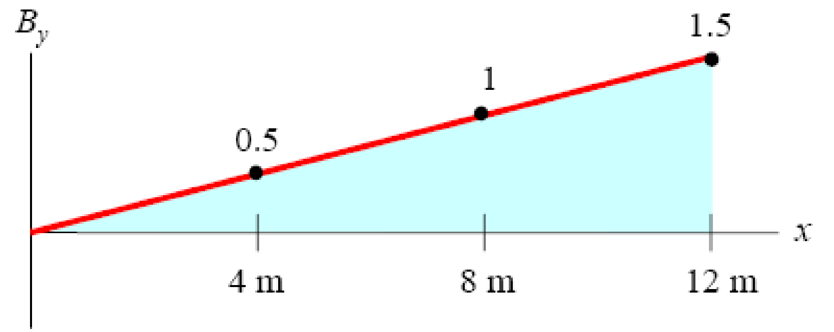
GREDA S PREPUSTOM

REAKCIJA U B



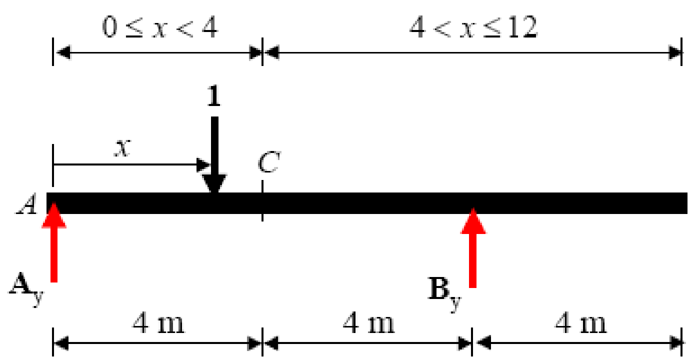
x	B_y
0	0
4	0.5
8	1
12	1.5

$$+\circlearrowleft \Sigma M_A = 0: \qquad B_y(8) - 1x = 0, \qquad B_y = \frac{1}{8}x$$

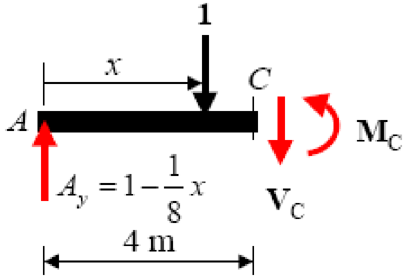


GREDA S PREPUSTOM

T SILA U C



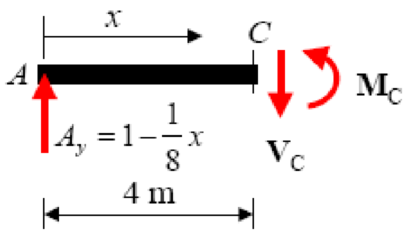
$0 \le x \le 4$



$$+\uparrow \Sigma F_y = 0: \quad 1 - \frac{1}{8}x - 1 - V_C = 0$$

$$V_C = -\frac{1}{8}x$$

$4 < x \le 12$

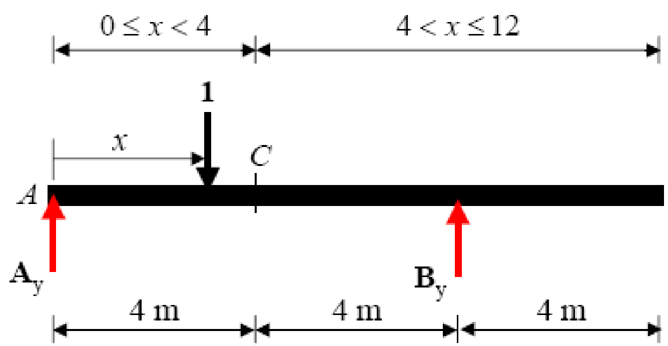


$$+\uparrow \Sigma F_y = 0: \quad 1 - \frac{1}{8}x - V_C = 0$$

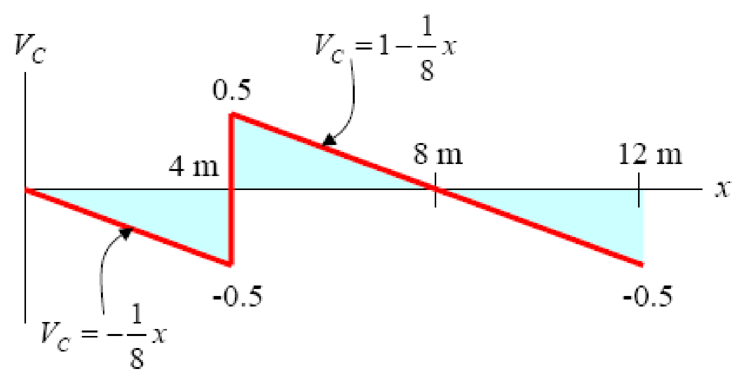
$$V_C = 1 - \frac{1}{8}x$$

GREDA S PREPUSTOM

T SILA U C

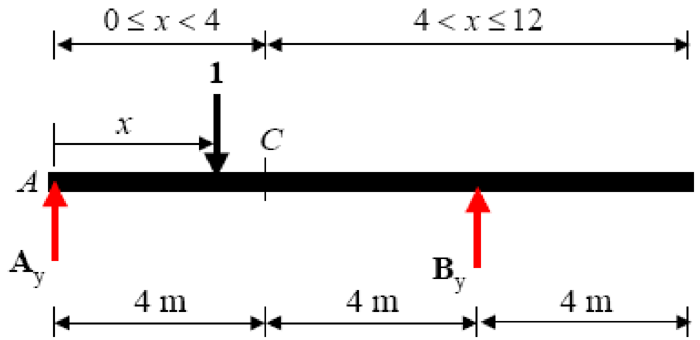


	x	V_C
$V_C = -\frac{1}{8}x$	0	0
	4^-	-0.5
$V_C = 1 - \frac{1}{8}x$	4^+	0.5
	8	0
	12	-0.5

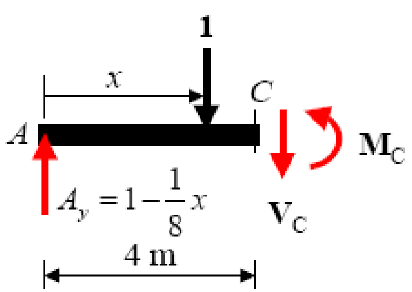


GREDA S PREPUSTOM

MOMENT U C



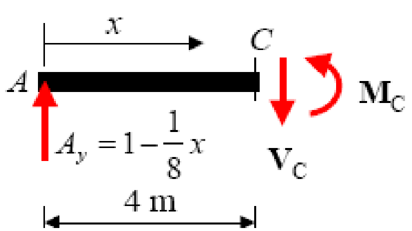
$0 \le x \le 4$



$$\circlearrowleft + \ \Sigma M_C = 0: \quad M_C + 1(4 - x) - (1 - \frac{1}{8}x)(4) = 0$$

$$M_C = \frac{1}{2}x$$

$4 < x \le 12$

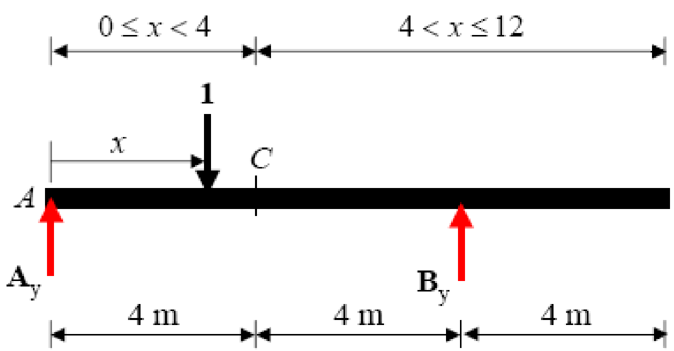


$$\circlearrowleft + \ \Sigma M_C = 0: \quad M_C - (1 - \frac{1}{8}x)(4) = 0$$

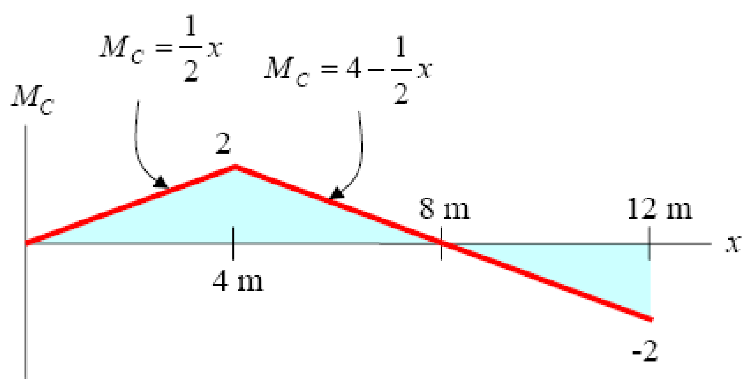
$$M_C = 4 - \frac{1}{2}x$$

GREDA S PREPUSTOM

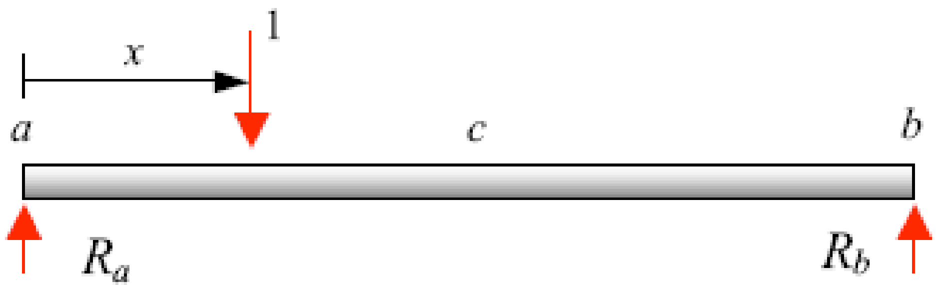
MOMENT U C



	x	M_C
$M_C = \frac{1}{2}x$	0	0
	4	2
$M_C = 4 - \frac{1}{2}x$	8	0
	12	-2



PROSTA GREDA

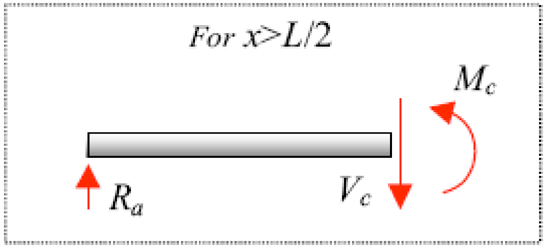


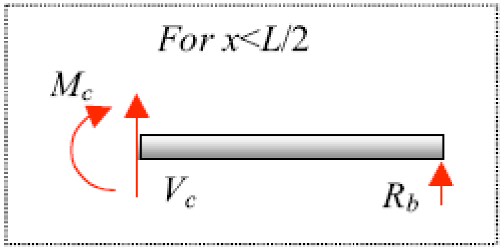
$\Sigma M_b = 0 \Rightarrow R_a = (L-x)/L$ $\Sigma M_a = 0 \Rightarrow R_b = x/L$

$\eta \; R_a$

$\eta \; R_b$

PROSTA GREDA





Sila desno od presjeka:

: $x>L/2$

$V_c = R_a$

$M_c = R_a L/2$

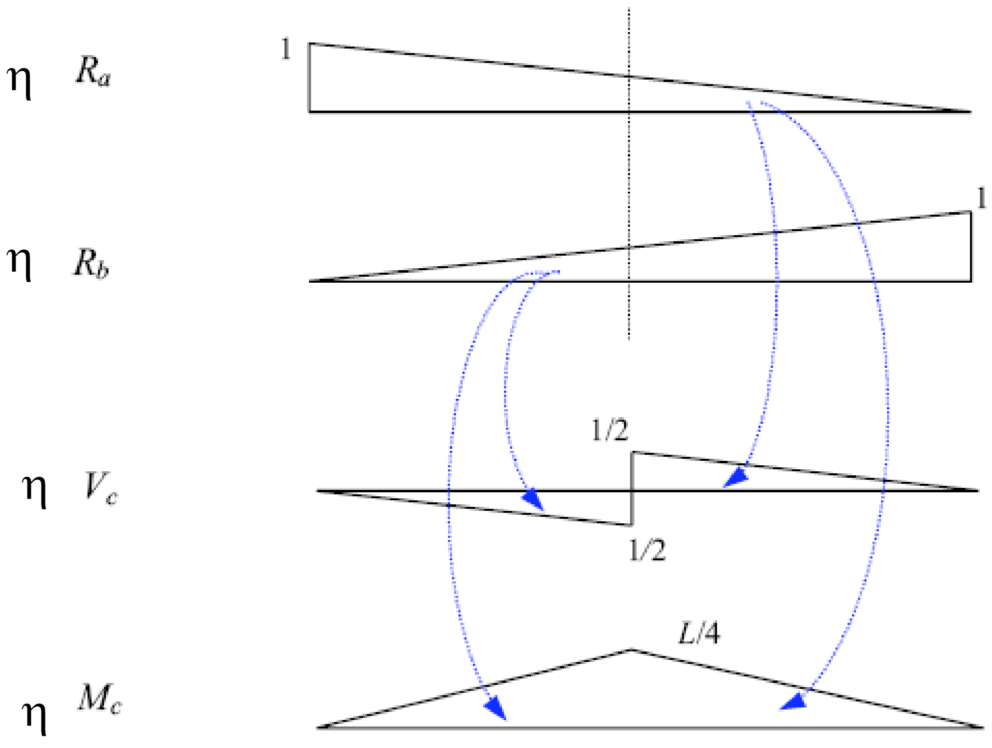
Sila lijevo od presjeka:

$x<L/2$

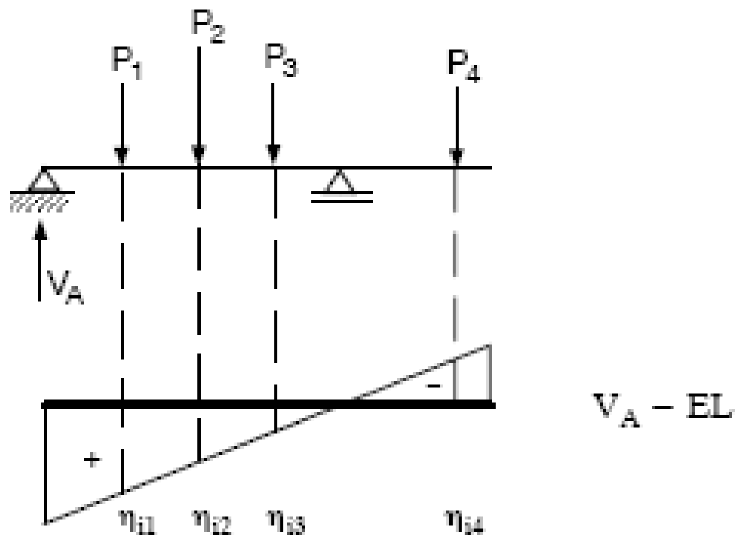
$V_c = -R_b$

$M_c = R_b L/2$

PROSTA GREDA



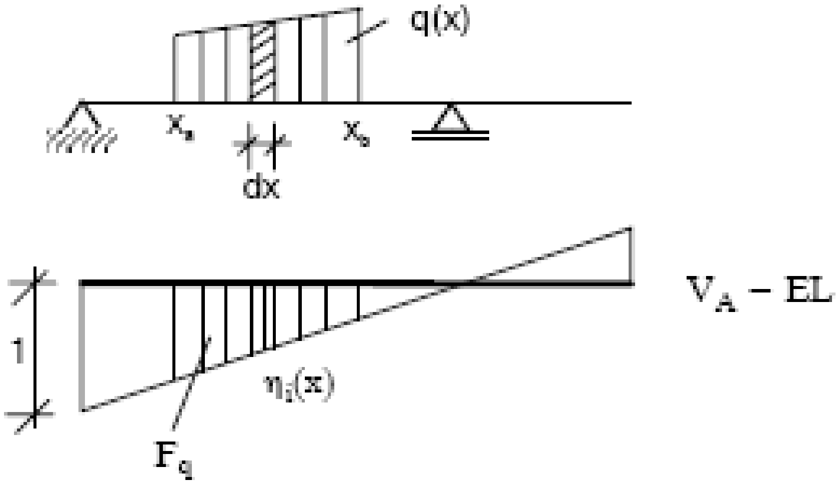
KORIŠTENJE U.L.



$$Z_i = \eta_{i1} P_1 + \eta_{i2} P_2 + \eta_{i3} P_3 + \eta_{i4} P_4$$

$$= \sum_m \eta_{im} P_m$$

KORIŠTENJE U.L.



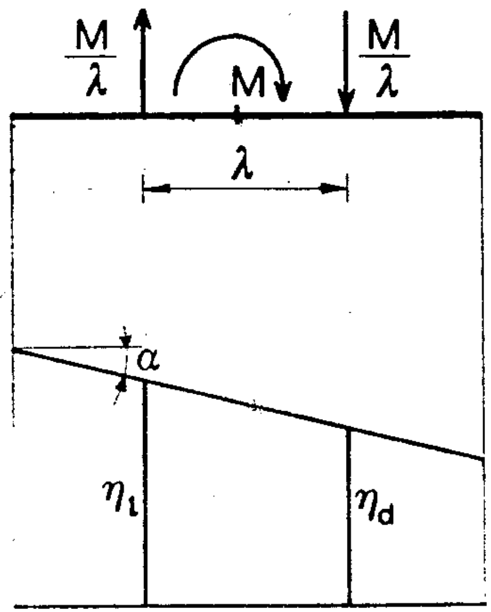
$$Z_i = \int_{x_a}^{x_b} \eta_i(x)\, q(x)\, dx$$

$q(x) = q = \text{konst.}$

$$Z_i = q \int_{x_a}^{x_b} \eta_i(x)\, dx = q\, F_q$$

Površina ispod u.l.

KORIŠTENJE U.L.



$$U_4 = \frac{M}{\lambda} \cdot \eta_l - \frac{M}{\lambda}\,\eta_d = M\,\frac{\eta_l - \eta_d}{\lambda}$$

odnosno:

$$U_4 = M \cdot \mathrm{tg}\alpha$$

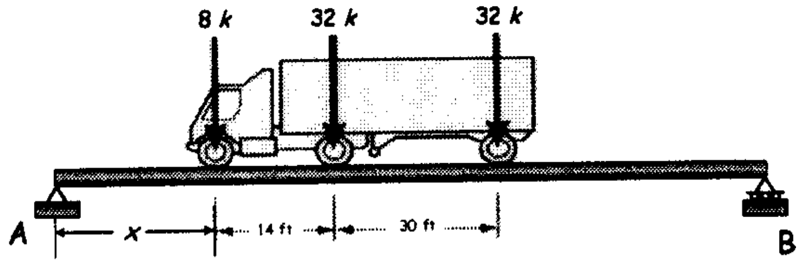
KRITIČNO OPTEREĆENJE

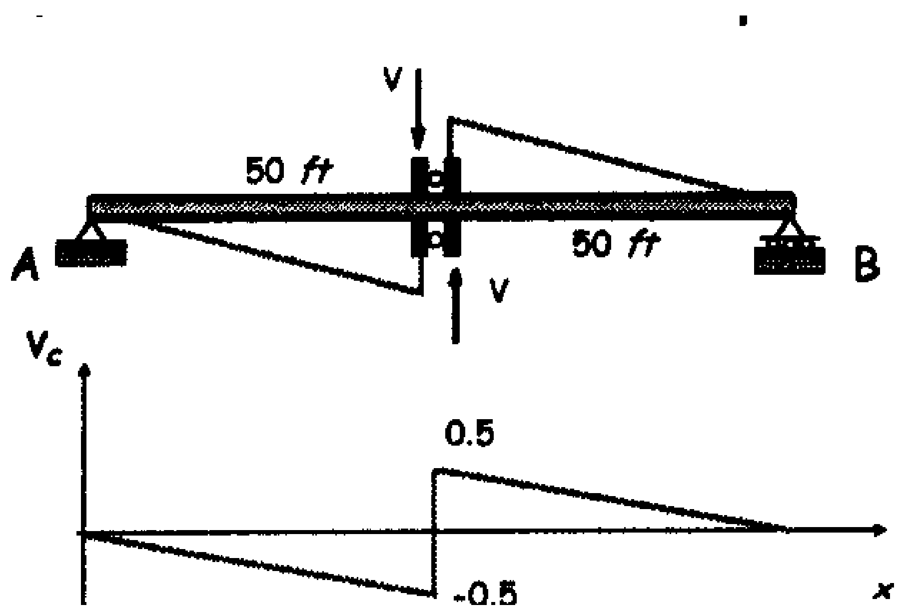
-Opterećenje koje je najveće-koje izračunamo tkzv. postupkom probanja je **kritično opterećenje**, a sila koja je tada na vrhu u.l. je **kritična sila**.

-Ako imamo zadano opterećenje i nacrtanu u.l. traži se položaj opterećenja za koji je veličina utjecaja ekstremna-prema tom utjecaju se dimenzionira.
-Ako imamo više sila koje se kreću na konstrukciji za koju je nacrtana u.l. ekstrem nekoga utjecaja pomoću njegove u.l. je moguće dobiti samo ako je jedna od sila na vrhu u.l.

-Samo 1 takav položaj postoji na konstrukciji.

KRITIČNO OPTEREĆENJE

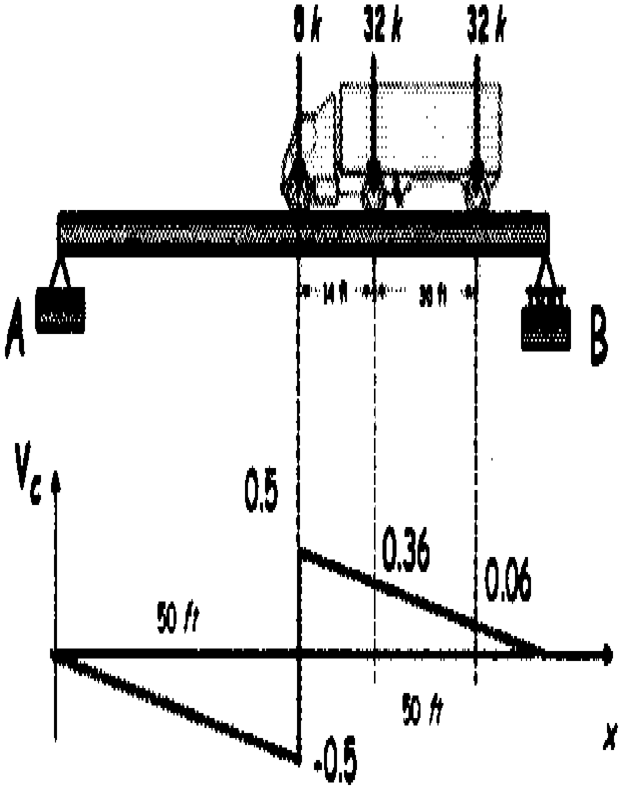




η Tc

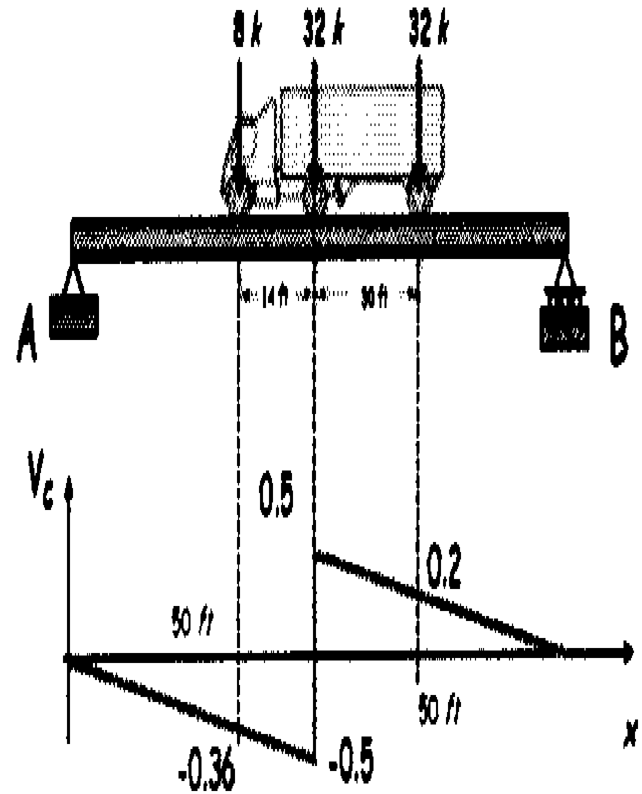
KRITIČNO OPTEREĆENJE

1.



$$(V_C)_{Case\ 1} = 8k(0.5) + 32k(0.36) + 32k(0.06) = 17.44k$$

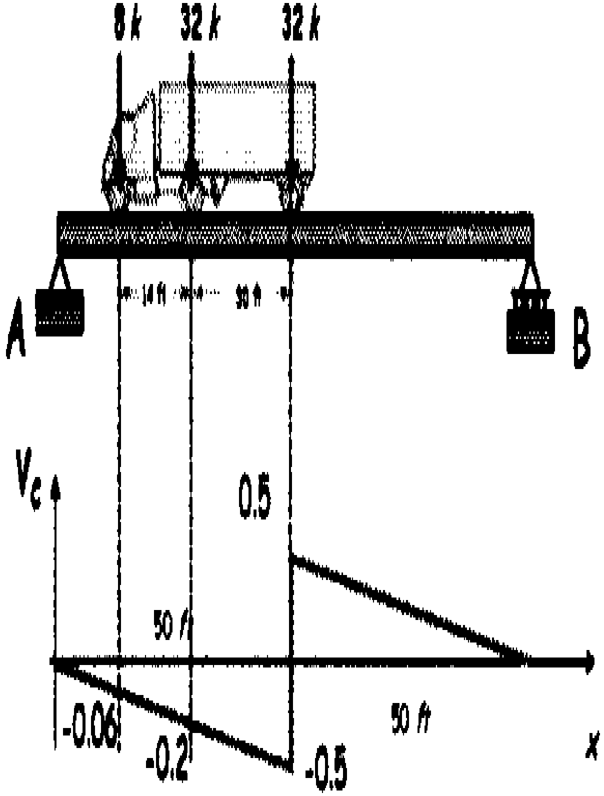
2.



$$(V_C)_{Case\ 2} = 8k(-0.36) + 32k(0.5) + 32k(0.2) = 19.52k$$

KRITIČNO OPTEREĆENJE

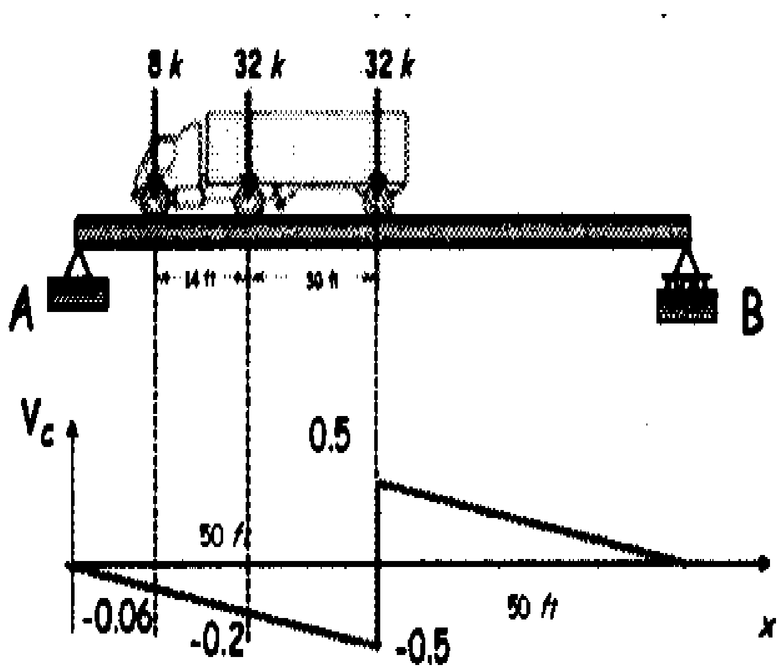
3.



$$(V_C)_{Case\ 3} = 8k(-0.06) + 32k(-0.3) + 32k(0.5) \;=\; 9.12k$$

Maksimalna pozitivna poprečna sila je 19,52 kN.

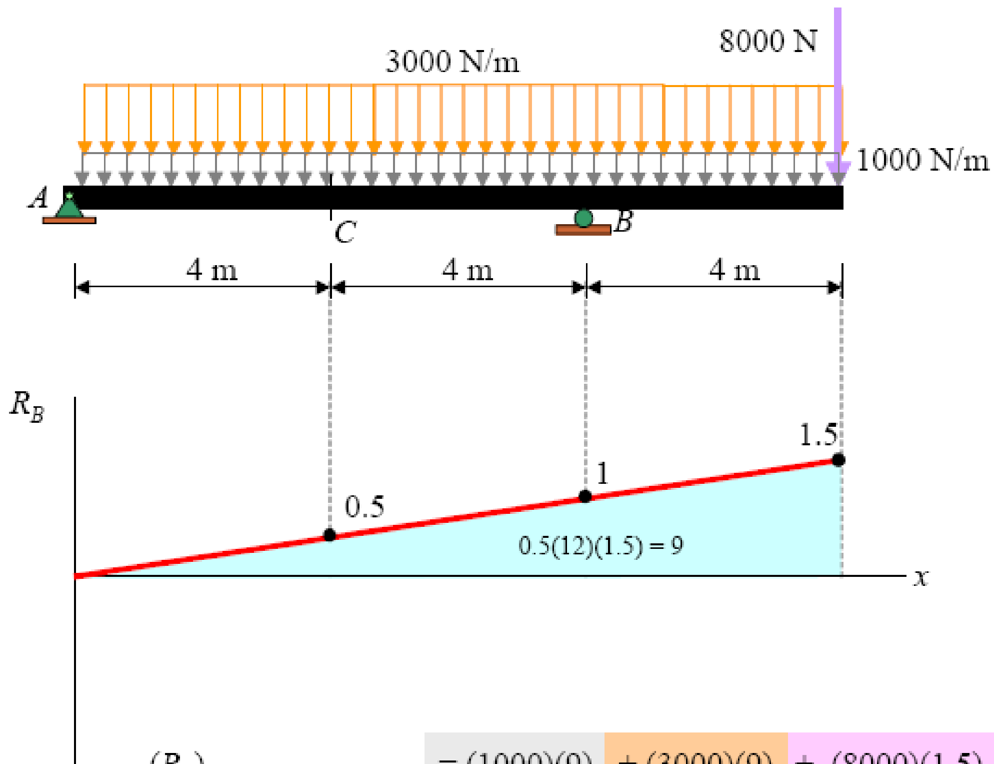
KRITIČNO OPTEREĆENJE



$$(V_C)_{Case\ 3} = 8k(-0.06) + 32k(-0.3) + 32k(-0.5) = -22.88k$$

Maksimalna negativna pop. sila je -22,88 kN.

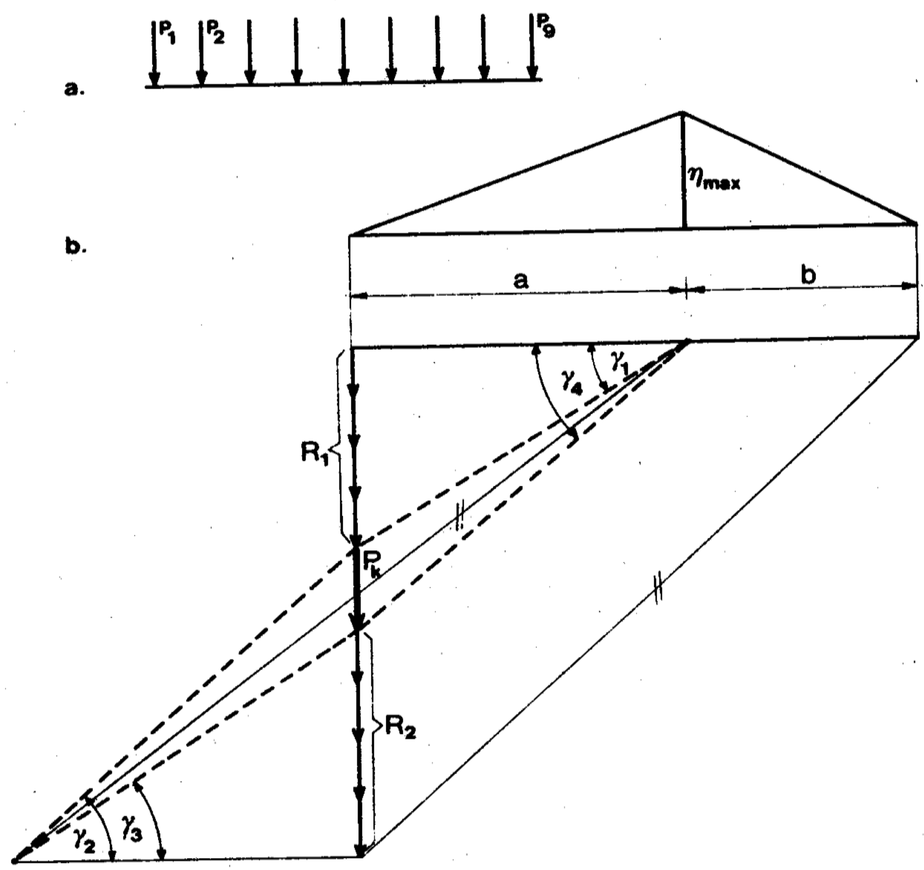
KRITIČNO OPTEREĆENJE



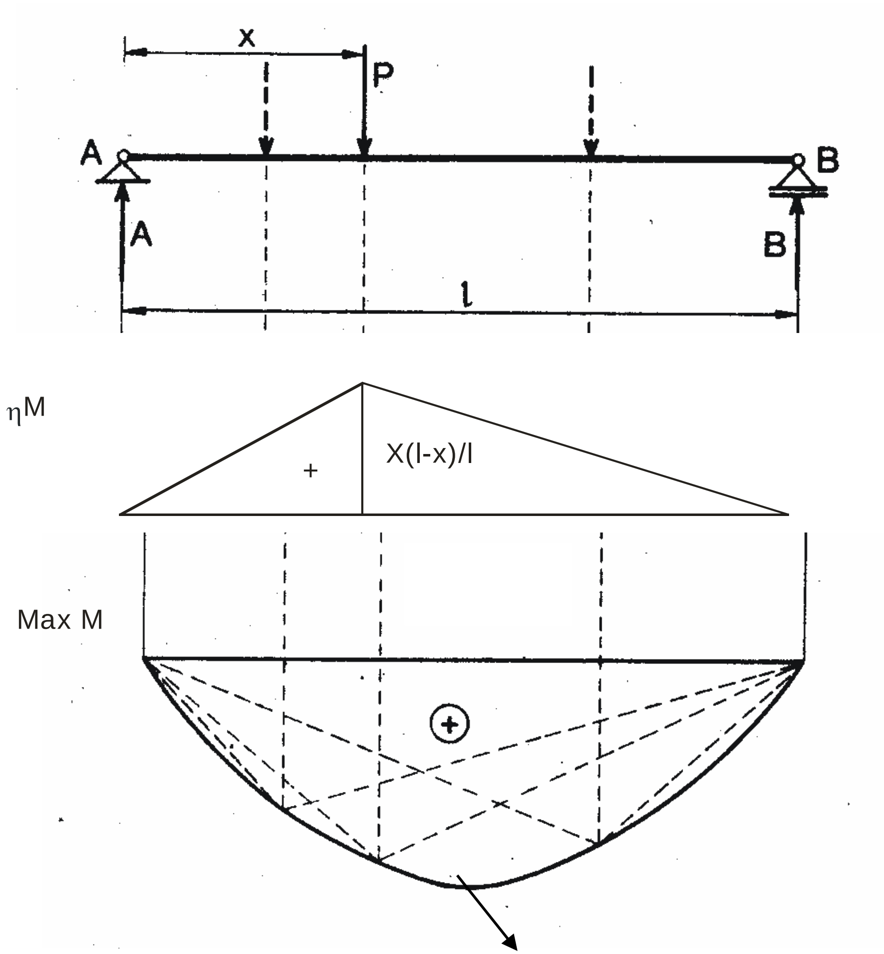
$$(R_B)_{max} = (1000)(9) + (3000)(9) + (8000)(1.5)$$

$$= 48000 \text{ N} = 48 \text{ kN}$$

KRITIČNO OPTEREĆENJE



Dijagrami ekstremnih un. sila



max M =P*ηmax= P*(l-x)*x/l.

max
M(l/2)=P*l/4

Dijagrami ekstremnih un. sila