Građevna statika 2 (21093) | Metoda proračuna 2/3
Akademska godina 2020./2021.

Metoda pomaka

Uvod, objašnjenje postupka, osnovni proračun, utjecaj prisilnih pomaka, utjecaj temperaturnih promjena

Doc. dr. sc. Marin Grubišić, mag. ing. aedif.

Sveučilište u Osijeku (UNIOS)
Građevinski i arhitektonski fakultet Osijek (GrAFOS)
Zavod za tehničku mehaniku (ZTM)
Katedra/Laboratorij za eksperimentalnu mehaniku
Vladimira Preloga 3, **Ured II.26**, HR–31 000 Osijek, Hrvatska

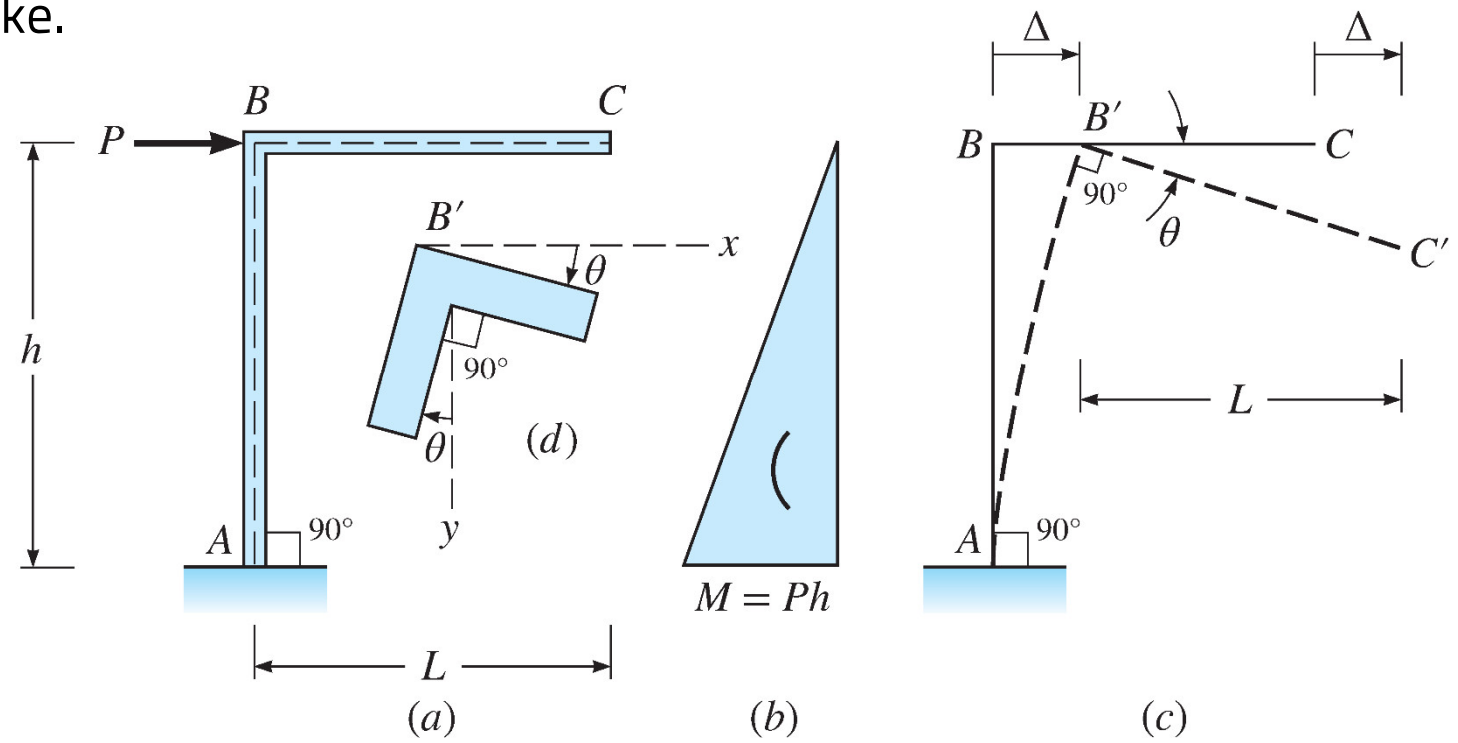
marin.grubisic@gfos.hr

Konzultacije: **srijedom 8:00 — 9:00 sati**
Google Classroom: **qmvjpo6**

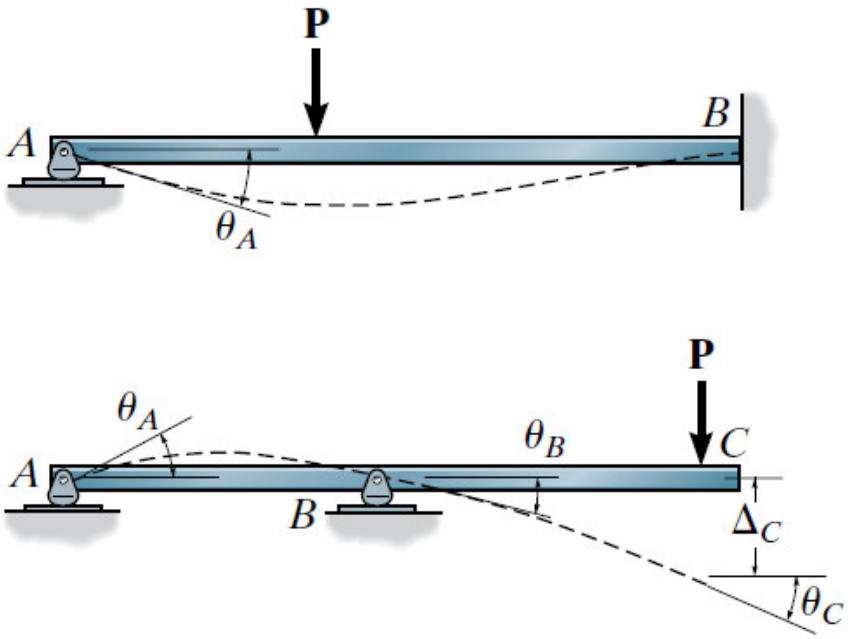
Metoda pomaka

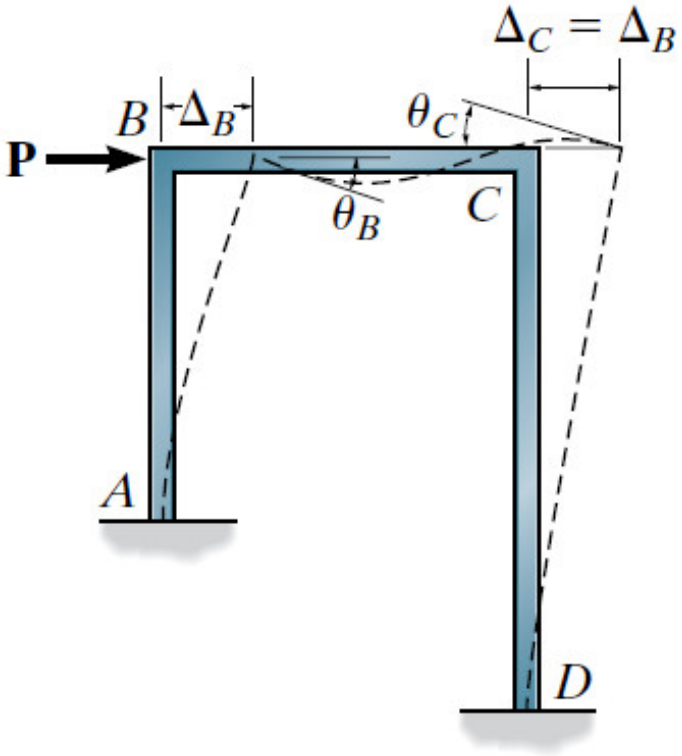
Metoda proračuna statički neodređenih sustava u kojoj su **nepoznanice translacijski i rotacijski pomaci** odabranih točaka nosača koje nazivamo **čvorovima.**

Svi štapovi koji su kruto spojeni u jednom čvoru imaju u njemu jednake pomake.

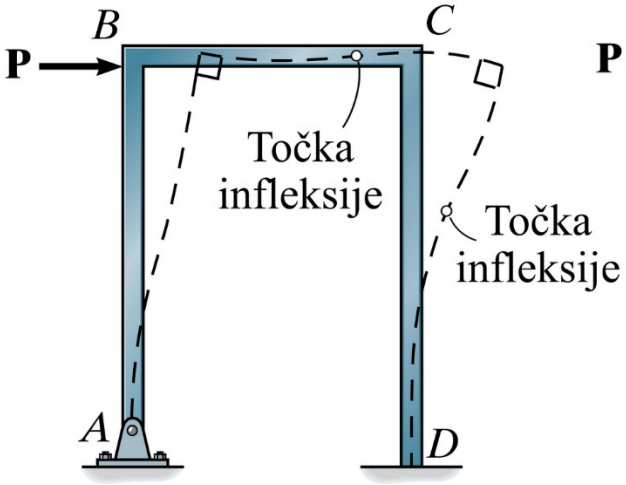


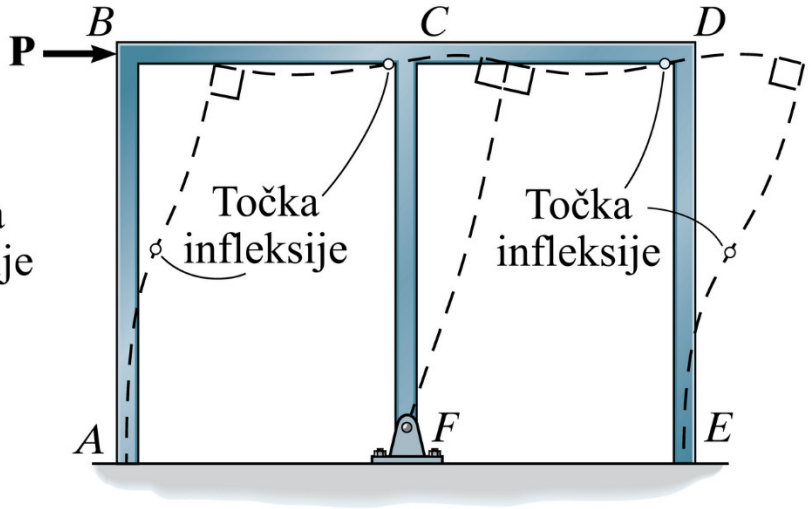
Metoda pomaka — Pomaci sustava

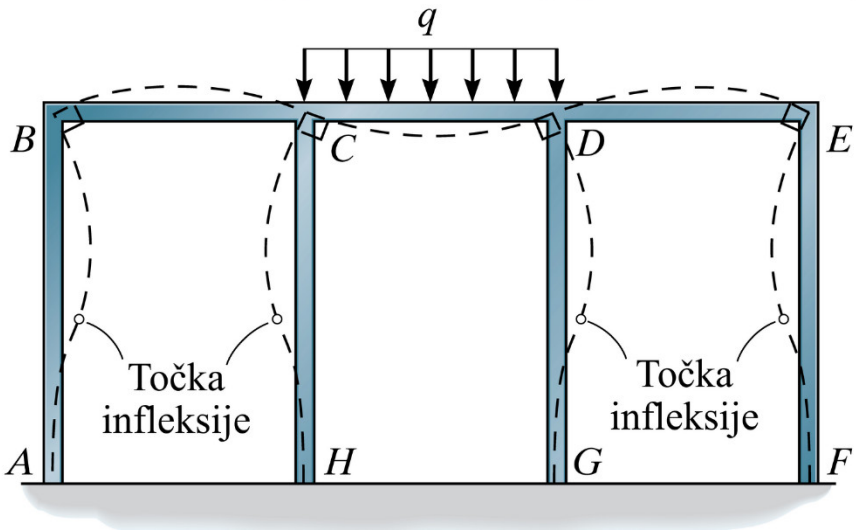




Metoda pomaka — Pomaci sustava

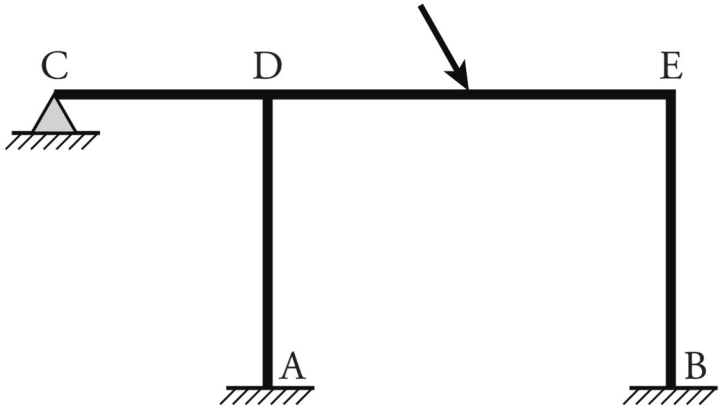


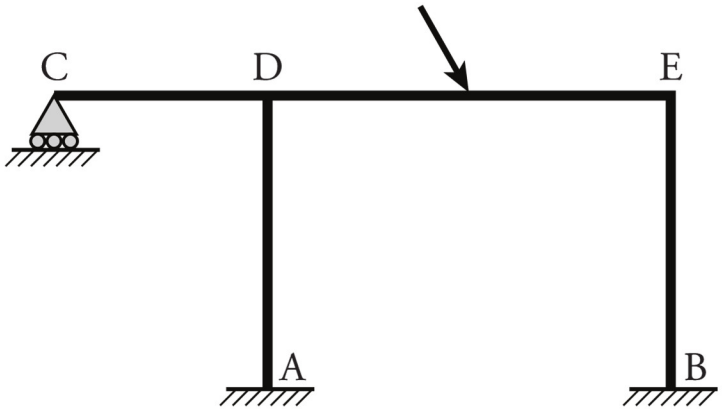


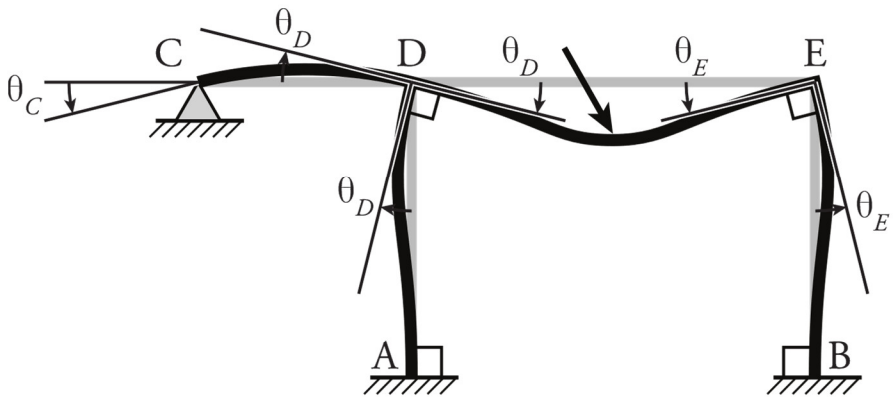


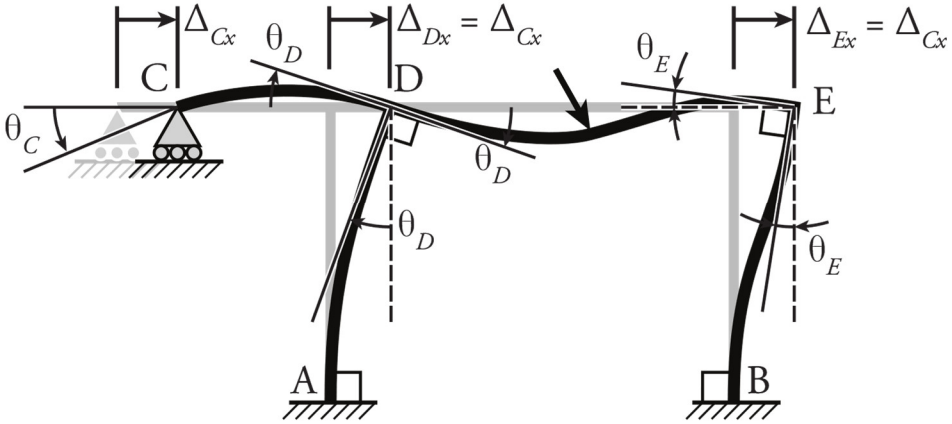
Svaki ravninski čvor ima **dva translatorna pomaka** (u_i,v_i) i **rotacijski pomak (rotacija / kut zaokreta)** θ_i

Metoda pomaka — Pomaci sustava

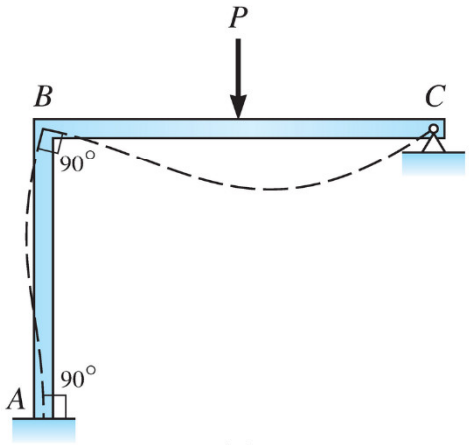


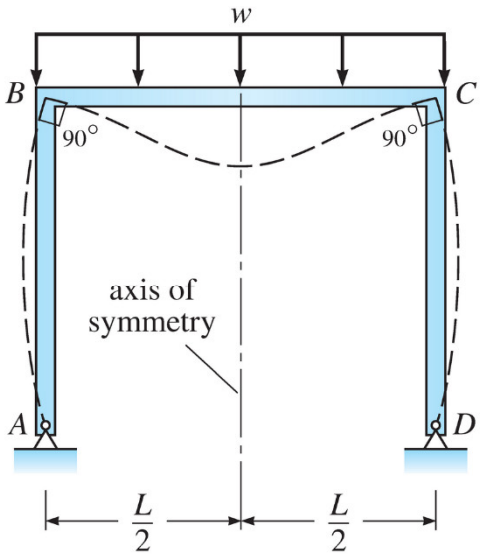


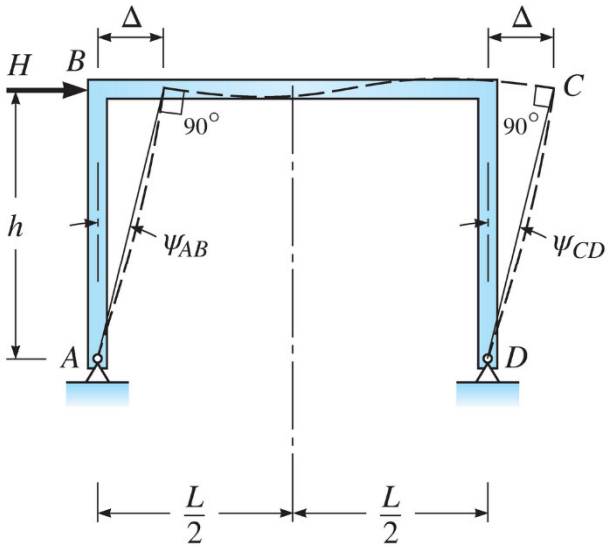


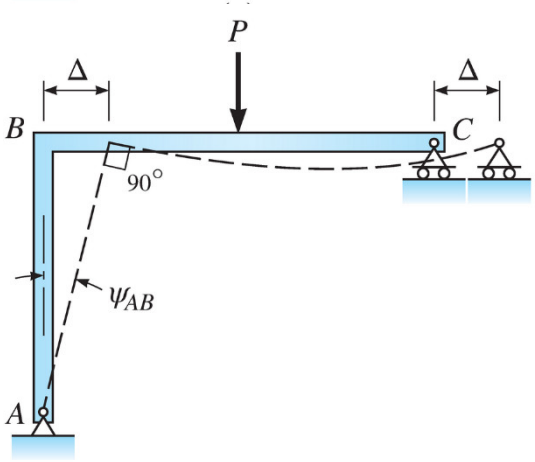


Metoda pomaka — Pomaci sustava









Metoda pomaka — Pretpostavke

Točna metoda pomaka

- uzima sva tri pomaka jednog čvora za nepoznanice
- rijetko se primjenjuje zbog velikog broja nepoznanica

Inženjerska metoda pomaka (IMP)

- zanemaruje uzdužne deformacije elemenata $EA \approx \infty$
- nepoznanice rotacije (kutovi zaokreta) čvorova i neovisni translatorni pomaci (smanjen broj nepoznanica u odnosu na točnu metodu).

Inženjerska metoda pomaka — Postupak proračuna

1. Nepoznanice
2. Proračun krutosti (i rotacija štapova – plan pomaka)
3. **Momenti upetosti**
4. **Jednadžbe momenata na krajevima štapova**
5. **Jednadžbe ravnoteže i /ili rada**
6. Konačni dijagram momenata savijanja

1. Nepoznanice — Rotacija (kut zaokreta) čvorova

$$n = n_r + n_d$$

Rotacije (kutovi zaokreta) čvorova jednak je broju krutih čvorova. Pod krutim čvorom podrazumijevamo svaki čvor u kojem su barem 2 elementa povezana krutom vezom.

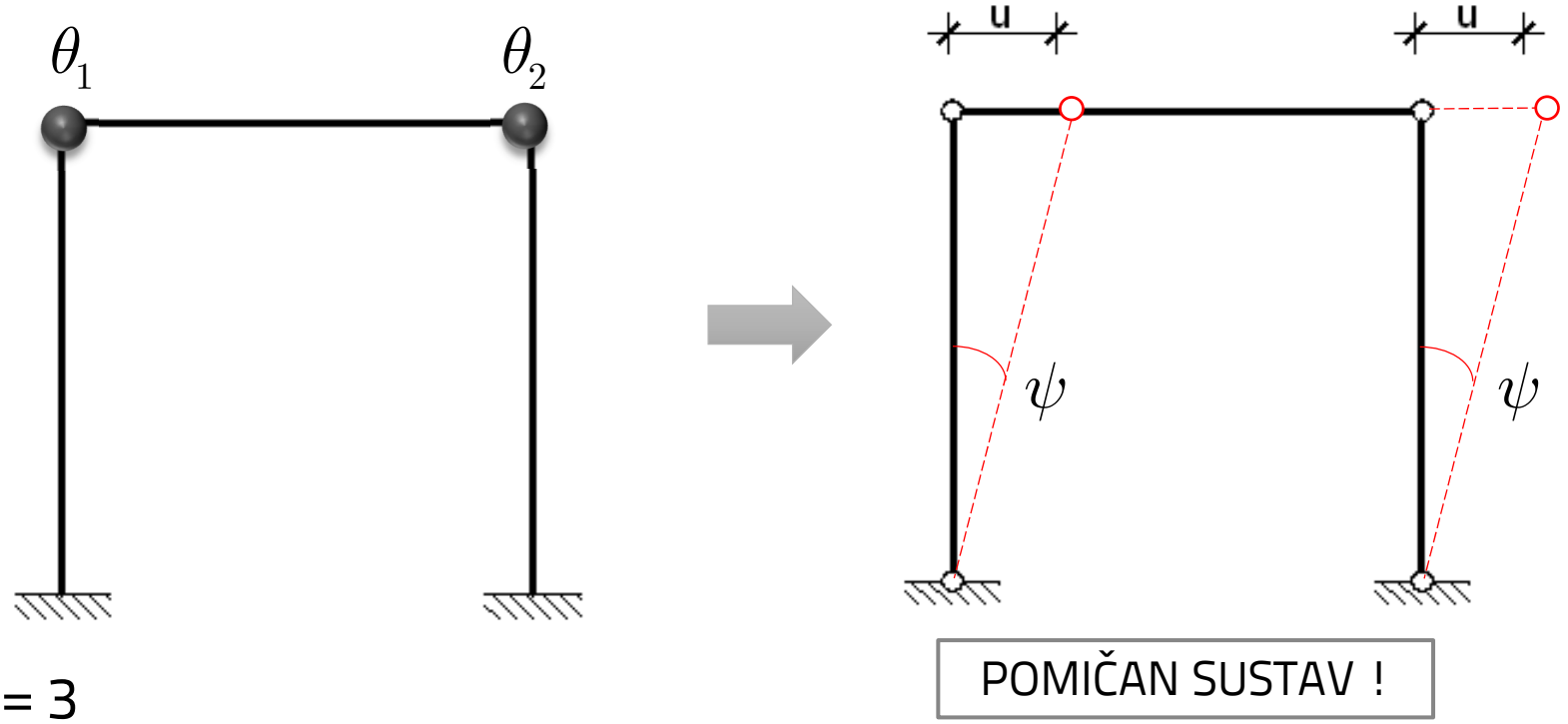
$$\theta_i$$

Neovisni translatorni pomaci

Za određivanje broja neovisnih translatornih pomaka koristi se tzv. **zglobna shema sustava** (stanje slobodnih pomaka čvorova), koju dobivamo tako da sve krute veze u čvorovima pretvorimo u zglobne.

$$u_i \;\&\; v_i$$

Primjer #1 — Određivanje nepoznanica



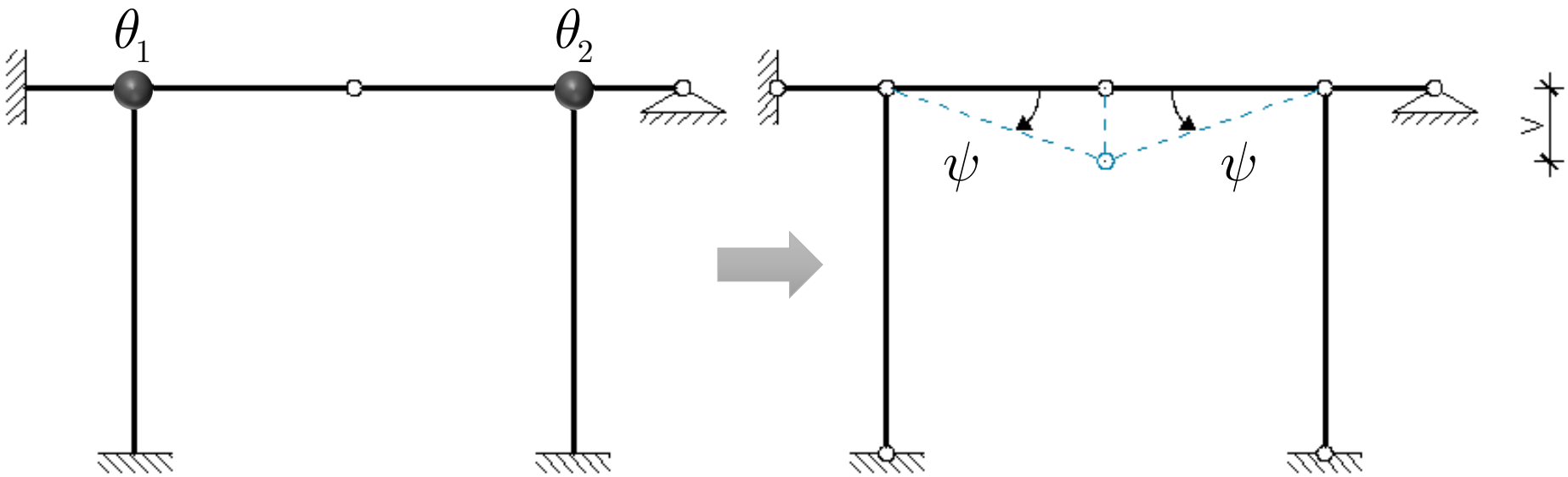
POMIČAN SUSTAV !

MS = 3

TMP = $3 \times 2 = 6$

IMP : n_r=2 ; n_d= $2 \times$ Č – Š = $2 \times 2 - 3 =$ **1** ! $\Rightarrow n = 3$ **u_i** ILI **v_i**

Primjer #2 — Određivanje nepoznanica



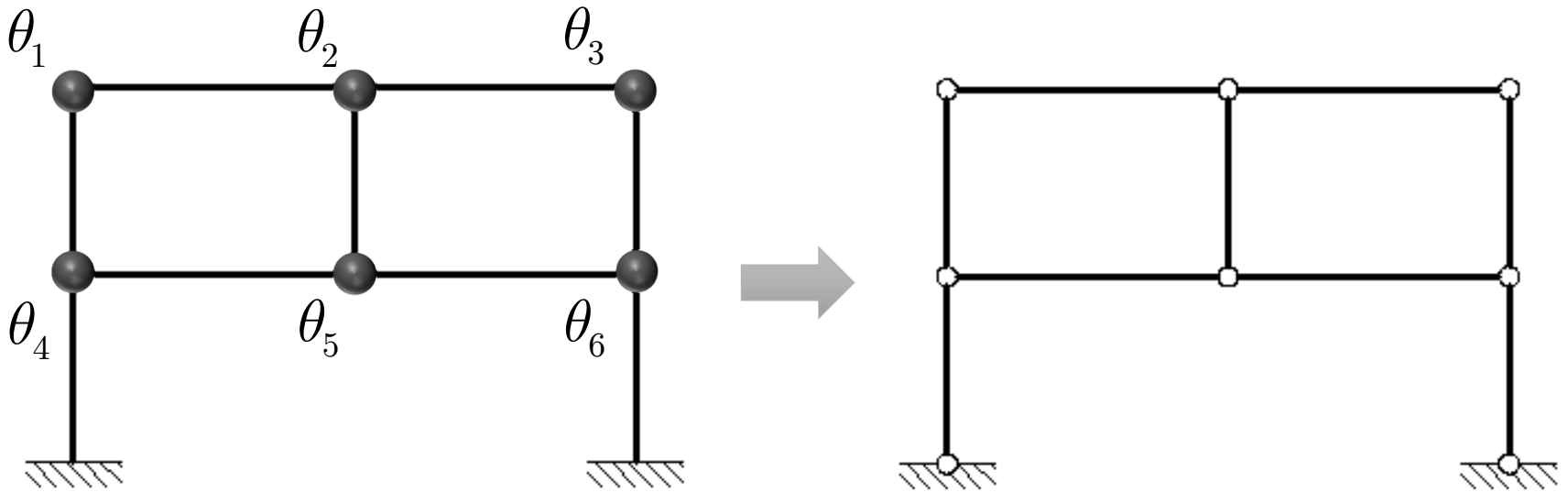
POMIČAN SUSTAV !

MS = 7

TMP = $3 \times 2 = 6$

IMP : $n_r=2$; $n_d= 2 \times \check{C} - \check{S} = 2 \times 3 - 6 = \mathbf{0}$! $\Rightarrow n = 3$ $\mathbf{u_i}$ ILI $\mathbf{v_i}$

Primjer #3 — Određivanje nepoznanica



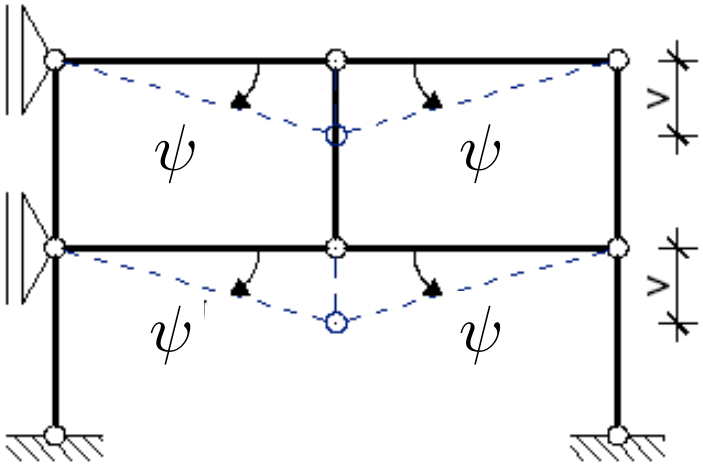
POMIČAN SUSTAV !

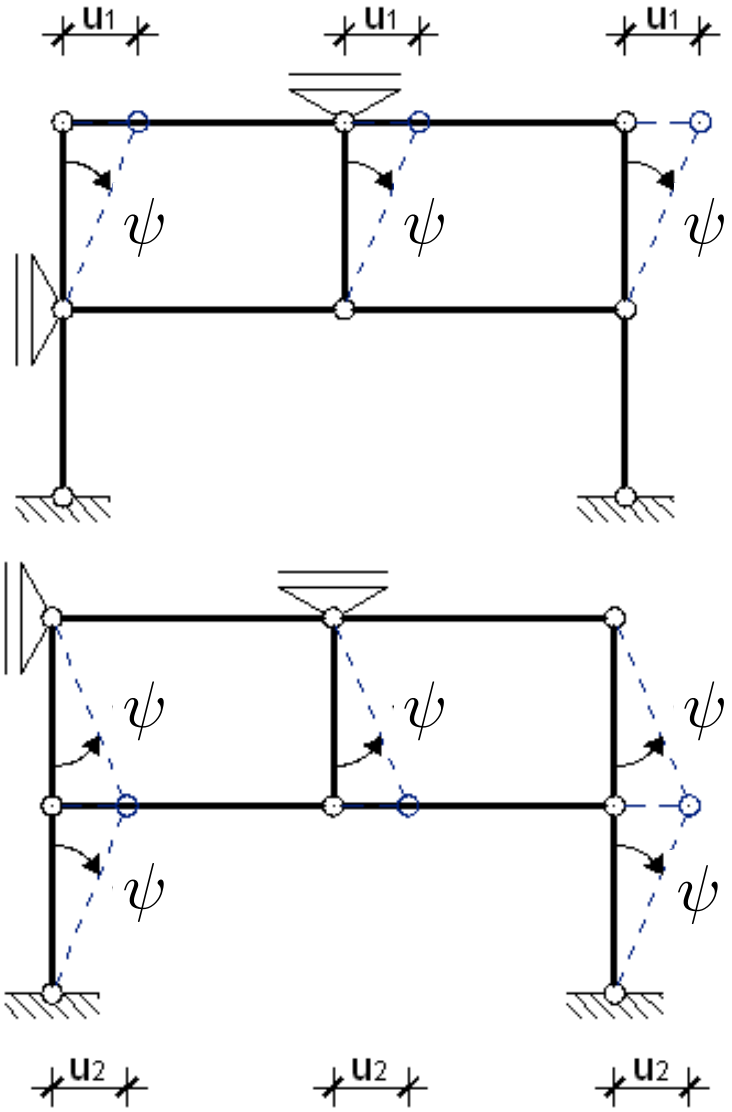
$MS = 9$

$TMP = 3 \times 6 = 18$

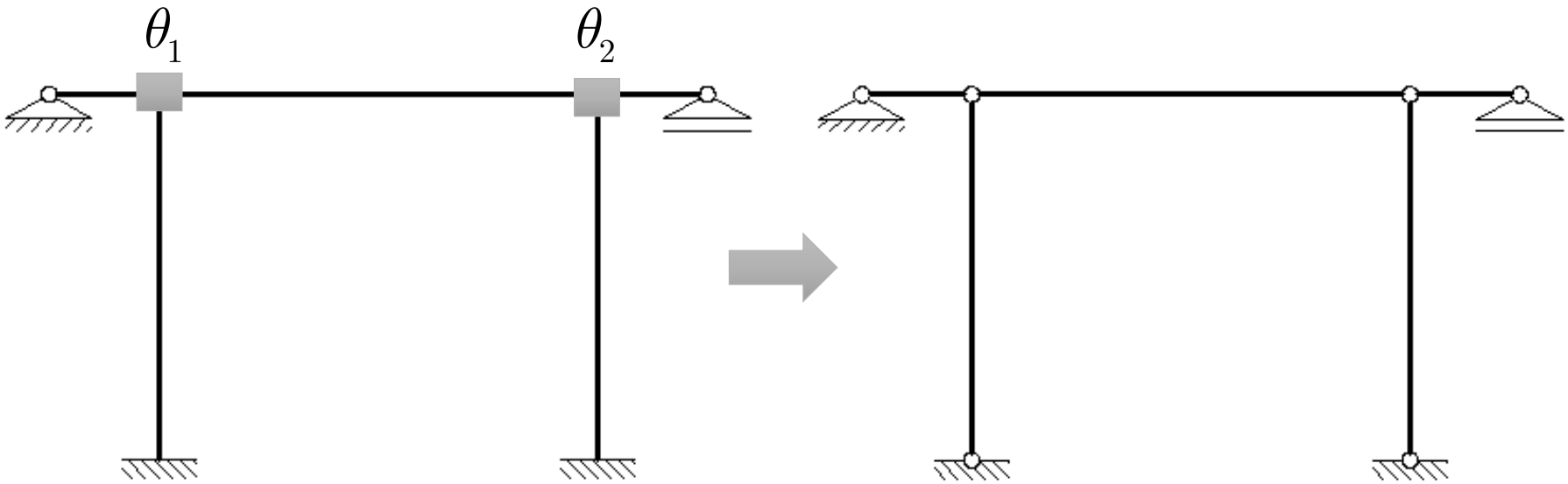
IMP : $n_r=6$; $n_d=2 \times Č - Š = 2 \times 6 - 9 = \mathbf{3}$! $\Rightarrow n_k = 9$ $\mathbf{u_i}$ ILI $\mathbf{v_i}$

Primjer #3 — Nastavak





Primjer #4 — Određivanje nepoznanica



NEPOMIČAN SUSTAV !

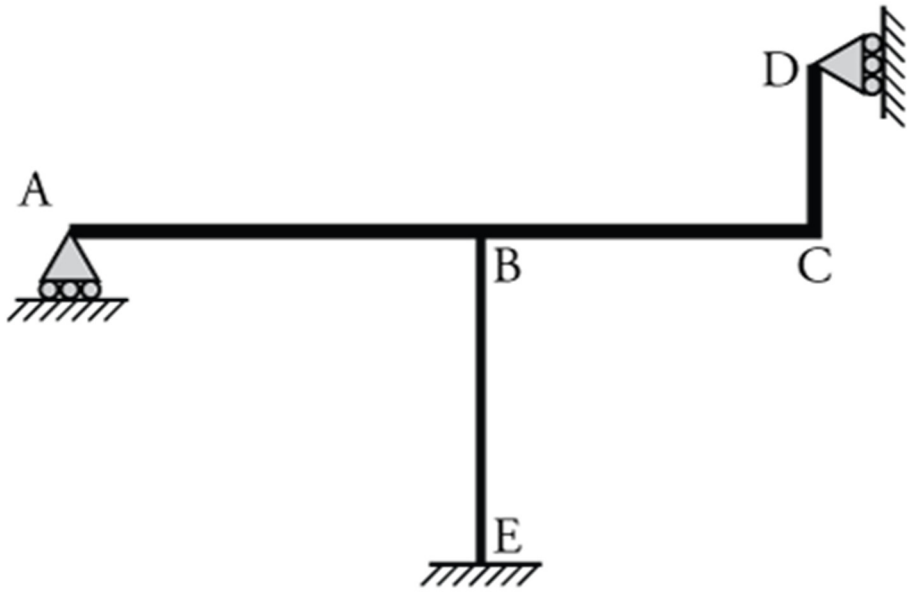
$MS = 6$

$TMP = 3 \times 2 = 6$ $\quad \mathbf{u_i}$ ILI $\mathbf{v_i}$

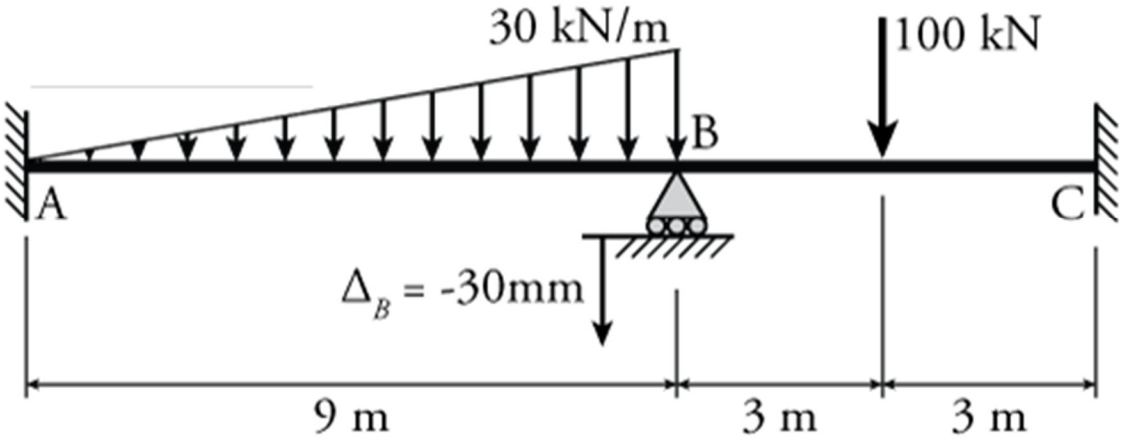
$IMP : n_r = 2 \; ; \; n_d = 2 \times Č - Š = 2 \times 2 - 5 = \mathbf{-1}$! $\quad \Rightarrow n_k = 2$

Doc. dr. sc. Marin Grubišić, mag. ing. aedif.
Ured II.26, Vladimira Preloga 3, marin.grubisic@gfos.hr | Google Classroom: **qmvjpo6**

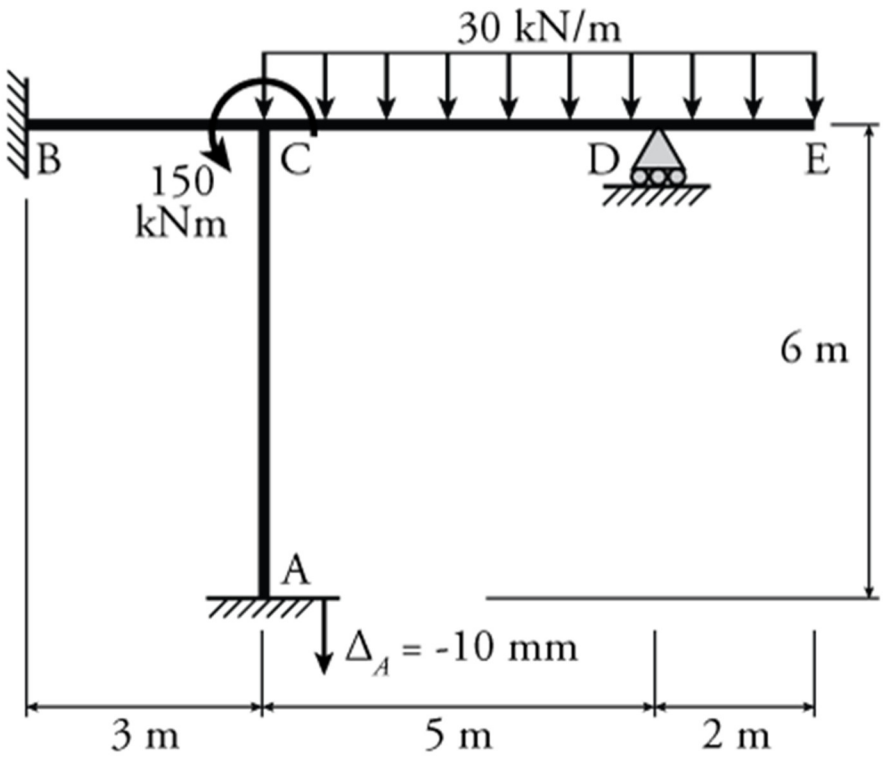
Primjer #5 — Određivanje nepoznanica



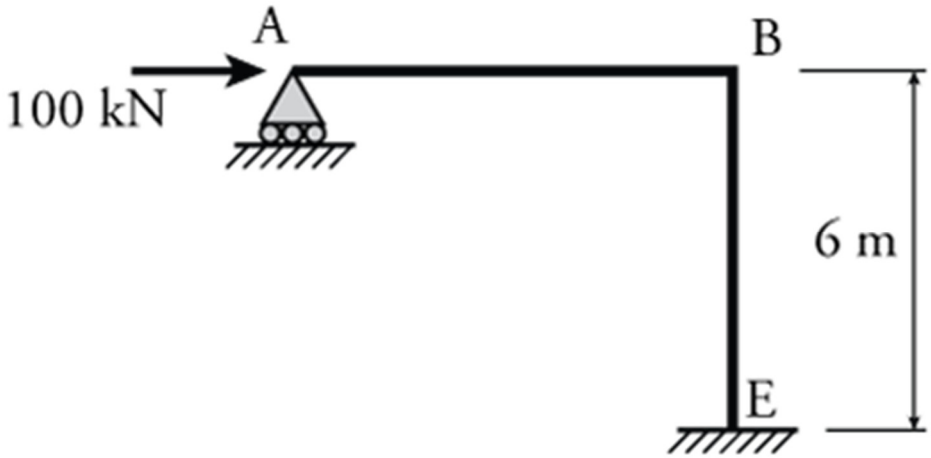
Primjer #6 — Određivanje nepoznanica



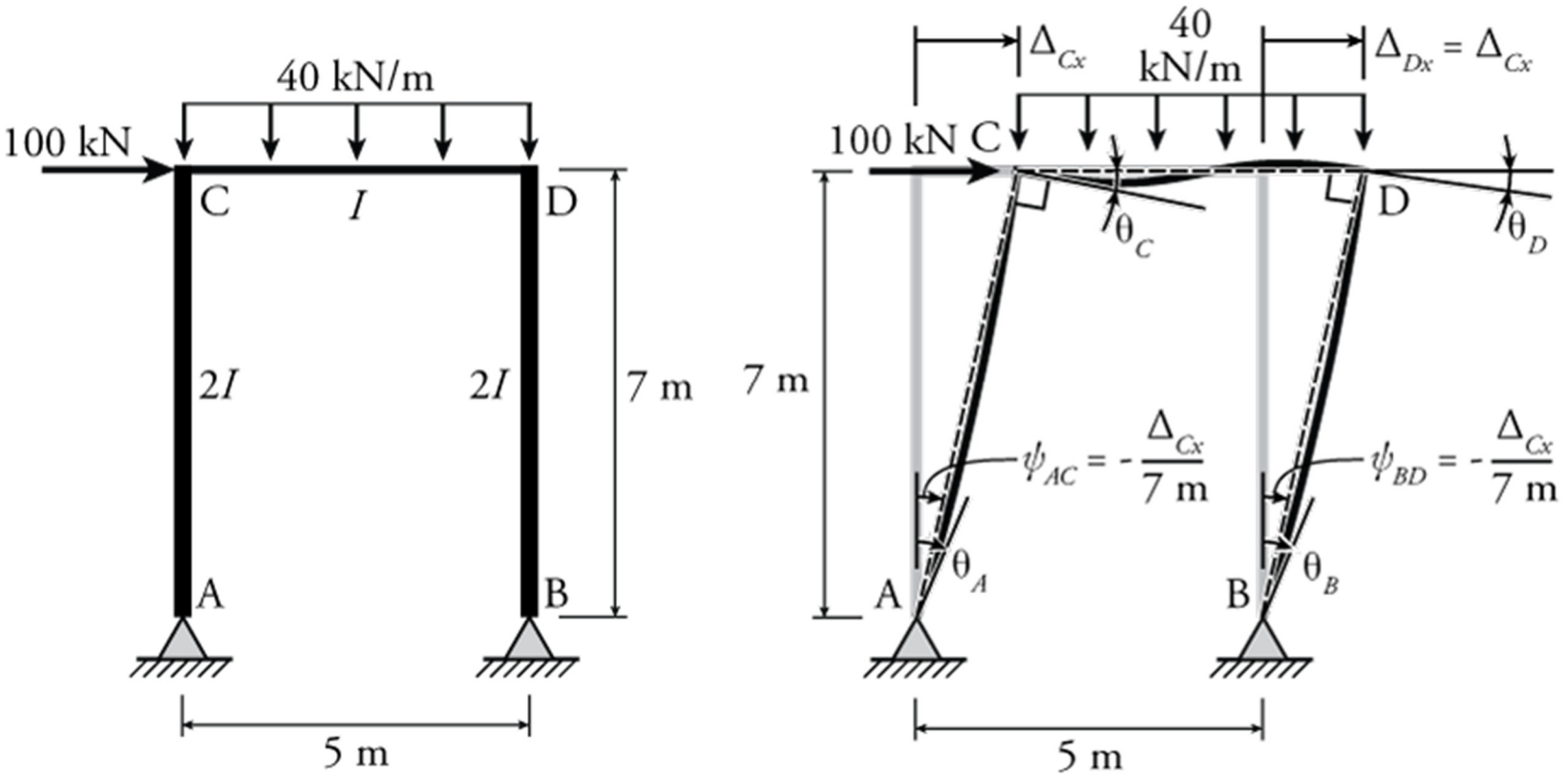
Primjer #7 — Određivanje nepoznanica



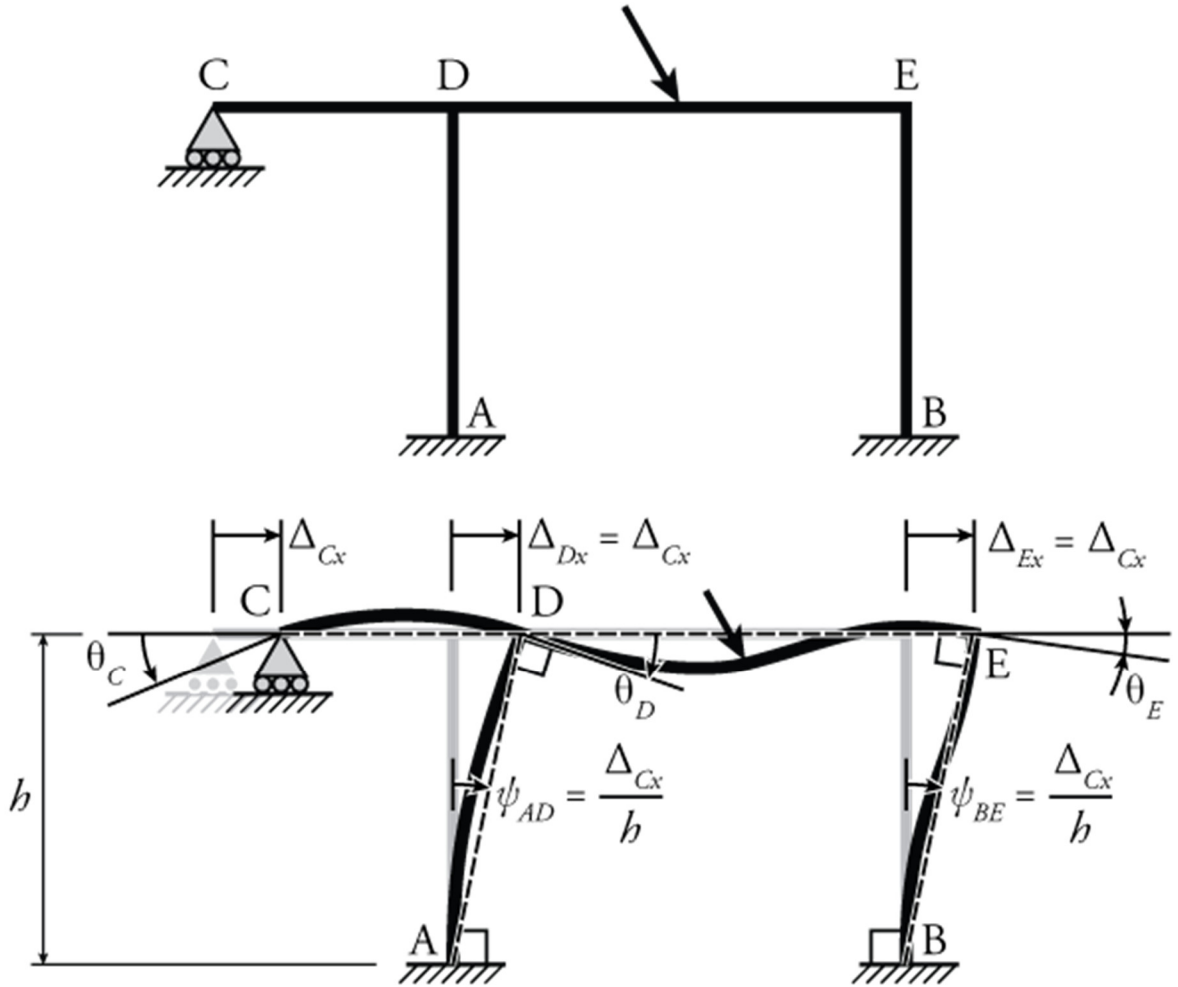
Primjer #8 — Određivanje nepoznanica



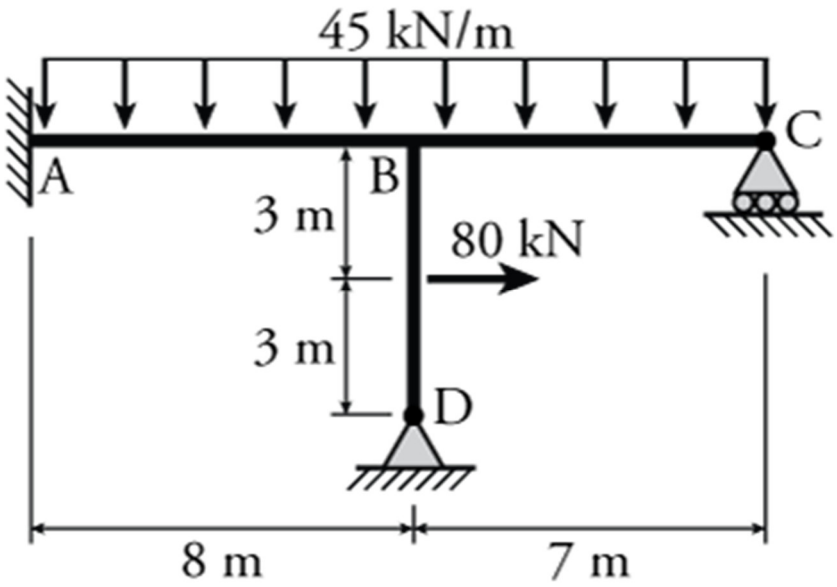
Primjer #9 — Određivanje nepoznanica



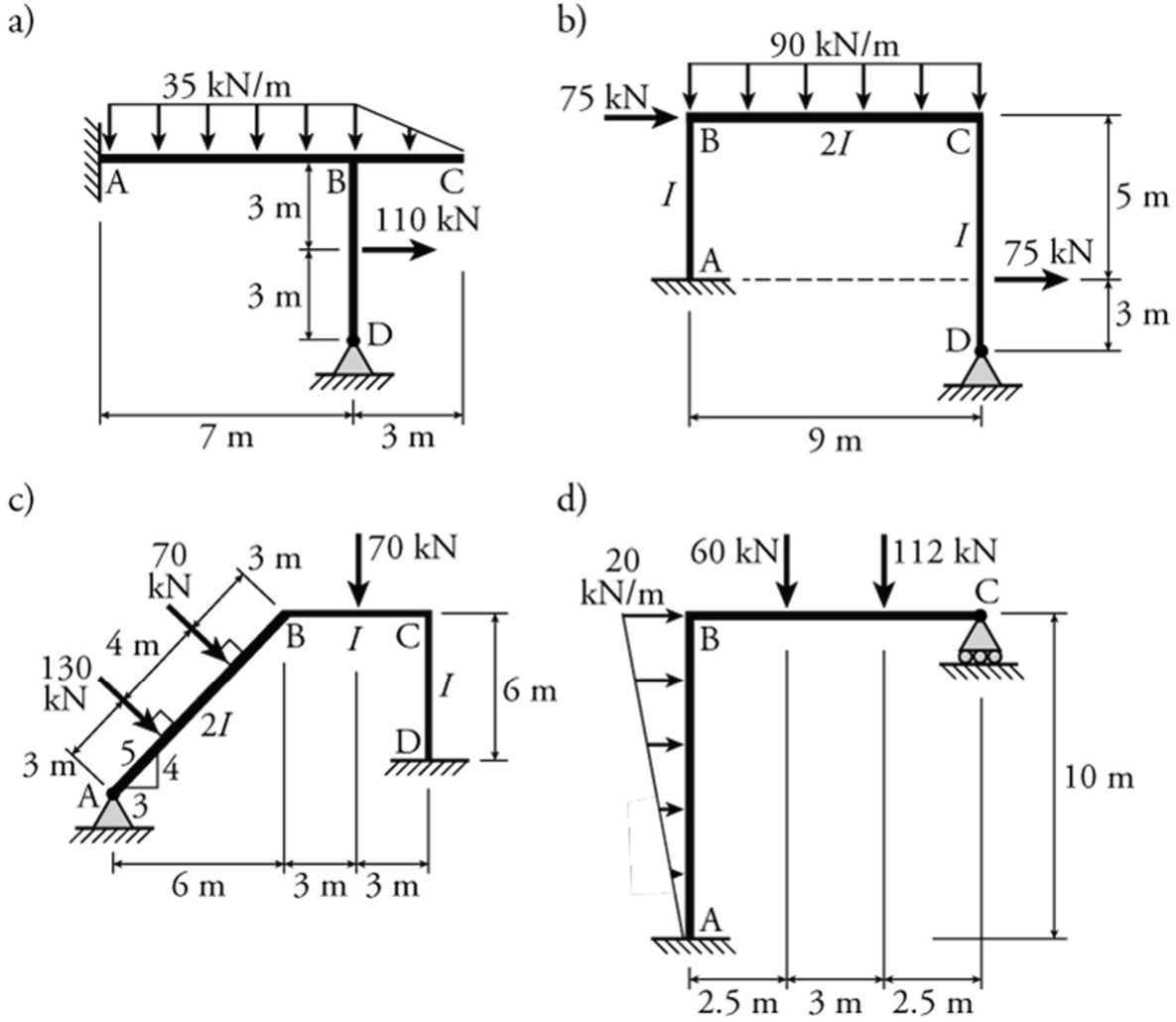
Primjer #10 — Određivanje nepoznanica



Primjer #11 — Određivanje nepoznanica



Primjer #12 — Određivanje nepoznanica

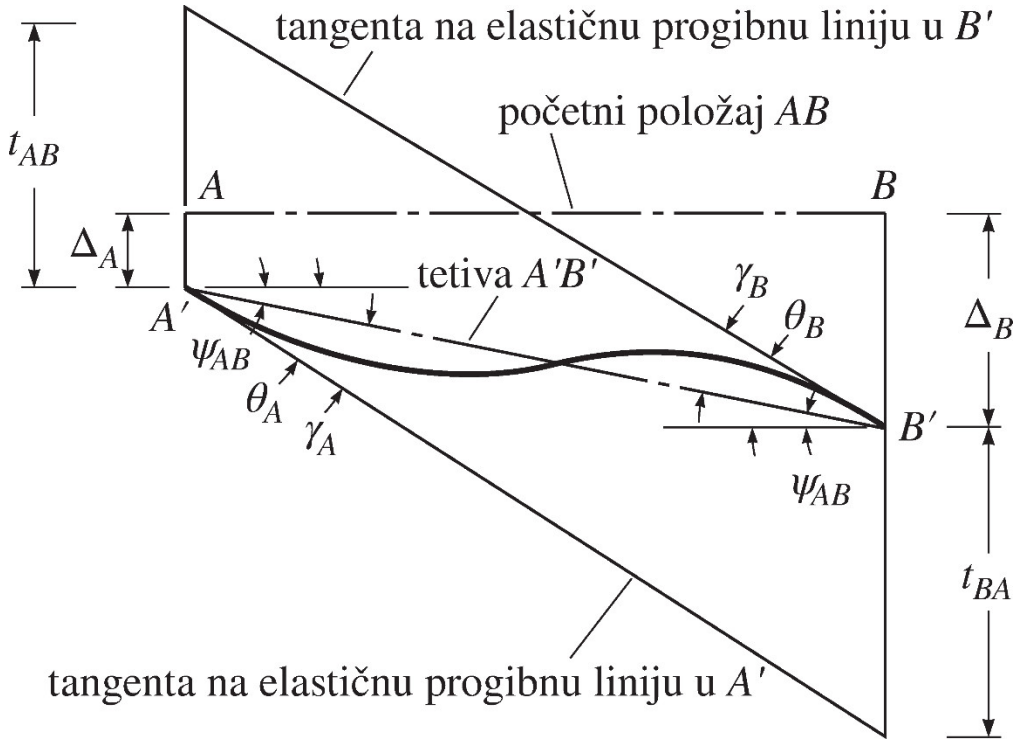


2. Proračun krutosti i rotacija (kutova zaokreta) štapova

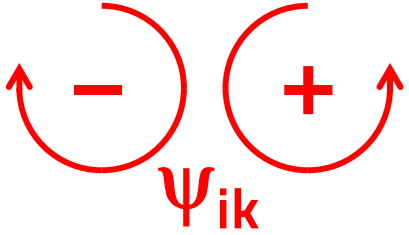
Stvarna krutost štapa koristi se za izračun momenata upetosti kada su nosači opterećeni prisilnim pomacima :

$$k_{ik} = \frac{E_{ik} \cdot I_{ik}}{L_{ik}}$$

Rotacija (kut zaokreta) štapa odnosno elementa (razlikovati od rotacije (kuta zaokreta) čvora **θ$_i$** nepoznanica): poznata vrijednost uz pretpostavku **u=1**:

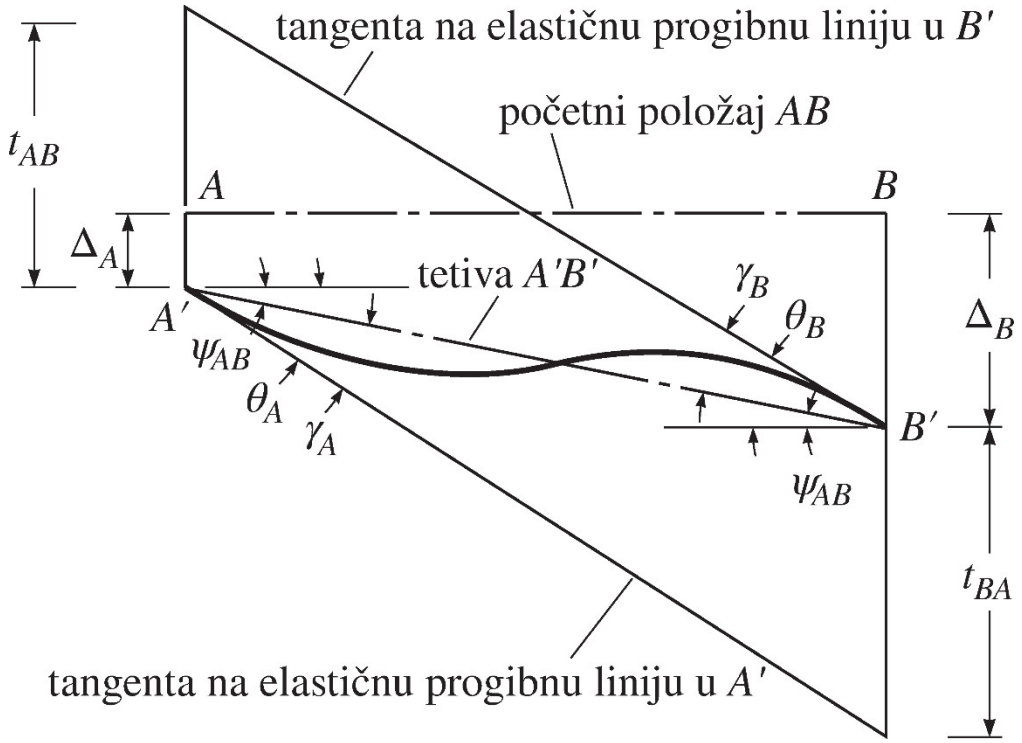


$$\psi_{ik} = \frac{\Delta_{ik}}{l_{ik}} = \psi_{ki}$$



$$\gamma_A = \frac{t_{BA}}{L} \quad \gamma_B = \frac{t_{AB}}{L}$$

2. Proračun krutosti i rotacija (kutova zaokreta) štapova

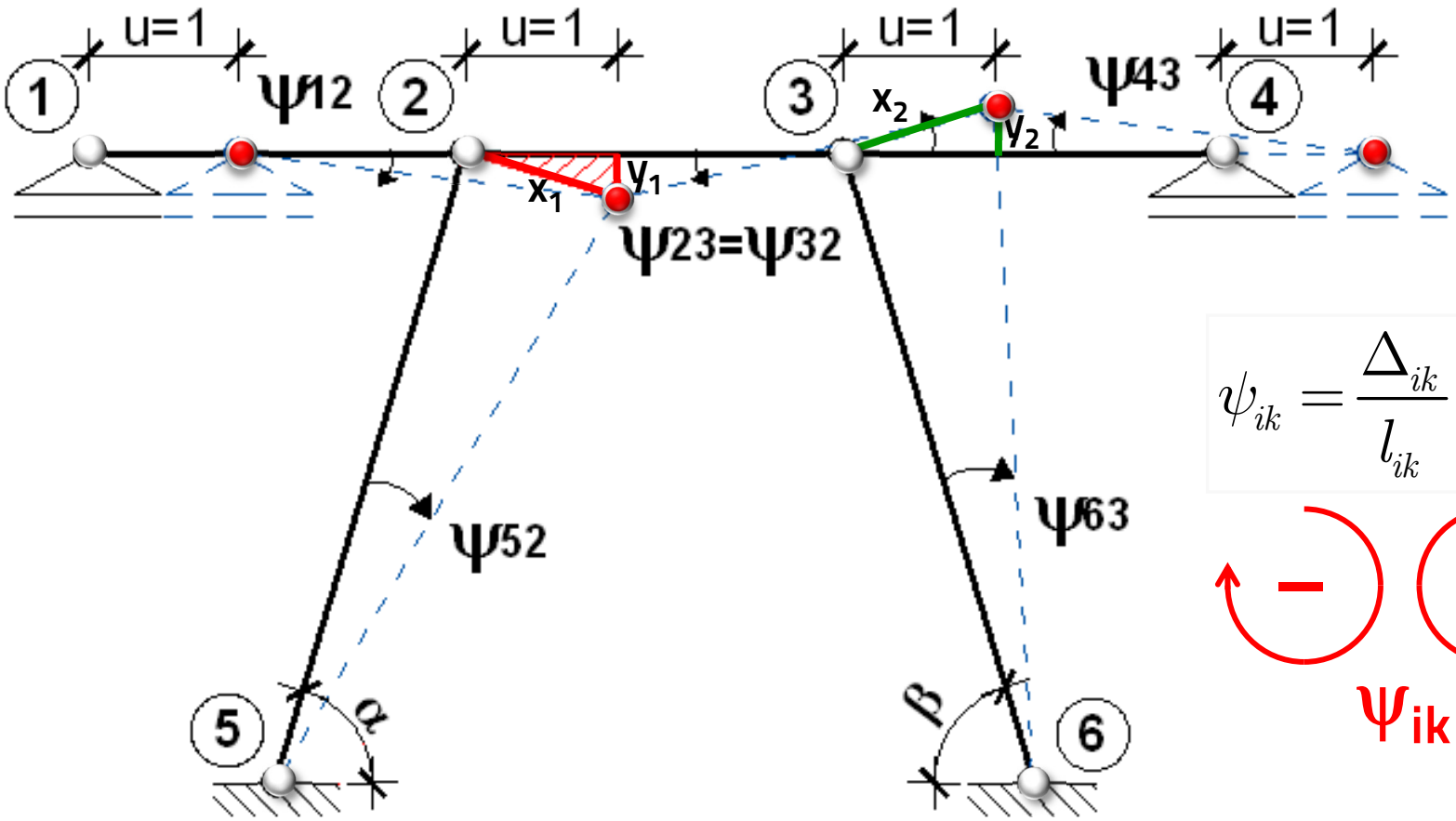


$$\gamma_A = \frac{t_{BA}}{L} \quad \gamma_B = \frac{t_{AB}}{L}$$

$$\theta_A - \psi_{AB} = \frac{t_{BA}}{L}$$

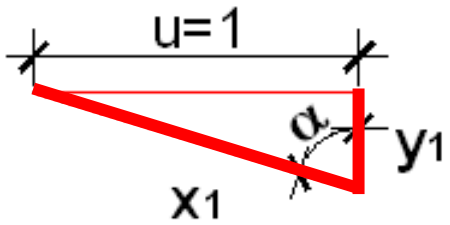
$$\theta_B - \psi_{AB} = \frac{t_{AB}}{L}$$

$$\psi_{AB} = \frac{\Delta_B - \Delta_A}{L}$$

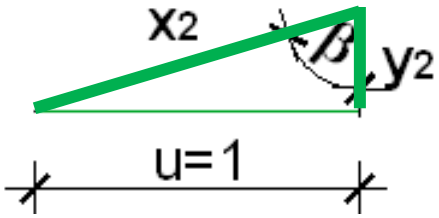


$$\psi_{ik} = \frac{\Delta_{ik}}{l_{ik}} = \psi_{ki}$$

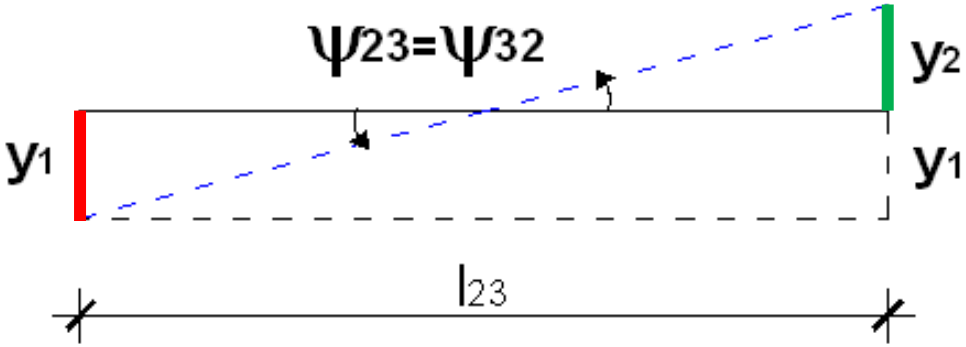
$$\psi_{12} = -\frac{y_1}{l_{12}} \qquad \psi_{52} = -\frac{x_1}{l_{52}} \qquad \psi_{43} = -\frac{y_2}{l_{34}} \qquad \psi_{63} = -\frac{x_2}{l_{63}}$$



$$x_1 = \frac{1}{\sin\alpha} \quad y_1 = \frac{1}{tg\alpha}$$

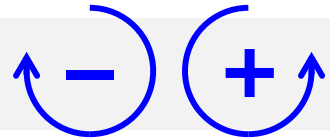


$$x_1 = \frac{1}{\sin\beta} \quad y_1 = \frac{1}{tg\beta}$$



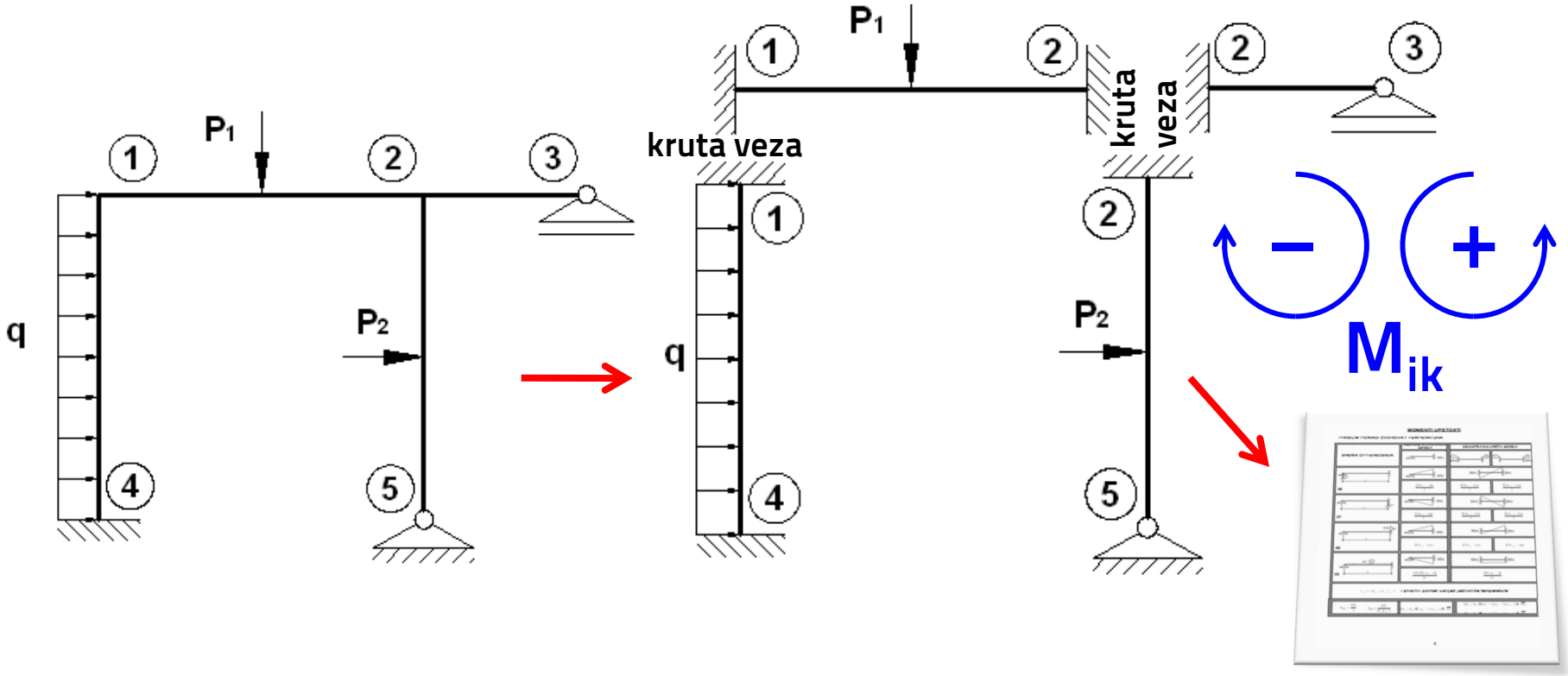
$$\psi_{23} = \psi_{32} = \frac{y_1 + y_2}{l_{23}}$$

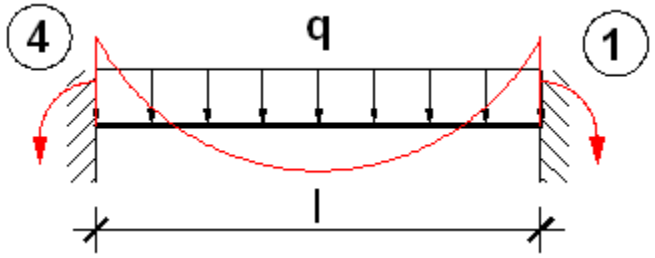
3. Momenti upetosti, $\overline{M}$



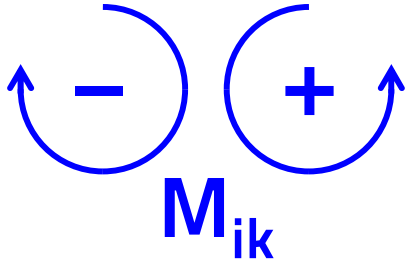
Sustav rastavljamo na niz **jednostrano i obostrano upetih greda.**

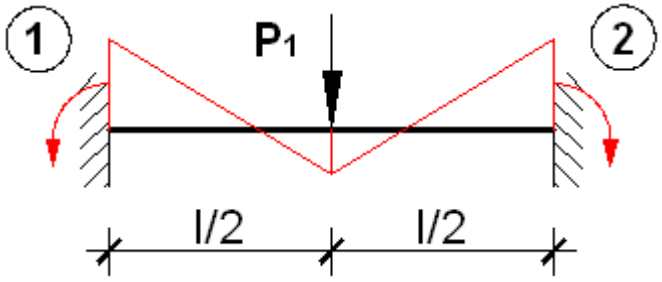
Momenti na krajevima elemenata od vanjskih opterećenja (**Tablice!**)



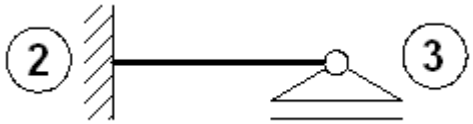


$$\bar{M}_{41} = \frac{q \cdot l^2}{12} \qquad \bar{M}_{14} = -\frac{q \cdot l^2}{12}$$

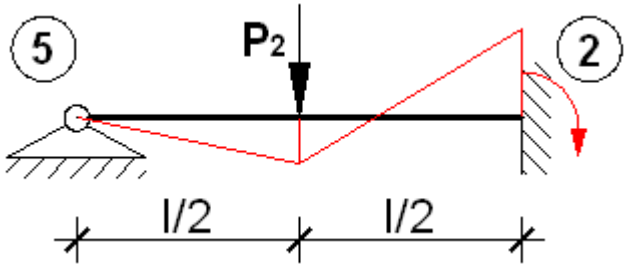




$$\bar{M}_{12} = \frac{P_1 \cdot l}{8} \qquad \bar{M}_{21} = -\frac{P_1 \cdot l}{8}$$

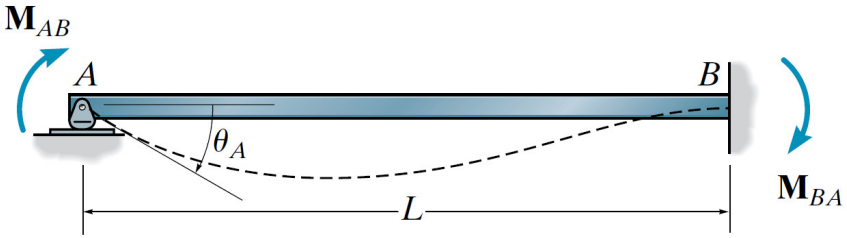


Nema opterećenja – nema momenata upetosti!

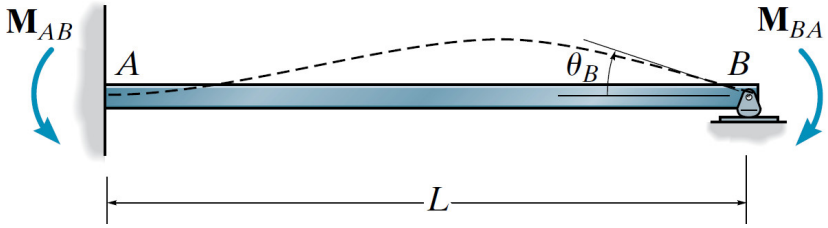


$$\bar{M}_{25} = -\frac{3}{16} \cdot P_2 \cdot l$$

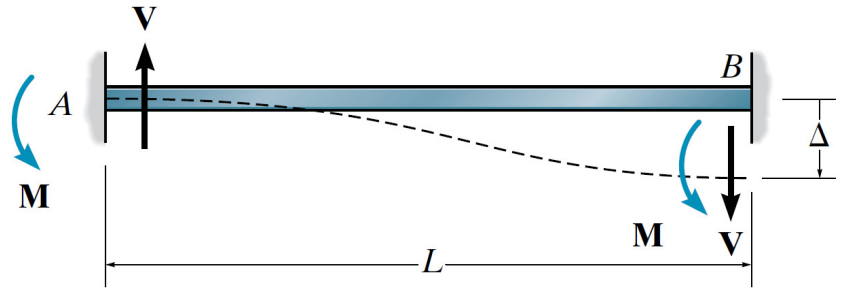
4. Jednadžbe momenata na krajevima štapova



$$M_{AB} = \frac{4EI}{L}\theta_A \qquad M_{BA} = \frac{2EI}{L}\theta_A$$



$$M_{AB} = \frac{2EI}{L}\theta_B \qquad M_{BA} = \frac{4EI}{L}\theta_B$$



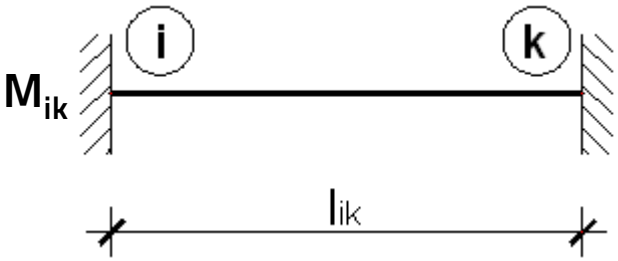
$$M_{AB} = M_{BA} = -\frac{6EI}{L^2}\Delta = -\frac{6EI}{L}\psi_{AB}$$

$$k = \frac{EI}{L} = \frac{M}{\theta} \qquad \psi_{AB} = \frac{\Delta}{L} \qquad \Delta = \Delta_B - \Delta_A$$

Doc. dr. sc. Marin Grubišić, mag. ing. aedif.
Ured II.26, Vladimira Preloga 3, marin.grubisic@gfos.hr | Google Classroom: qmvjpo6

4. Jednadžbe momenata na krajevima štapova

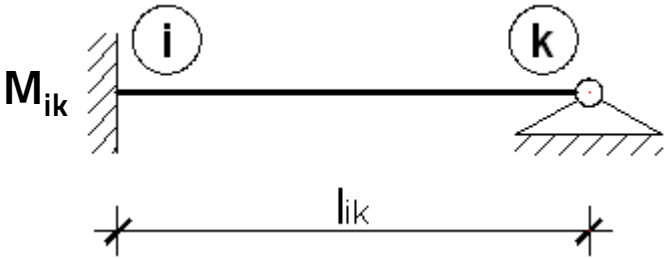
a) Obostrano upeta greda



$$M_{ik} = \frac{EI}{L} \cdot \left(4\theta_i + 2\theta_k - 6\psi_{ik} \cdot u\right) + \bar{M}_{ik}$$

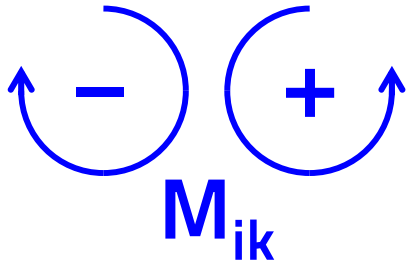
$$M_{ki} = \frac{EI}{L} \cdot \underbrace{\left(4\theta_k + 2\theta_i - 6\psi_{ik} \cdot u\right)}_{\text{stanje slobodnih pomaka (iz plana pomaka)}} + \underbrace{\bar{M}_{ki}}_{\text{stanje upetosti}}$$

b) Jednostrano upeta greda



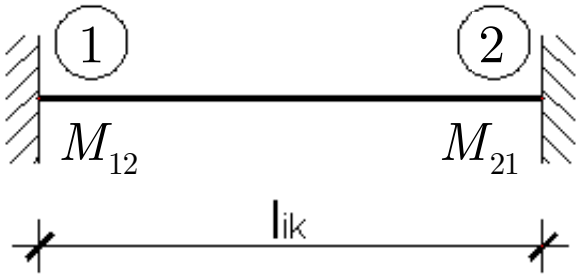
$$M_{ik} = \frac{EI}{L} \cdot \left(3\theta_i - 3\psi_{ik} \cdot u\right) + \bar{M}_{ik}$$

$$k = \frac{EI}{L} = \frac{M}{\theta} \qquad \psi_{AB} = \frac{\Delta}{L} \qquad \Delta = \Delta_B - \Delta_A$$



4. Jednadžbe momenata i poprečnih sila na krajevima štapova

a) Obostrano upeta greda



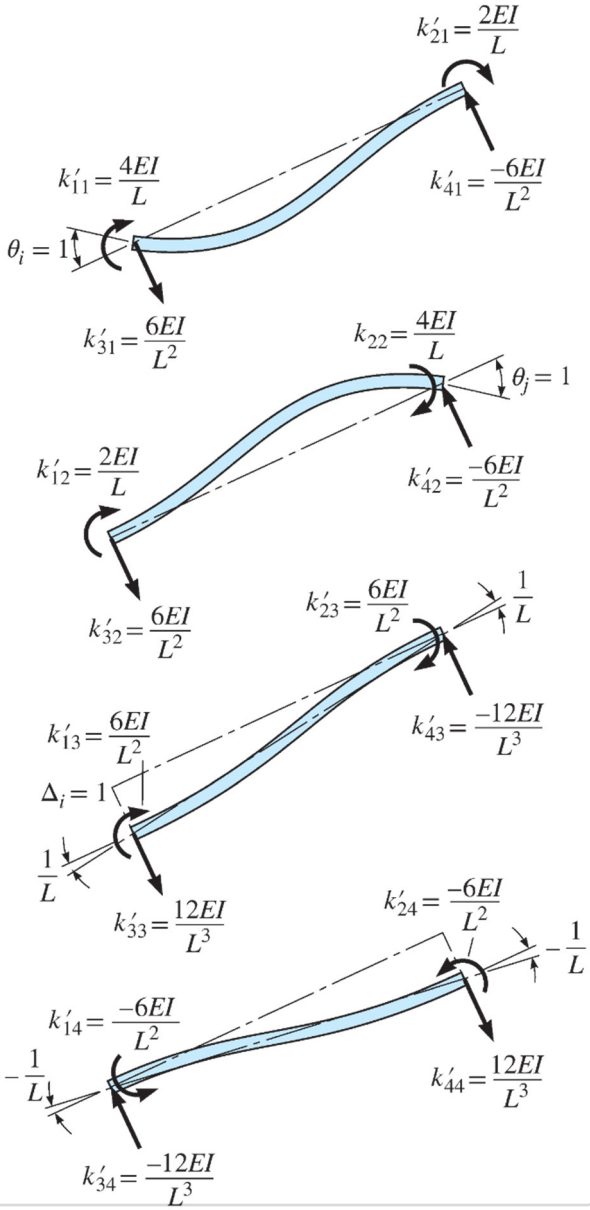
$$M_{12} = \frac{4EI}{L}\theta_1 + \frac{2EI}{L}\theta_2 + \frac{6EI}{L^2}\Delta_1 - \frac{6EI}{L^2}\Delta_2$$

$$M_{21} = \frac{2EI}{L}\theta_1 + \frac{4EI}{L}\theta_2 + \frac{6EI}{L^2}\Delta_1 - \frac{6EI}{L^2}\Delta_2$$

$$V_{12} = \frac{6EI}{L^2}\theta_1 + \frac{6EI}{L^2}\theta_2 + \frac{12EI}{L^3}\Delta_1 - \frac{12EI}{L^3}\Delta_2$$

$$V_{21} = -\frac{6EI}{L^2}\theta_1 - \frac{6EI}{L^2}\theta_2 - \frac{12EI}{L^3}\Delta_1 + \frac{12EI}{L^3}\Delta_2$$

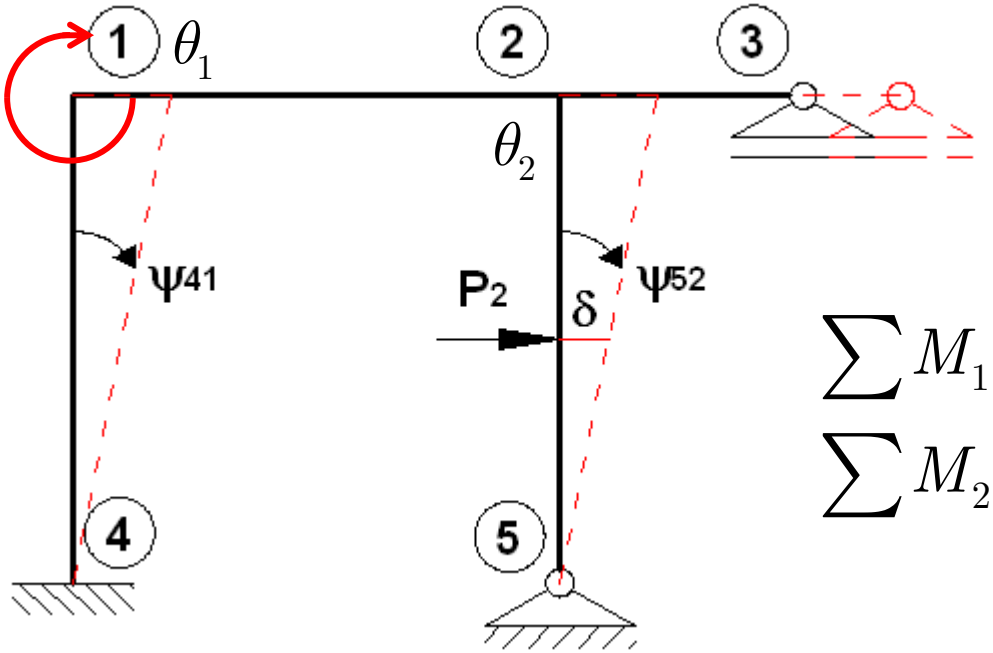
$$\begin{Bmatrix} V_{12} \\ M_{12} \\ V_{21} \\ M_{21} \end{Bmatrix} = \left[\begin{array}{cc|cc} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ \hline -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{array}\right] \cdot \begin{Bmatrix} \Delta_1 \\ \theta_1 \\ \Delta_2 \\ \theta_2 \end{Bmatrix}$$



5. Jednadžbe ravnoteže i/ili rada

Pomoću sustava jednadžbi ravnoteže i/ili rada određujemo vrijednosti nepoznanica pomoću jednadžbi momenata na krajevima štapova M_{ik}.

Koliko ima **nepoznatih rotacija (kutova zaokreta) čvora** toliko ima **jednadžbi ravnoteže**!
Koliko ima **neovisnih translatornih pomaka** toliko ima **jednadžbi rada**!



Jednadžbe ravnoteže

$$\sum M_i = 0$$

$$\sum M_1 = 0 \qquad M_{14} + M_{12} = 0$$

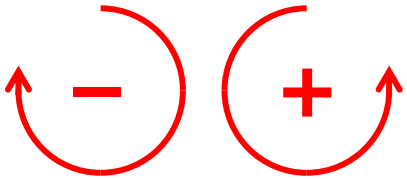
$$\sum M_2 = 0 \qquad M_{21} + M_{23} + M_{25} = 0$$

$M_{14} + M_{12} =$ –50 (npr.)

Jednadžba rada

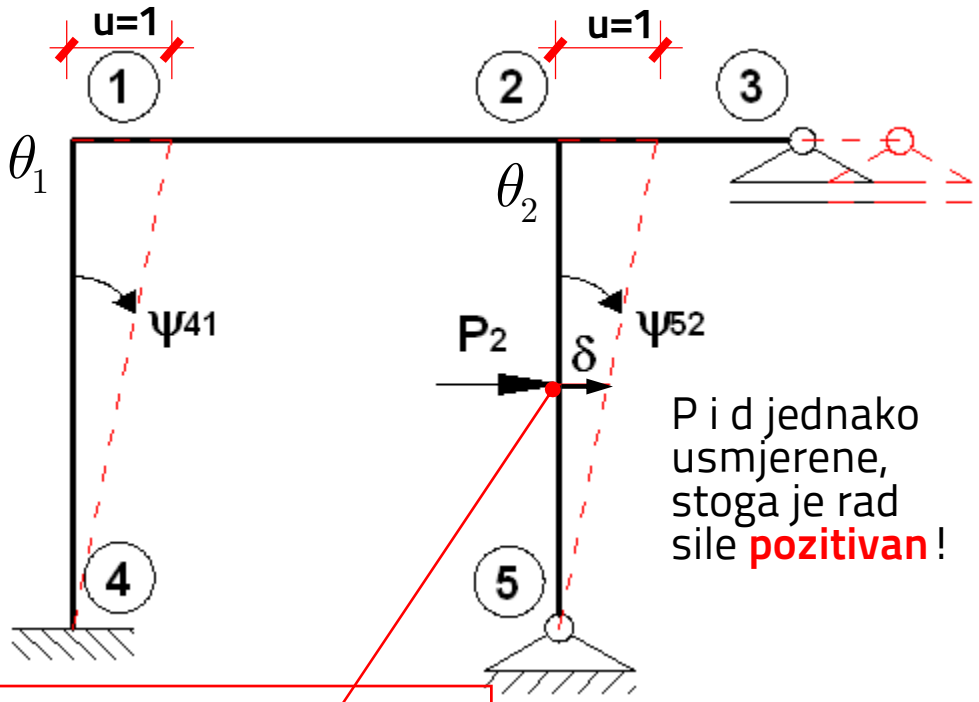
Plan pomaka!

$$\sum M_{ik} \cdot \psi_{ik} + \textbf{rad sila} = 0$$



ψ_{ik}

- $P \cdot \Delta$
- $M \cdot \psi$
- $q \cdot L \cdot \Delta$



P i d jednako usmjerene, stoga je rad sile **pozitivan**!

Lijevi stup koji se zaokreće

Desni stup koji se zaokreće

$$(M_{14} + M_{41}) \cdot \psi_{41} + M_{25} \cdot \psi_{52} + P \cdot \delta = 0$$

Rad vanjske sile P

ψ_{ik} u jednadžbe rada uvijek uvrštavamo s predznakom!

6. Konačni momentni dijagram

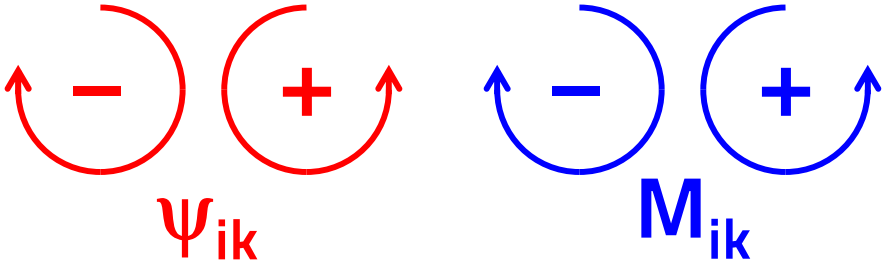
Konačne vrijednosti momenata savijanja dobivamo iz **jednadžbi momenata na krajevima štapova** uvrštavanjem dobivenih vrijednosti nepoznanica u **korak 4**.

$$M_{ik} = \frac{EI}{L} \cdot \left(4\theta_i + 2\theta_k - 6\psi_{ik} \cdot u\right) + \bar{M}_{ik}$$

$$M_{ki} = \frac{EI}{L} \cdot \left(4\theta_k + 2\theta_i - 6\psi_{ik} \cdot u\right) + \bar{M}_{ki}$$

$$M_{ik} = \frac{EI}{L} \cdot \left(3\theta_i - 3\psi_{ik} \cdot u\right) + \bar{M}_{ik}$$

Dijagrame momenata savijanja **crtamo s obzirom na pravilo vrtnje momenta!**



Dijagram poprečnih sila se određuju pomoću **diferencijalnih odnosa** iz dijagrama momenata savijanja.

Pregled

Sustave prema metodi pomaka možemo podijeliti na:

1. **POMIČNE SUSTAVE** – postoje translatorni pomaci;

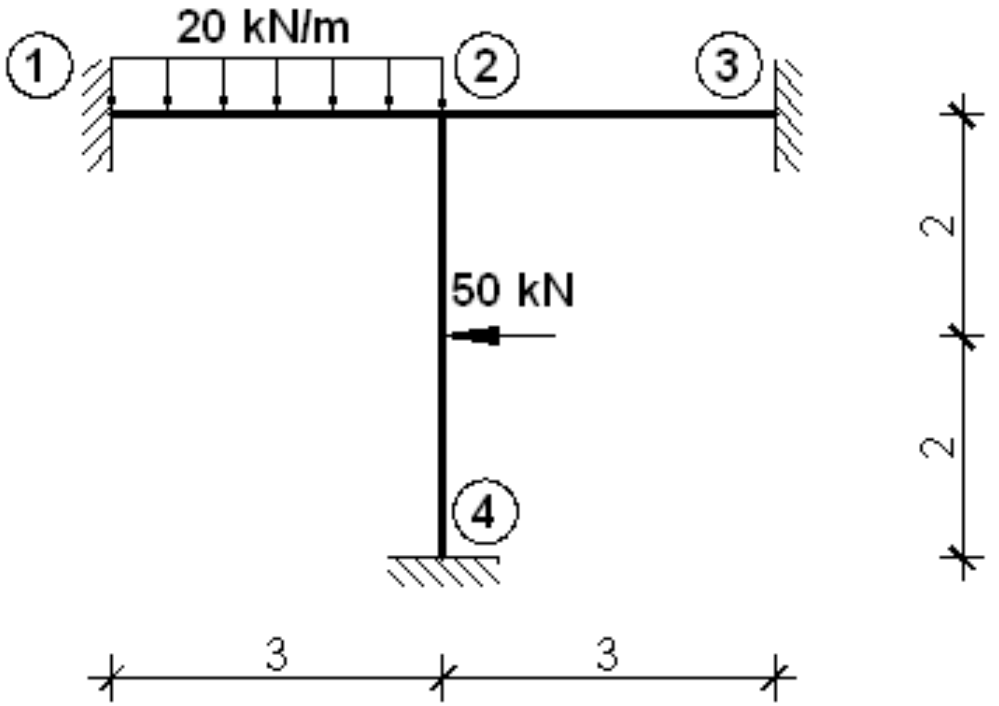
   Nepoznanice – rotacije (kutovi zaokreta) i translatorni pomaci čvorova
   Jednadžbe – jednadžbe ravnoteže čvorova i jednadžbe rada

2. **NEPOMIČNE SUSTAVE** – nema translatornih pomaka;

   Nepoznanice – rotacije (kutovi zaokreta) čvorova
   Jednadžbe – samo jednadžbe ravnoteže čvorova

Zadatak #1

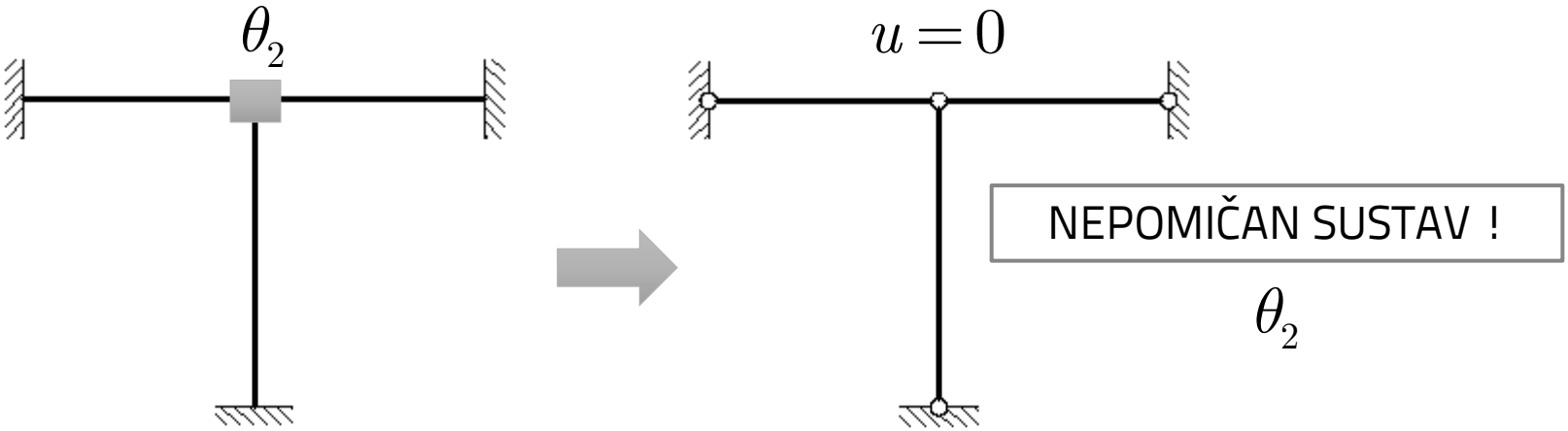
Za prikazani sustav odrediti dijagram momenata savijanja.



STUP/GREDA:

b/h = 30/30 cm
E = 30 GPa

1. Nepoznanice



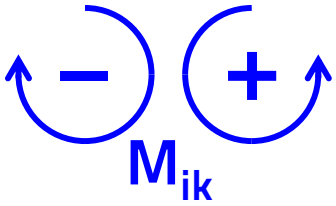
2. Krutosti štapova (elemenata)

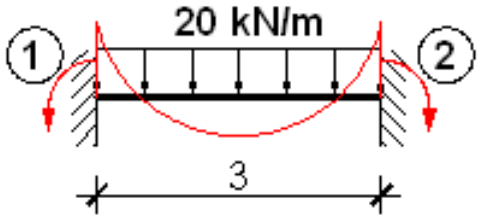
$$k_{ik} = \frac{E_{ik} I_{ik}}{L_{ik}}$$

$$\text{STUP/GREDA}: \quad EI = 20\,250 \ \text{kNm}^2$$

3. Momenti upetosti

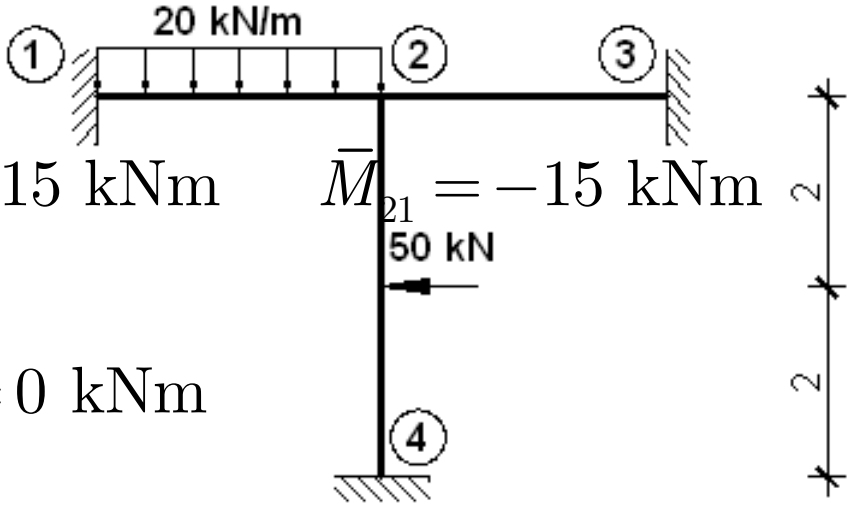
$$k_{12} = \frac{EI}{3} \qquad k_{23} = \frac{EI}{3} \qquad k_{24} = \frac{EI}{4}$$

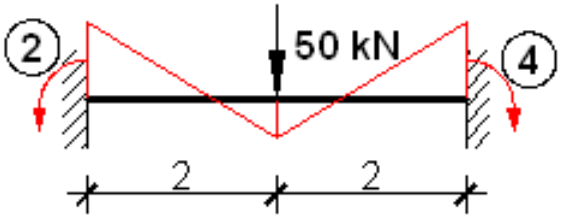




$$\bar{M}_{12} = \frac{qL^2}{12} = 15 \text{ kNm} \qquad \bar{M}_{21} = -15 \text{ kNm}$$

$$\bar{M}_{23} = \bar{M}_{32} = 0 \text{ kNm}$$





$$\bar{M}_{24} = \frac{PL}{8} = 25 \text{ kNm} \qquad \bar{M}_{42} = -25 \text{ kNm}$$

4. Jednadžbe momenata na krajevima štapova

$$M_{12} = k_{12} \cdot (4\overset{0}{\cancel{\theta_1}} + 2\theta_2 - 6\overset{0}{\cancel{\psi_{12}}} \cdot u) + \overset{0}{\cancel{\bar{M}_{12}}} = \frac{2EI}{3}\theta_2 + 15$$

$$M_{21} = k_{12} \cdot (4\theta_2 + 2\overset{0}{\cancel{\theta_1}} - 6\overset{0}{\cancel{\psi_{12}}} \cdot u) + \overset{0}{\cancel{\bar{M}_{21}}} = \frac{4EI}{3}\theta_2 - 15$$

$$M_{23} = k_{23} \cdot (4\theta_2 + 2\overset{0}{\cancel{\theta_3}} - 6\overset{0}{\cancel{\psi_{23}}} \cdot u) + \overset{0}{\cancel{\bar{M}_{23}}} = \frac{4EI}{3}\theta_2$$

$$M_{32} = k_{23} \cdot (4\overset{0}{\cancel{\theta_3}} + 2\theta_2 - 6\overset{0}{\cancel{\psi_{23}}} \cdot u) + \overset{0}{\cancel{\bar{M}_{32}}} = \frac{2EI}{3}\theta_2$$

$$M_{24} = k_{24} \cdot (4\theta_2 + 2\overset{0}{\cancel{\theta_4}} - 6\overset{0}{\cancel{\psi_{24}}} \cdot u) + \bar{M}_{24} = \frac{4EI}{4}\theta_2 + 25$$

$$M_{42} = k_{24} \cdot (4\overset{0}{\cancel{\theta_4}} + 2\theta_2 - 6\overset{0}{\cancel{\psi_{24}}} \cdot u) + \bar{M}_{42} = \frac{2EI}{4}\theta_2 - 25$$

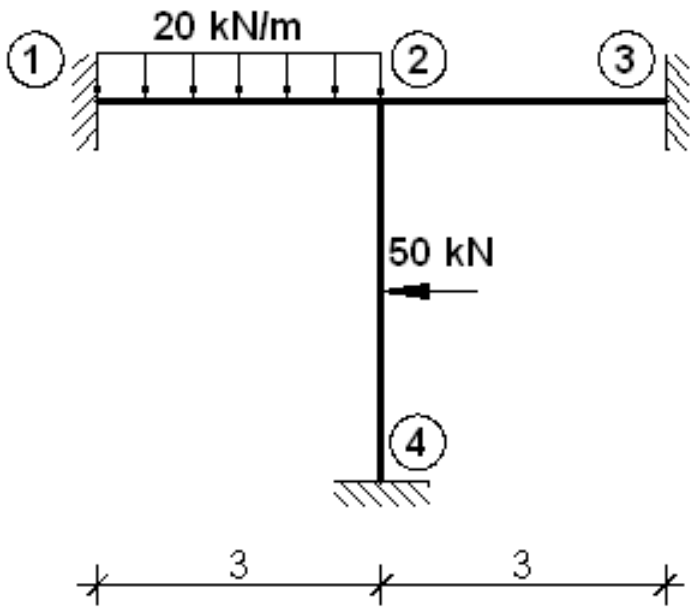
Doc. dr. sc. Marin Grubišić, mag. ing. aedif.
Ured II.26, Vladimira Preloga 3, marin.grubisic@gfos.hr | Google Classroom: **qmvjpo6**

5. Jednadžba ravnoteže

$$\sum M_2 = 0 \qquad M_{21} + M_{23} + M_{24} = 0$$

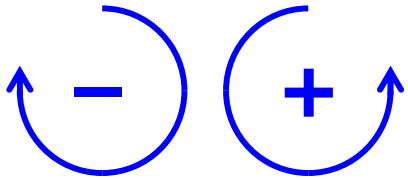
$$EI = 20\,250 \ \text{kNm}^2$$

$$\underbrace{\frac{4EI}{3}\theta_2 - 15}_{M_{21}} + \underbrace{\frac{4EI}{3}\theta_2}_{M_{23}} + \underbrace{\frac{\cancel{4}EI}{\cancel{4}}\theta_2 + 25}_{M_{24}} = 0$$

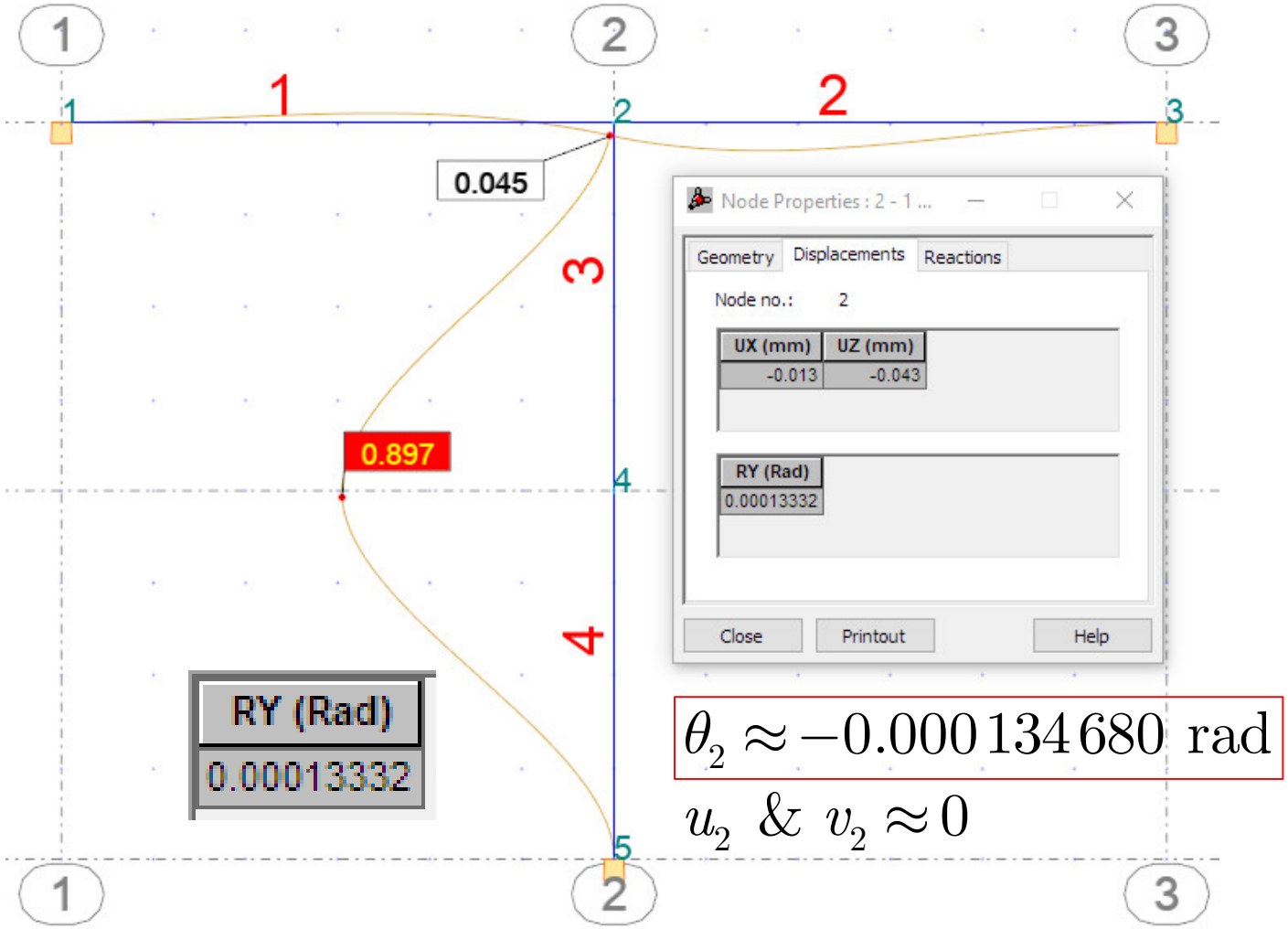


$$\theta_2 = -\frac{30}{11EI}$$

$$\theta_2 \approx -0.000\,134\,680 \ \text{rad}$$



5. Jednadžba ravnoteže



$$\theta_2 \approx -0.000\,134\,680 \text{ rad}$$

$$u_2 \ \& \ v_2 \approx 0$$

6. Konačni momentni dijagram

$$M_{12} = \frac{2EI}{3}\left(-\frac{30}{11EI}\right) + 15 \approx 13.19 \text{ kNm}$$

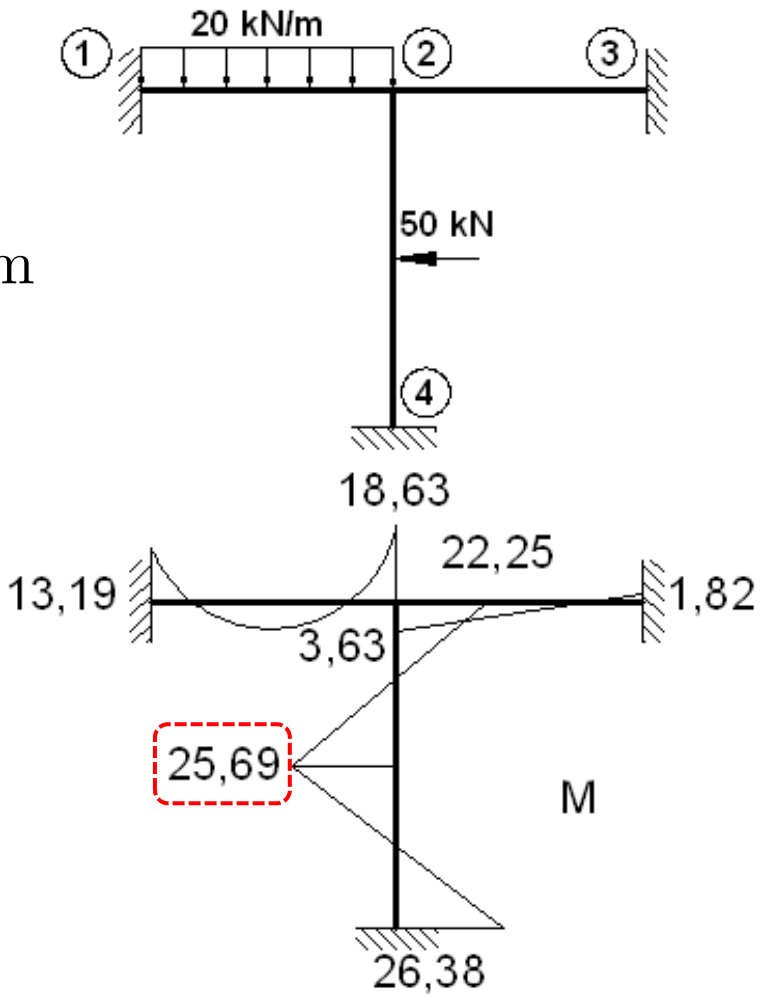
$$M_{21} = \frac{4EI}{3}\left(-\frac{30}{11EI}\right) - 15 \approx -18.63 \text{ kNm}$$

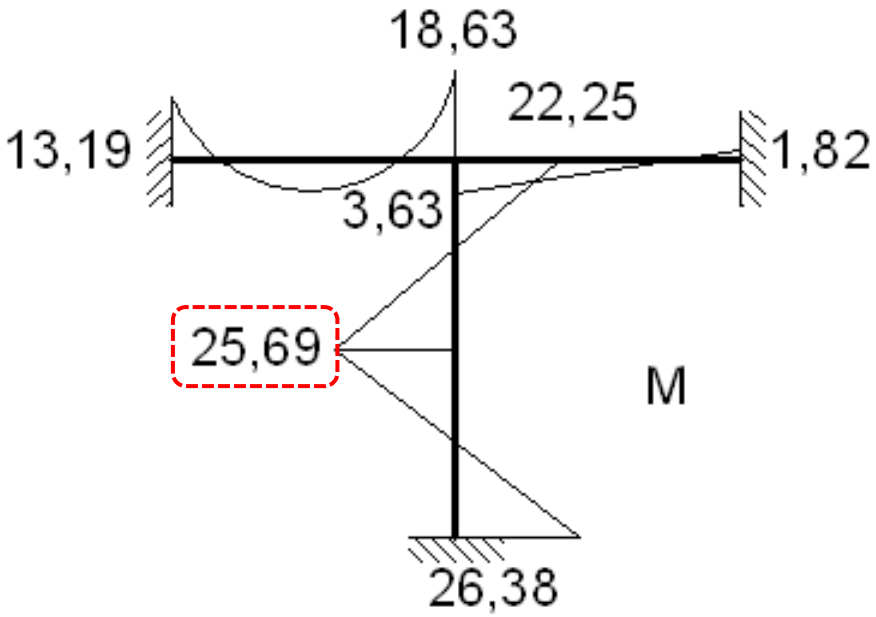
$$M_{23} = \frac{4EI}{3}\left(-\frac{30}{11EI}\right) \approx -3.63 \text{ kNm}$$

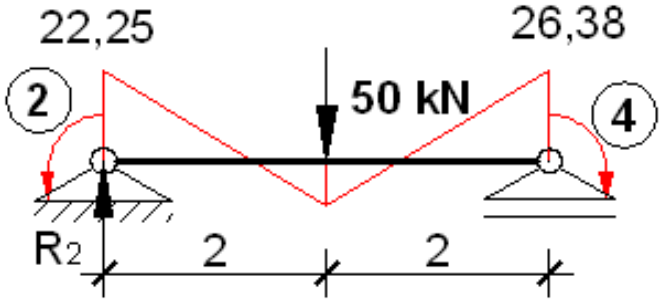
$$M_{32} = \frac{2EI}{3}\left(-\frac{30}{11EI}\right) \approx -1.82 \text{ kNm}$$

$$M_{24} = \frac{4EI}{4}\left(-\frac{30}{11EI}\right) + 25 \approx 22.25 \text{ kNm}$$

$$M_{42} = \frac{2EI}{4}\left(-\frac{30}{11EI}\right) - 25 \approx 26.38 \text{ kNm}$$







$$\sum M_4 = 0$$

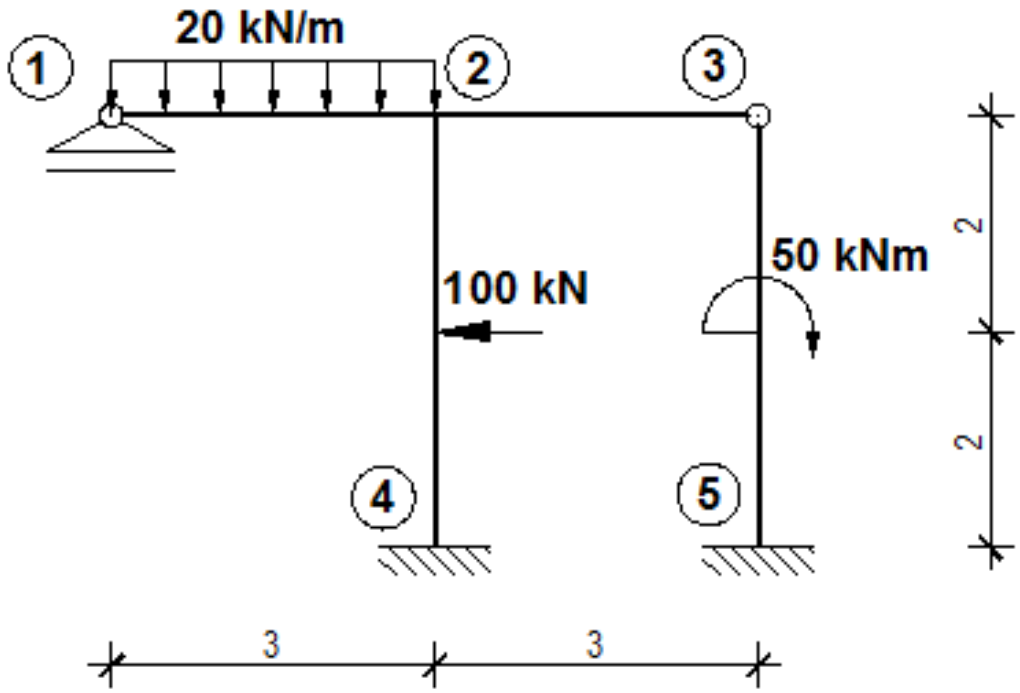
$$22.25 - 26.38 + 50 \cdot 2 - R_2 \cdot 4 = 0$$

$$R_2 = +23.97 \text{ kN} \uparrow$$

$$M_5 = -22.25 + R_2 \cdot 2 = +25.69 \text{ kNm}$$

Zadatak #2

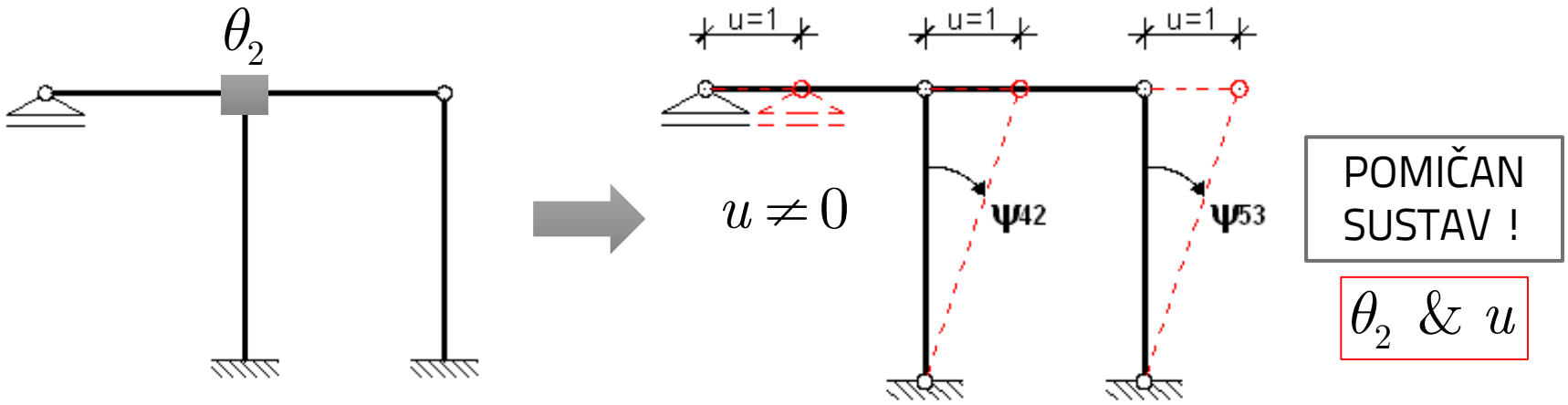
Za prikazani sustav metodom pomaka odrediti dijagram momenata savijanja.



STUP/GREDA:

b/h = 30/30 cm
E = 30 GPa

1. Nepoznanice



POMIČAN SUSTAV !

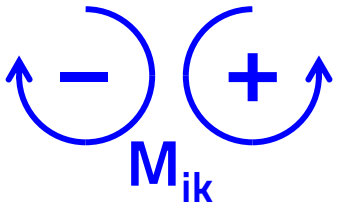
θ_2 & u

$$\psi_{42} = -\frac{1}{4} = -0.25 \qquad \psi_{53} = -\frac{1}{4} = -0.25$$

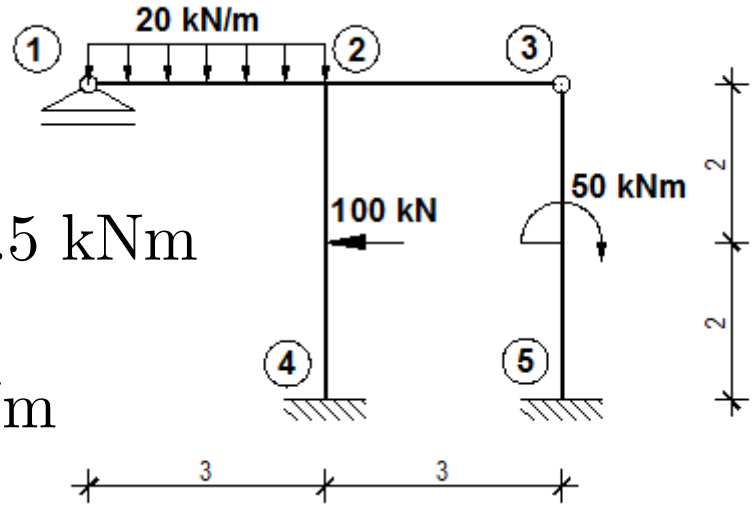
2. Krutosti štapova (elemenata)

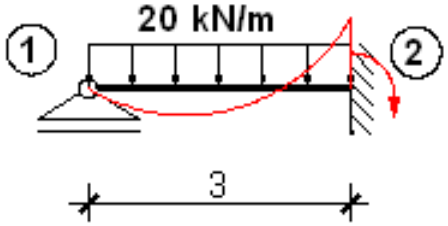
$$k_{ik} = \frac{E_{ik} I_{ik}}{L_{ik}} \qquad EI = 20\,250 \ \mathrm{kNm}^2$$

3. Momenti upetosti

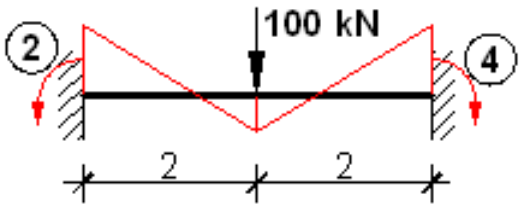


$$k_{12} = \frac{EI}{3} = k_{23} \qquad k_{24} = \frac{EI}{4} = k_{25}$$



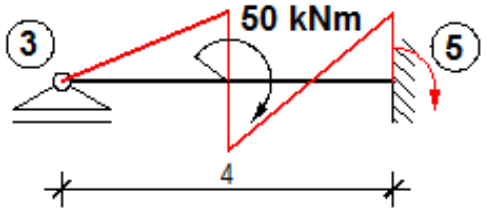


$$\bar{M}_{21} = -\frac{qL^2}{8} = -22.5 \text{ kNm}$$



$$\bar{M}_{24} = \frac{PL}{8} = 50 \text{ kNm}$$

$$\bar{M}_{42} = -50 \text{ kNm}$$



$$\bar{M}_{53} = -\frac{M}{8} = -6.25 \text{ kNm}$$

4. Jednadžbe momenata na krajevima štapova

$$M_{21} = k_{12} \cdot \left(3\theta_2 - 3\psi_{12} \cdot u\right) + \bar{M}_{21} = \frac{3EI}{3}\theta_2 - 22.5$$

$$M_{23} = k_{23} \cdot \left(3\theta_2 - 3\psi_{23} \cdot u\right) + \bar{M}_{23} = \frac{3EI}{3}\theta_2$$

$$M_{24} = k_{24} \cdot \left(4\theta_2 + 2\theta_4 - 6\psi_{24} \cdot u\right) + \bar{M}_{24} = \frac{4EI}{4}\theta_2 + \frac{1.5EI}{4}u + 50$$

$$M_{42} = k_{24} \cdot \left(4\theta_4 + 2\theta_2 - 6\psi_{24} \cdot u\right) + \bar{M}_{42} = \frac{2EI}{4}\theta_2 + \frac{1.5EI}{4}u - 50$$

$$M_{53} = k_{35} \cdot \left(3\theta_5 - 3\psi_{53} \cdot u\right) + \bar{M}_{35} = \frac{0.75EI}{4}u - 6.25$$

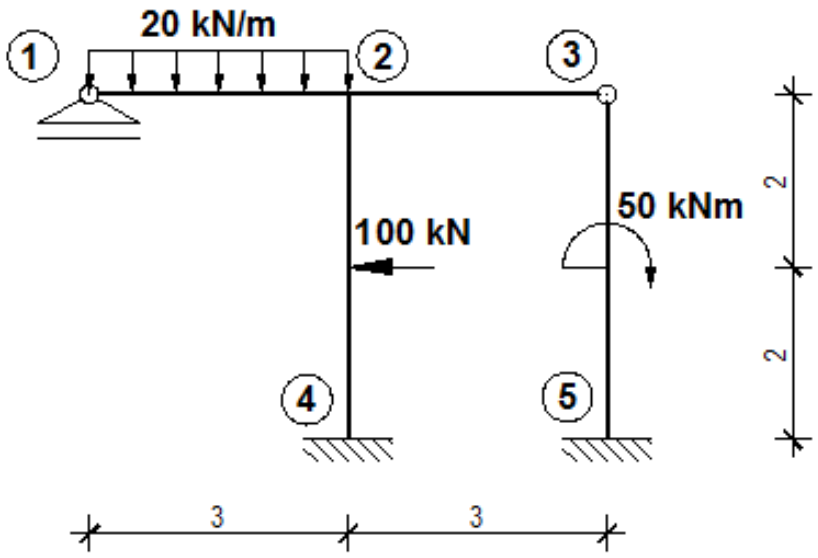
5. Jednadžba ravnoteže i jednadžba rada

$$\sum M_2 = 0 \qquad M_{21} + M_{23} + M_{24} = 0$$

θ_2 & u

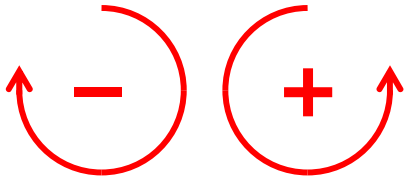
$$\underbrace{\frac{\cancel{3}EI}{\cancel{3}}\theta_2 - 22.5}_{M_{21}} + \underbrace{\frac{\cancel{3}EI}{\cancel{3}}\theta_2}_{M_{23}} + \underbrace{\frac{\cancel{4}EI}{\cancel{4}}\theta_2 + \frac{1.5EI}{4}u + 50}_{M_{24}} = 0$$

$$3EI\theta_2 + \frac{1.5EI}{4}u = -27.5$$



$\theta_2 \;\&\; u$

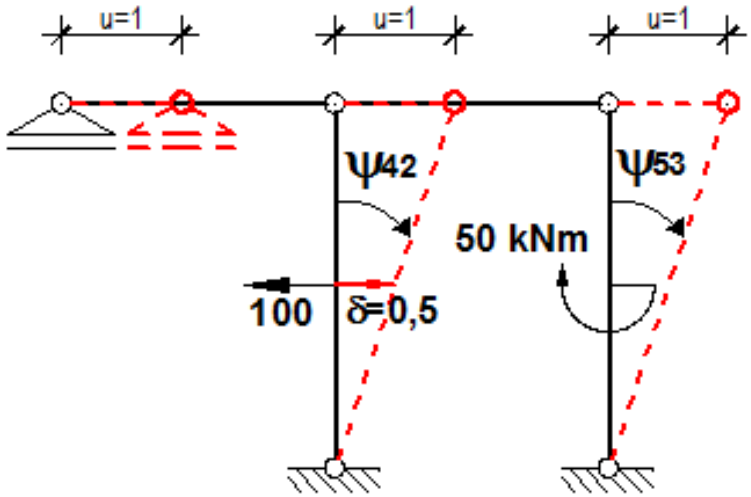
$$\sum M_{ik} \cdot \psi_{ik} + \textbf{rad sila} = 0$$



$P \cdot \Delta$

$M \cdot \psi$

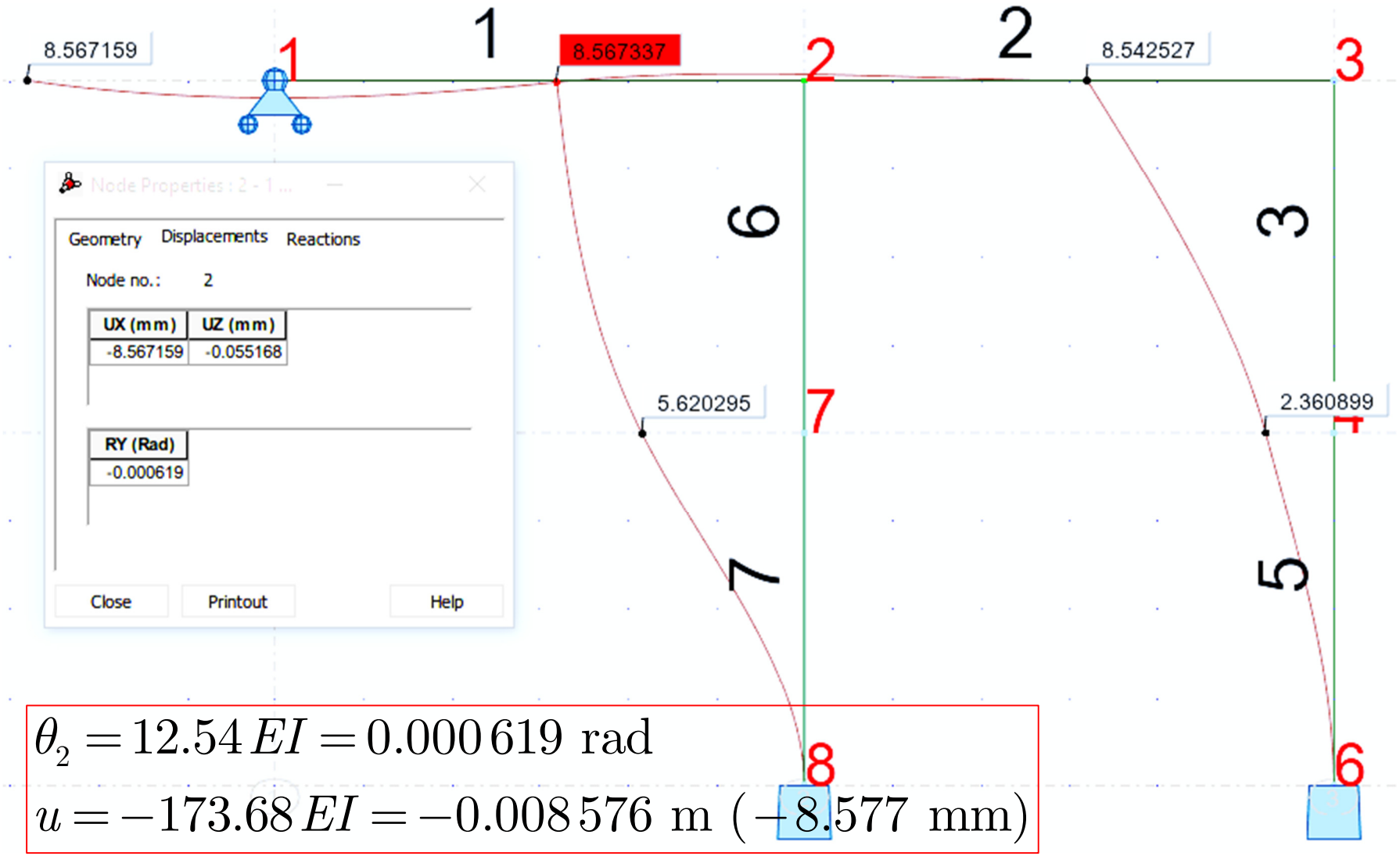
$q \cdot L \cdot \Delta$



$$\left(M_{24} + M_{42}\right) \cdot \psi_{42} + M_{53} \cdot \psi_{53} + \underbrace{50 \cdot 0.25}_{M \cdot \psi} - \underbrace{100 \cdot 0.5}_{P \cdot \delta} = 0$$

$$\left(1.5EI\,\theta_2 + \frac{3EI}{4}u\right) \cdot (-0.25) + \left(\frac{0.75EI}{4}u - 6.25\right) \cdot (-0.25) + 12.5 - 50 = 0$$

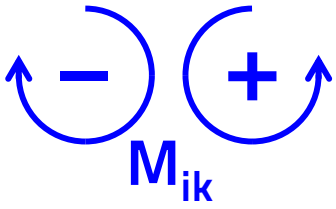
$$-\frac{1.5EI}{4}\theta_2 - 0.234EI\,u = 35.938$$



$$\theta_2 = 12.54\,EI = 0.000\,619 \text{ rad}$$

$$u = -173.68\,EI = -0.008\,576 \text{ m } (-8.577 \text{ mm})$$

6. Konačni dijagram momenata savijanja



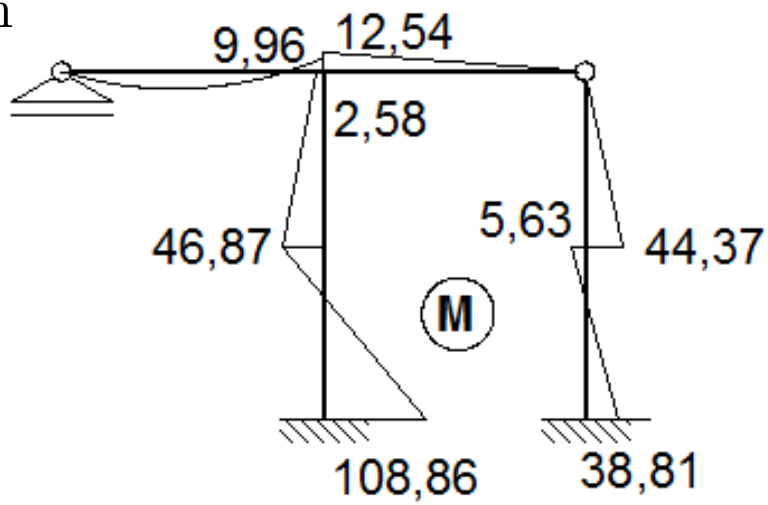
$$M_{21} = \frac{3EI}{3}\theta_2 - 22.5 = -9.96 \text{ kNm}$$

$$M_{23} = \frac{3EI}{3}\theta_2 = 12.54 \text{ kNm}$$

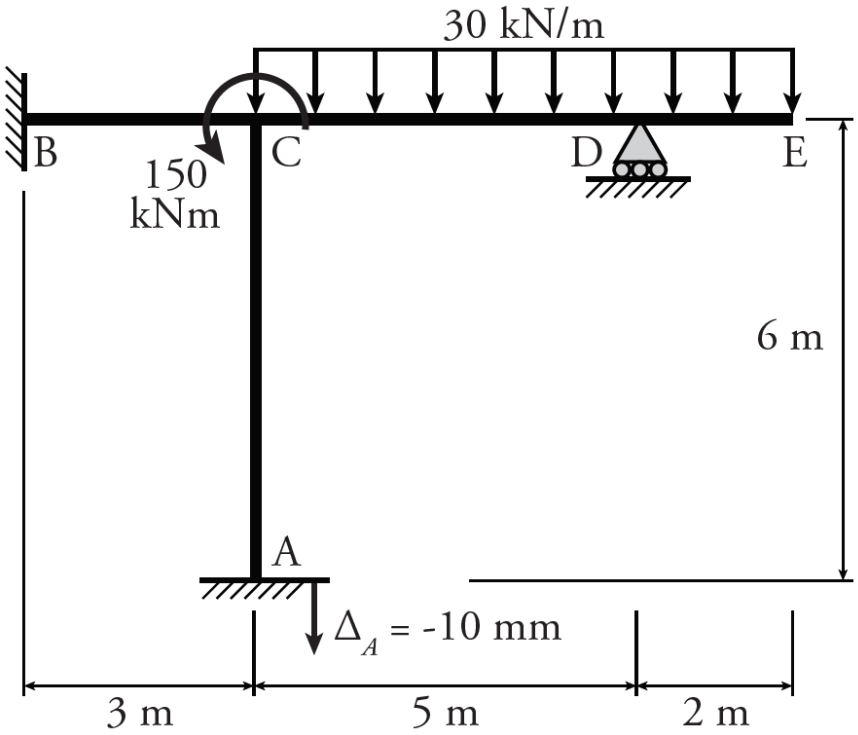
$$M_{24} = \frac{4EI}{4}\theta_2 + \frac{1.5EI}{4}u + 50 = -2.59 \text{ kNm}$$

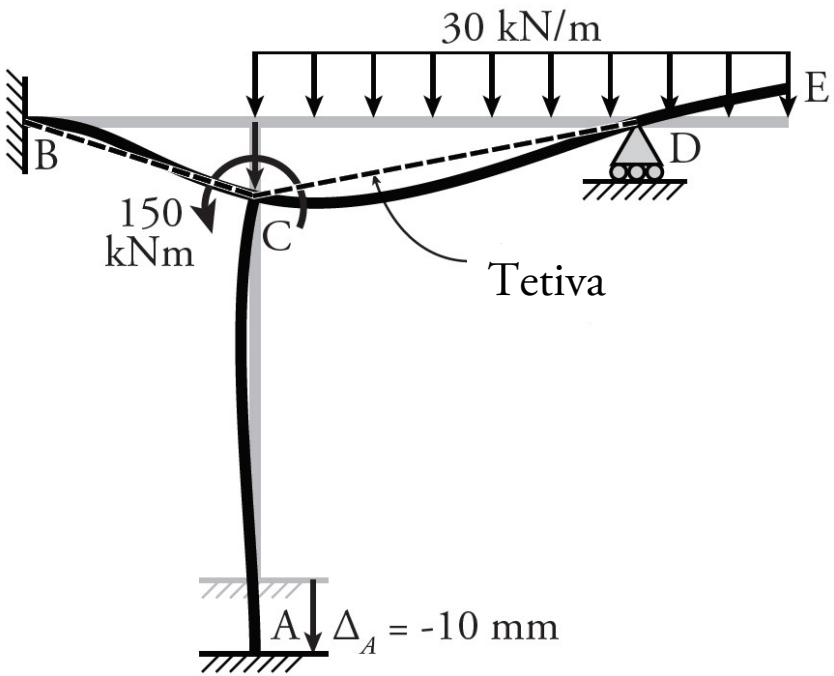
$$M_{42} = \frac{2EI}{4}\theta_2 + \frac{1.5EI}{4}u - 50 = -108.86 \text{ kNm}$$

$$M_{53} = \frac{0.75EI}{4}u - 6.25 = -38.81 \text{ kNm}$$



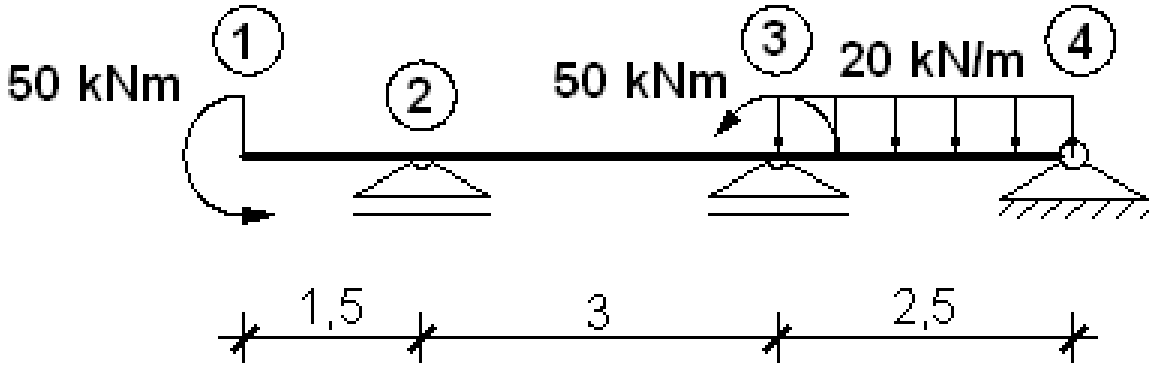
Metoda pomaka | Prepoznavanje nepoznanica





Zadatak #3

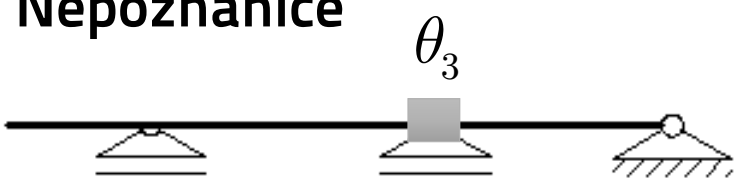
Za prikazani sustav metodom pomaka odrediti dijagram momenata savijanja.



GREDA:

b/h = 30/30 cm
E = 30 GPa

1. Nepoznanice



θ_3

NEPOMIČAN SUSTAV !

θ_2 nije nepoznanica, jer na tom mjestu znamo vrijednost momenta!

2. Krutosti štapova

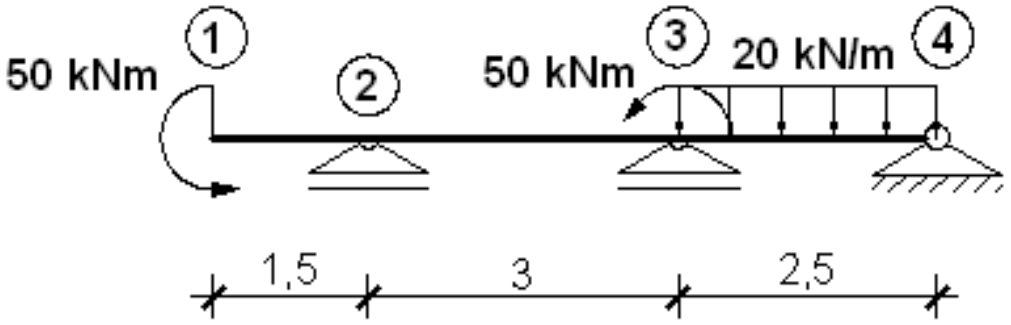
$$k_{ik} = \frac{E_{ik} I_{ik}}{L_{ik}}$$

$$\text{GREDA}: \quad EI = 20\,250 \ \text{kNm}^2$$

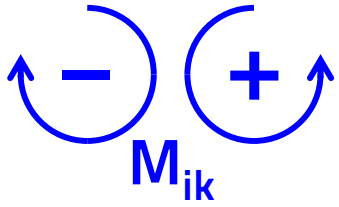
$$k_{12} = \frac{EI}{1.5}$$

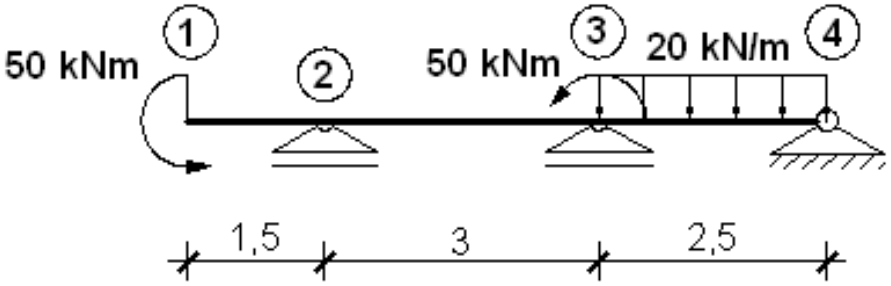
$$k_{23} = \frac{EI}{3}$$

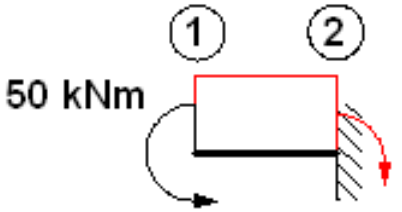
$$k_{34} = \frac{EI}{2.5}$$



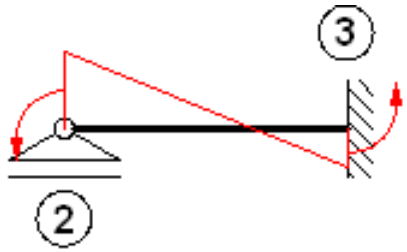
3. Momenti upetosti





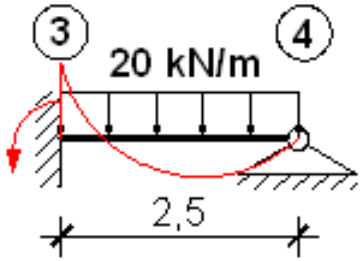


$$\bar{M}_{21} = -50 \text{ kNm}$$



$$\bar{M}_{23} = +50 \text{ kNm}$$

$$\bar{M}_{32} = +\frac{\bar{M}_{23}}{2} = +25 \text{ kNm}$$

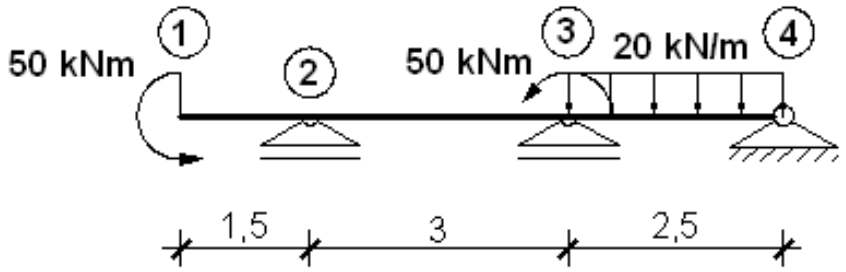


$$\bar{M}_{34} = +\frac{qL^2}{8} = +15.625 \text{ kNm}$$

4. Jednadžbe momenata na krajevima štapova

$$M_{32} = k_{23}\cdot\left(3\theta_3 - 3\psi_{23}^{\,0}\cdot u\right) + \bar{M}_{32}$$
$$= \frac{3EI}{3}\theta_3 + 25 = EI\theta_3 + 25$$

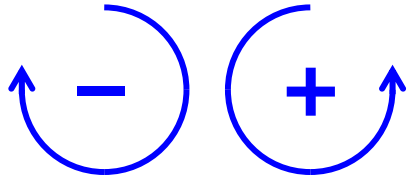
$$M_{34} = k_{34}\cdot\left(3\theta_3 - 3\psi_{34}^{\,0}\cdot u\right) + \bar{M}_{34}$$
$$= \frac{3EI}{2.5}\theta_3 + 15.625 = 1.2EI\theta_3 + 15.625$$



5. Jednadžba ravnoteže

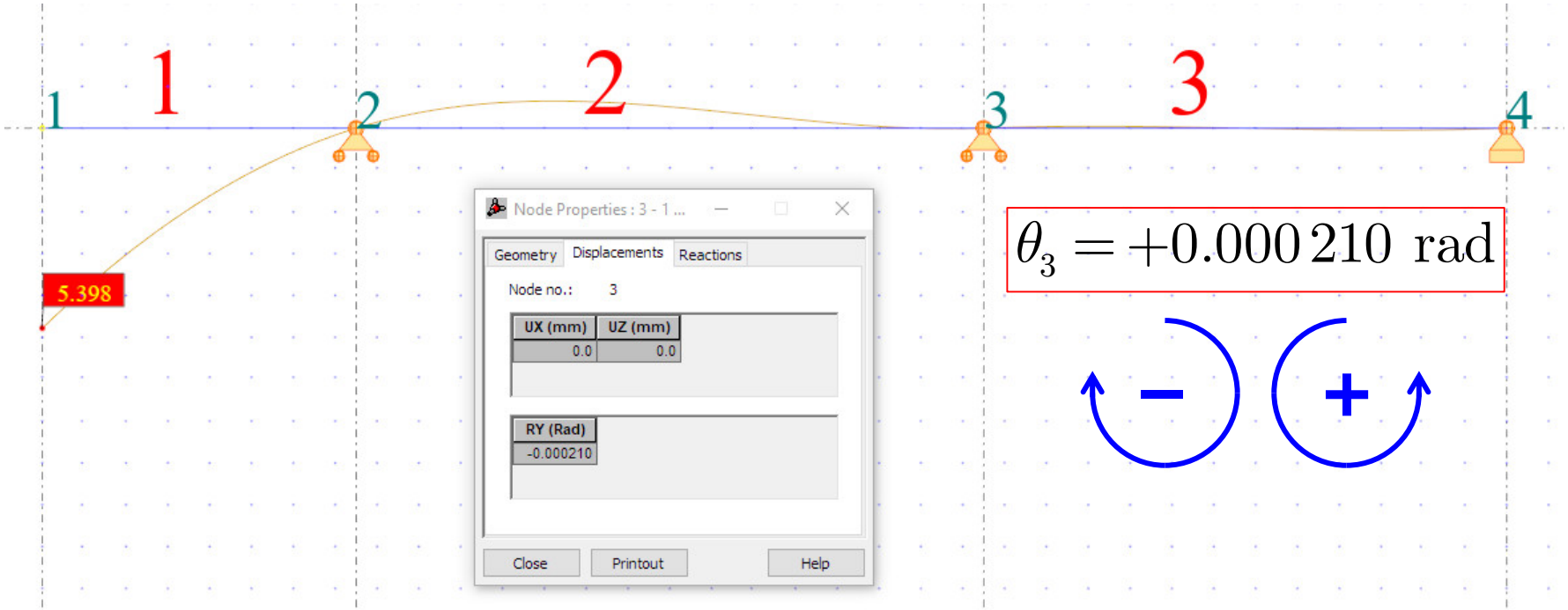
$$\sum M_3 = +50 \qquad M_{32} + M_{34} = +50$$

$$\underbrace{EI\theta_3 + 25}_{M_{32}} + \underbrace{1.2EI\theta_3 + 15.625}_{M_{34}} = +50$$

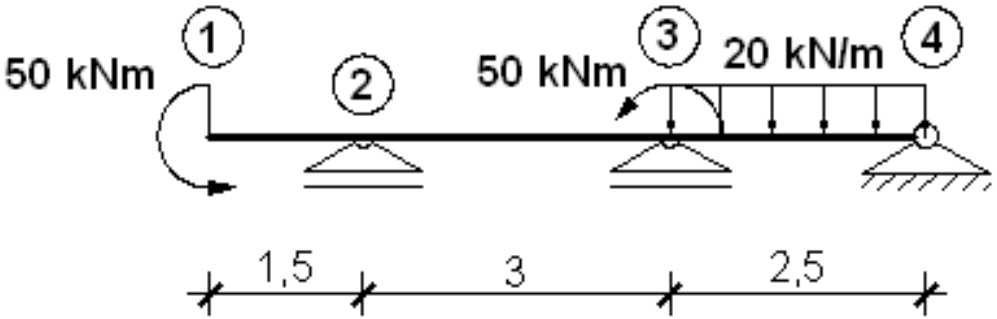


$$2.2EI\theta_3 = 9.375 \;\Rightarrow\; \theta_3 = +4.26/EI$$

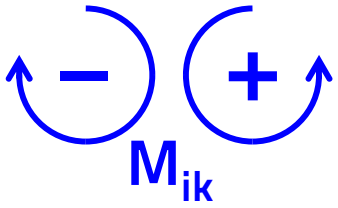
$$\boxed{\theta_3 = +0.000\,210\ \text{rad}}$$



$$\theta_3 = +0.000\,210 \text{ rad}$$

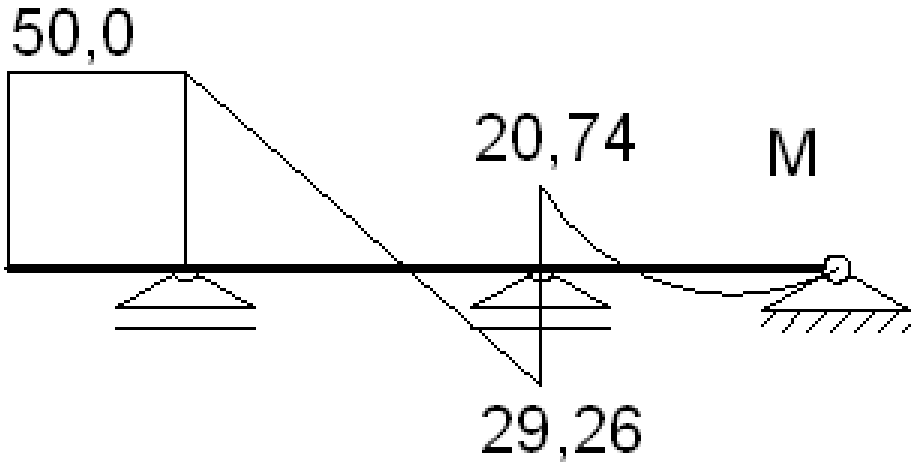


6. Konačni momentni dijagram



$$M_{32} = EI\left(4.26/EI\right)+25 = +29.26 \text{ kNm}$$

$$M_{21} = 1.2EI\left(4.26/EI\right)+15.625 = +20.74 \text{ kNm}$$



UTJECAJ PRISILNIH POMAKA LEŽAJA I TEMPERATURE

Utjecaj prisilnih pomaka ležaja i temperature kod metode pomaka uzima se pomoću **momenata upetosti**.

1. PRISILNI POMACI: Δv, Δh, Δθ

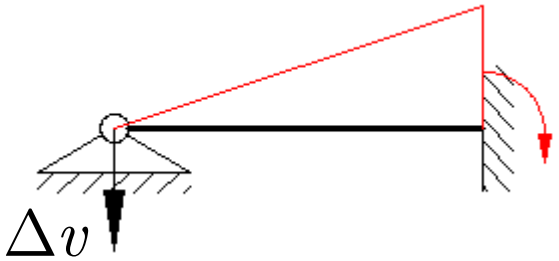
Prisilni pomaci se pri izračunu momenata upetosti množe sa **stvarnom krutošću** elementa na koji djeluje pomak:

$$k_{ik} = \frac{E_{ik} I_{ik}}{L_{ik}}$$

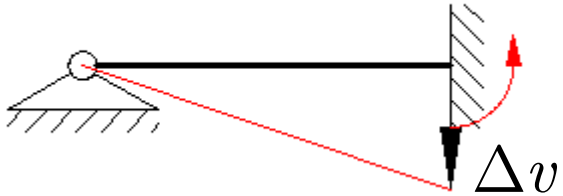
a) Translacijski pomaci: Δv, Δh

Skica momenata upetosti se crta prema djelovanju pomaka, analogno djelovanju koncentrirane sile!

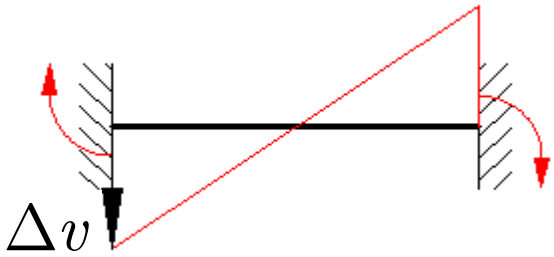
JEDNOSTRANO UPETA GREDA



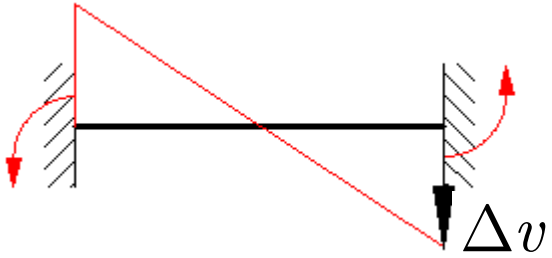
$$\bar{M}_{ki} = \pm \frac{3 \cdot k_{ik} \cdot \Delta v}{L_{ik}}$$



OBOSTRANO UPETA GREDA



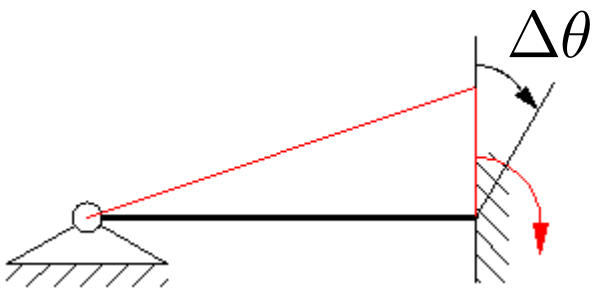
$$\bar{M}_{ik} = \bar{M}_{ki} = \pm \frac{6 \cdot k_{ik} \cdot \Delta v}{L_{ik}}$$



b) Rotacijski pomak: Δθ

Vrtnja momenata upetosti jednaka je vrtnji **Δθ**! Kod obostrano upete grede moment na čijem kraju djeluje **Δθ** biti će pomnožen sa faktorom 4, a suprotni kraj sa 2!

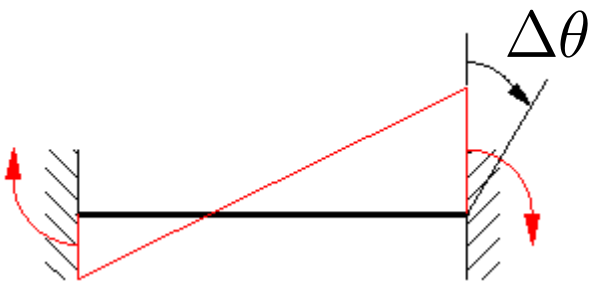
JEDNOSTRANO UPETA GREDA



$$\bar{M}_{ki} = \pm 3 \cdot k_{ik} \cdot \Delta\theta$$

OBOSTRANO UPETA GREDA

$$\bar{M}_{ik} = \pm 2 \cdot k_{ik} \cdot \Delta\theta$$



$$\bar{M}_{ki} = \pm 4 \cdot k_{ik} \cdot \Delta\theta$$

2. UTJECAJ TEMPERATURNIH PROMJENA

a) Jednolika temperatura, t_s

Jednolika temperatura uzrokuje **promjenu duljine štapa za** δ_{ts} koji se određuje prema izrazu:

$$\delta_{ts} = \alpha_T \cdot t_s \cdot L \quad [\mathrm{m}]$$

gdje je:

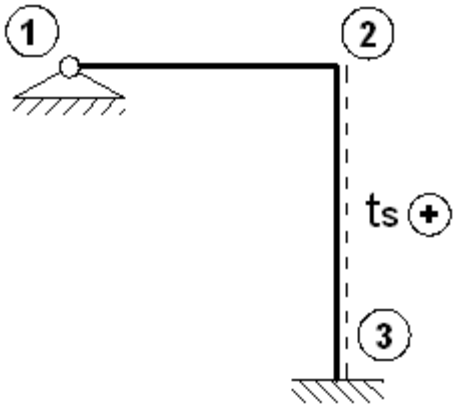
α_T – temperaturni koeficijent (α_t =1·10^{-5}C^{-1} za beton i čelik)

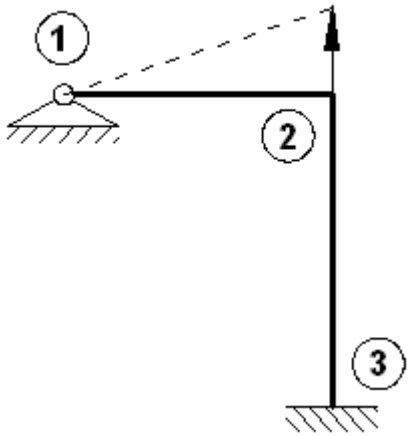
t_s – jednolika temperatura

L – duljina elementa **na kojem djeluje** jednolika temperatura.

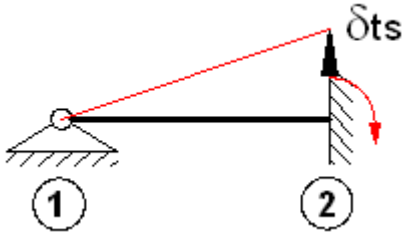
Pomak δ_{ts} uzrokuje momente upetosti **samo na susjednim elementima** (!) u odnosu na element na koji djeluje temperatura.

Primjer #1





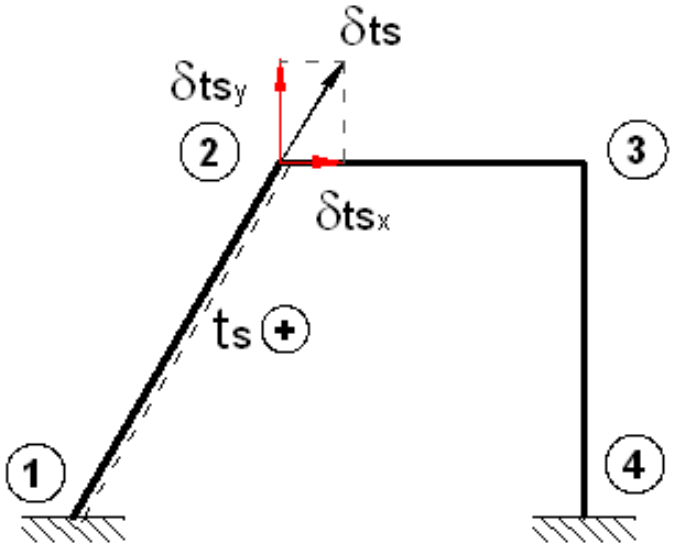
$$\delta_{ts} = \alpha_T \cdot t_s \cdot L_{23}$$

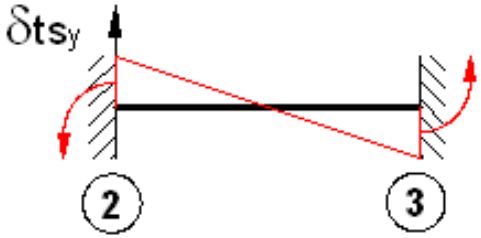


$$\bar{M}_{21} = \pm \frac{3 \cdot k_{12} \cdot \delta_{ts}}{L_{12}}$$

Primjer #2

Uzima se u obzir samo ona **projekcija pomaka δ_{ts} koja djeluje okomito na susjedni element** jer samo ona izaziva moment savijanja!

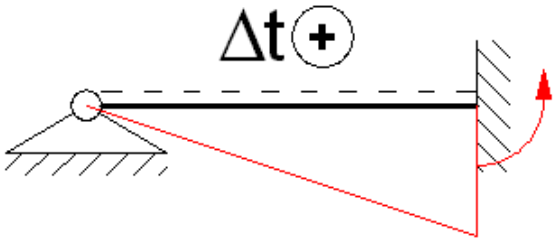




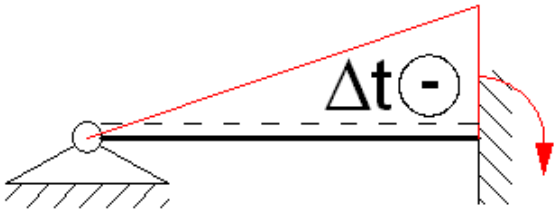
$$\bar{M}_{23} = \bar{M}_{32} = \frac{6 \cdot k_{23} \cdot \delta_{ts}^{Y}}{L_{23}}$$

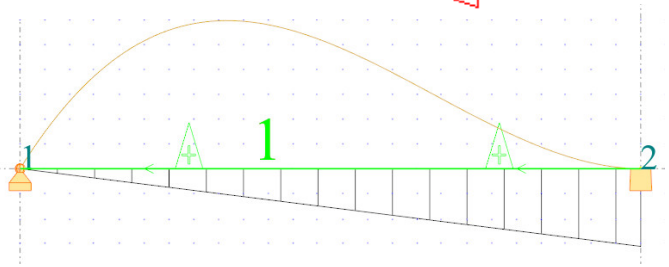
b) Nejednolika temperatura, Δt

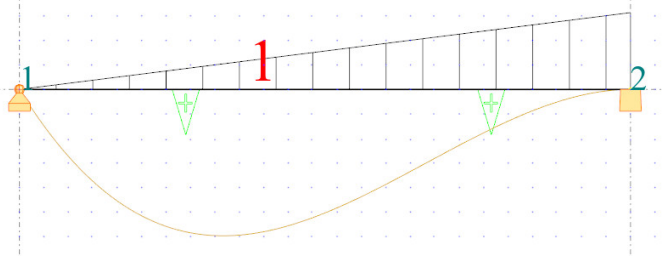
JEDNOSTRANO UPETA GREDA



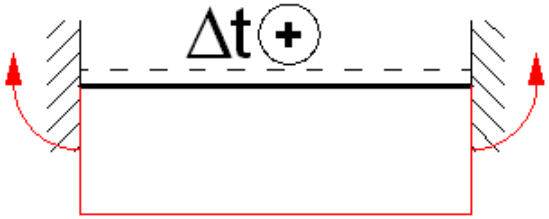
$$\bar{M}_{ki} = \frac{1.5 \cdot EI \cdot \alpha_T \cdot \Delta t}{h}$$



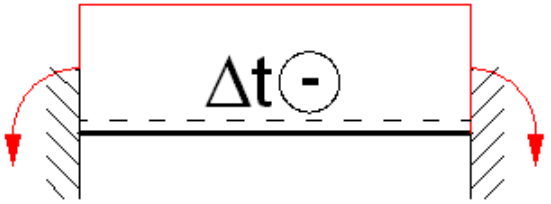




OBOSTRANO UPETA GREDA

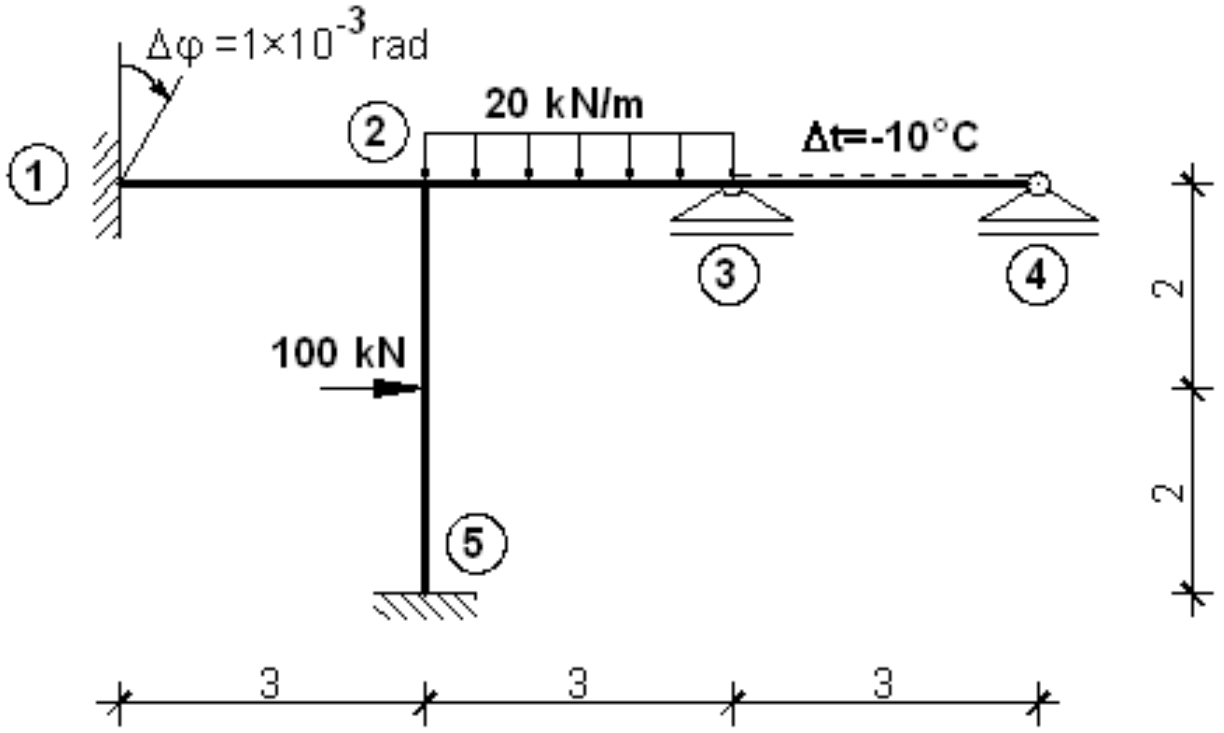


$$\bar{M}_{ik} = -\bar{M}_{ki} = \frac{EI \cdot \alpha_T \cdot \Delta t}{h}$$



Zadatak #4

Za prikazani sustav metodom pomaka odrediti dijagram momenata savijanja.



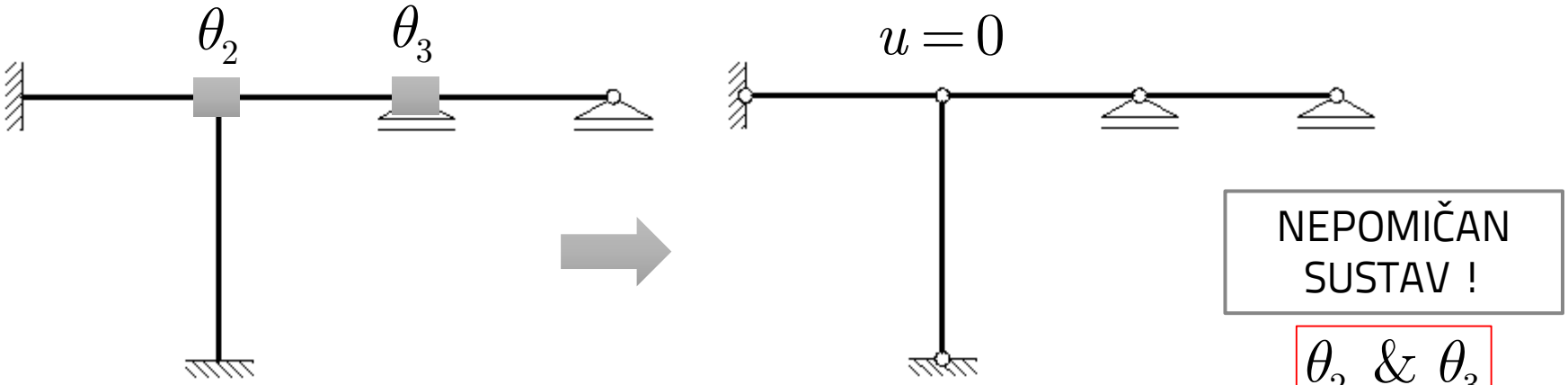
STUP/GREDA

$b/h = 40/40$ cm

$E = 30$ GPa

$$\alpha_T = \frac{1 \cdot 10^{-5}}{1°\mathrm{C}}$$

1. Nepoznanice

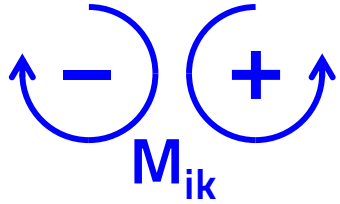


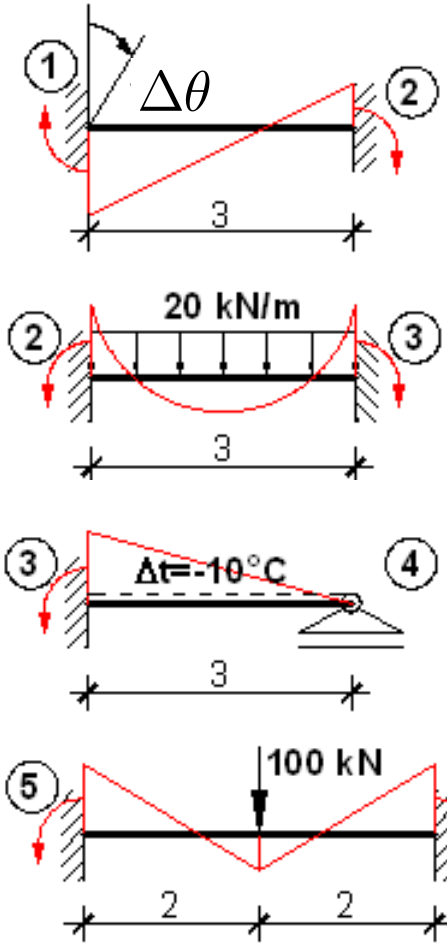
2. Krutosti štapova

$$k_{ik} = \frac{E_{ik} I_{ik}}{L_{ik}} \qquad \text{STUP/GREDA}: \quad EI = 64\,000 \ \text{kNm}^2$$

$$k_{12} = \frac{EI}{3} = 0.333EI = k_{23} = k_{34} \qquad k_{25} = \frac{EI}{4} = 0.25EI$$

3. Momenti upetosti





$$\bar{M}_{12} = -4 \cdot k_{12} \cdot \Delta\theta = -85.33 \text{ kNm}$$

$$\bar{M}_{21} = -2 \cdot k_{12} \cdot \Delta\theta = -42.67 \text{ kNm}$$

$$\bar{M}_{23} = +\frac{qL^2}{12} = +15 \text{ kNm}$$

$$\bar{M}_{32} = -15 \text{ kNm}$$

$$\bar{M}_{34} = +\frac{1.5 \cdot EI \cdot \alpha_T \cdot \Delta t}{h} = +24 \text{ kNm}$$

$$\bar{M}_{52} = +\frac{PL}{8} = +50 \text{ kNm}$$

$$\bar{M}_{25} = -50 \text{ kNm}$$

4. Jednadžbe momenata na krajevima štapova

$$M_{12} = k_{12} \cdot (4\theta_1 + 2\theta_2 - 6\psi_{12} \cdot u) + \bar{M}_{12} = \frac{2EI}{3}\theta_2 - 85.33$$

$$M_{21} = k_{12} \cdot (4\theta_2 + 2\theta_1 - 6\psi_{12} \cdot u) + \bar{M}_{21} = \frac{4EI}{3}\theta_2 - 42.67$$

$$M_{23} = k_{23} \cdot (4\theta_2 + 2\theta_3 - 6\psi_{23} \cdot u) + \bar{M}_{23} = \frac{4EI}{3}\theta_2 + \frac{2EI}{3}\theta_3 + 15$$

$$M_{32} = k_{23} \cdot (4\theta_3 + 2\theta_2 - 6\psi_{23} \cdot u) + \bar{M}_{32} = \frac{2EI}{3}\theta_2 + \frac{4EI}{3}\theta_3 - 15$$

$$M_{34} = k_{34} \cdot (3\theta_3 - 3\psi_{34} \cdot u) + \bar{M}_{34} = EI\theta_3 + 24$$

$$M_{25} = k_{25} \cdot (4\theta_2 + 2\theta_5 - 6\psi_{25} \cdot u) + \bar{M}_{25} = EI\theta_2 - 50$$

$$M_{52} = k_{25} \cdot (4\theta_5 + 2\theta_2 - 6\psi_{25} \cdot u) + \bar{M}_{52} = \frac{EI}{2}\theta_2 + 50$$

5. Jednadžbe ravnoteže

$$\sum M_2 = 0 \qquad M_{21} + M_{23} + M_{25} = 0$$

$$\underbrace{\frac{4EI}{3}\theta_2 - 42.67}_{M_{21}} + \underbrace{\frac{4EI}{3}\theta_2 + \frac{2EI}{3}\theta_3 + 15}_{M23} + \underbrace{EI\theta_2 - 50}_{M_{25}} = 0$$

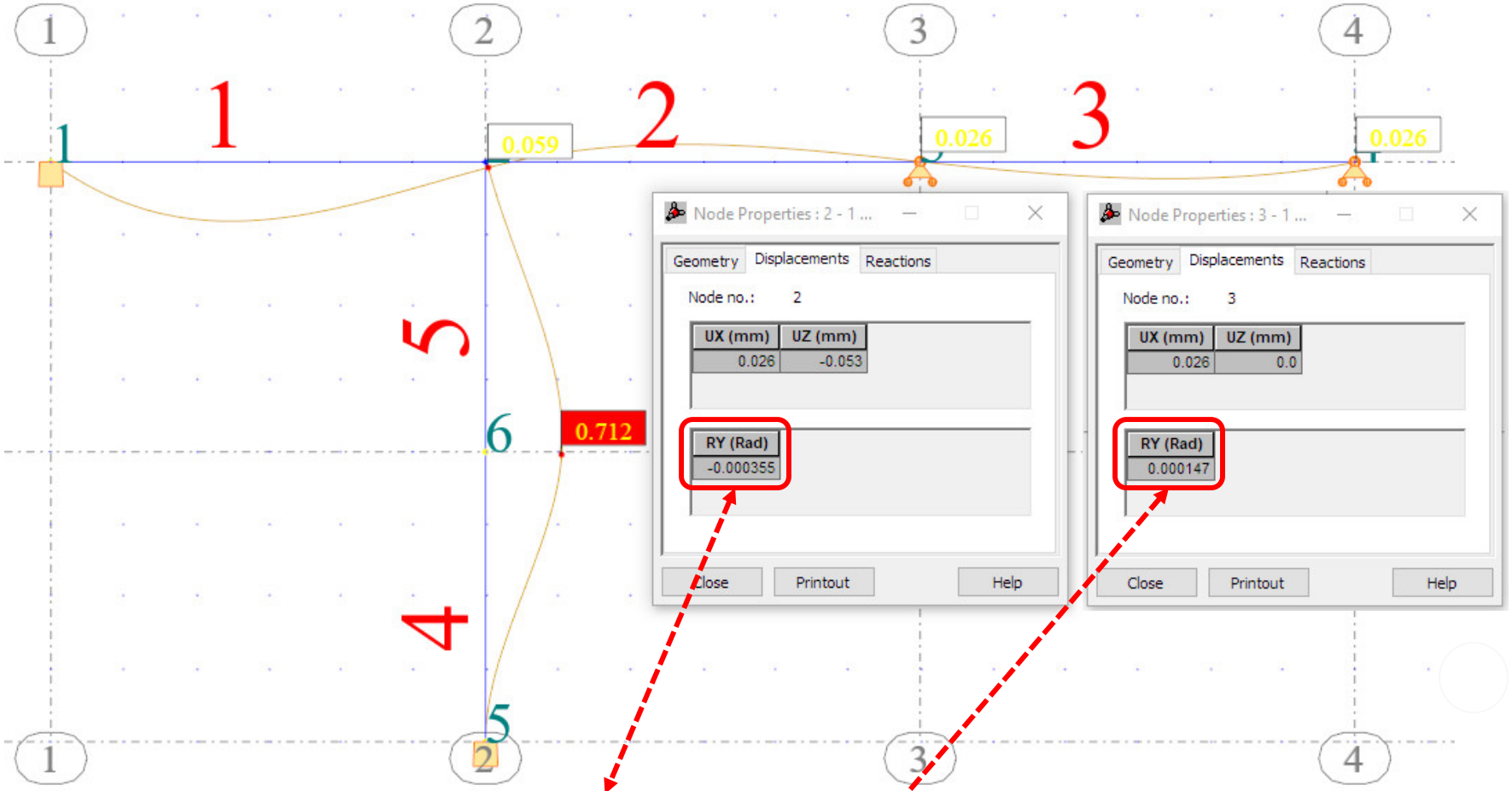
$$\sum M_3 = 0 \qquad M_{32} + M_{34} = 0$$

$$\underbrace{\frac{2EI}{3}\theta_2 + \frac{4EI}{3}\theta_3 - 15}_{M_{32}} + \underbrace{EI\theta_3 + 24}_{M_{34}} = 0$$

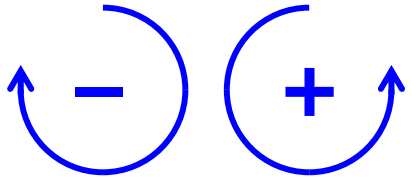
$$\frac{11EI}{3}\theta_2 + \frac{2EI}{3}\theta_3 = 77.67$$

$$\frac{2EI}{3}\theta_2 + \frac{7EI}{3}\theta_3 = -9$$

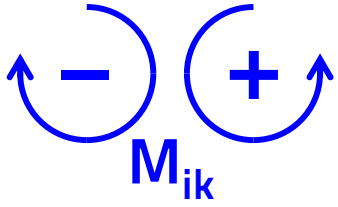
$$\theta_2 = +23.24/EI \approx +0.000\,363 \text{ rad}$$
$$\theta_3 = -10.49/EI \approx -0.000\,164 \text{ rad}$$



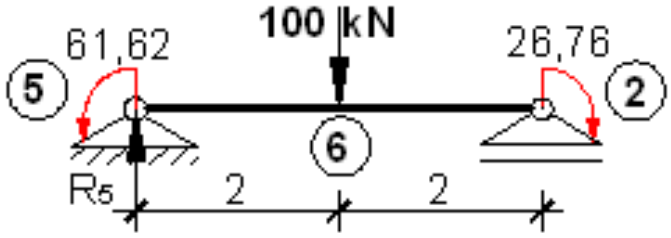
$$\theta_2 = +23.24/EI \approx +0.000\,363 \text{ rad}$$
$$\theta_3 = -10.49/EI \approx -0.000\,164 \text{ rad}$$

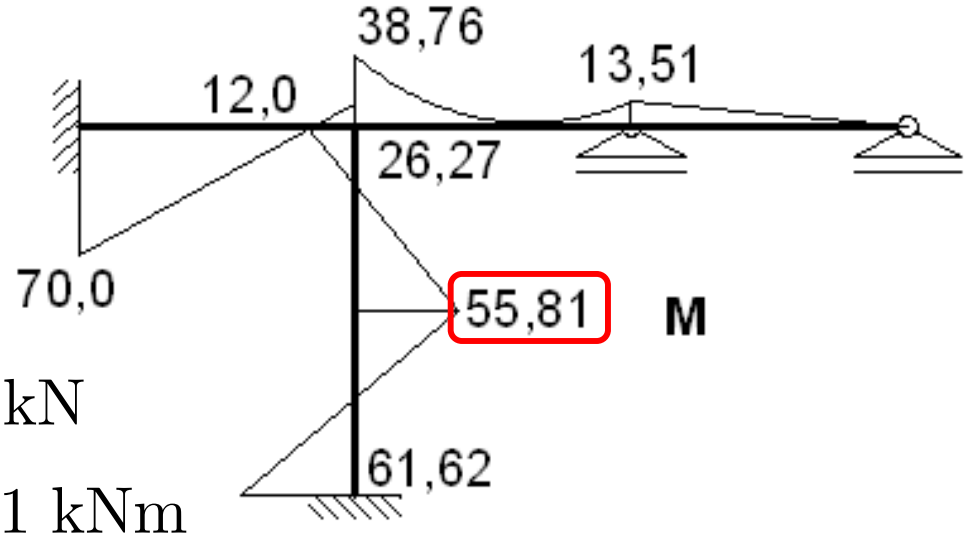


6. Konačni momentni dijagram



$$M_{12} = -70.00 \text{ kNm} \quad M_{32} = -13.51 \text{ kNm}$$
$$M_{21} = -12.00 \text{ kNm} \quad M_{34} = +13.51 \text{ kNm}$$
$$M_{23} = +38.76 \text{ kNm} \quad M_{25} = -26.76 \text{ kNm}$$
$$M_{52} = +61.62 \text{ kNm}$$

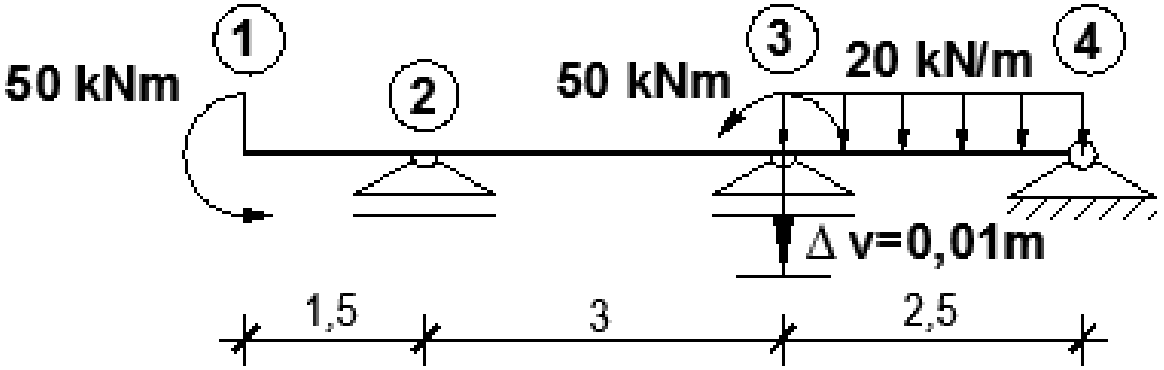




$$\sum M_2 = 0 \quad \Rightarrow \quad R_5 = 58.72 \text{ kN}$$
$$M_6 = 58.72 \cdot 2 - 61.62 = 55.81 \text{ kNm}$$

Zadatak #5

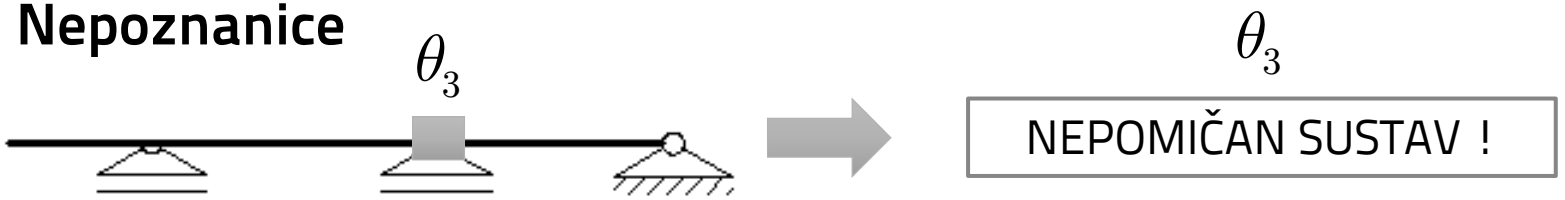
Za prikazani sustav metodom pomaka odrediti dijagram momenata savijanja.



GREDA:

b/h = 30/30 cm
E = 30 GPa

1. Nepoznanice



θ_2 nije nepoznanica, jer na tom mjestu znamo vrijednost momenta!

2. Krutosti štapova

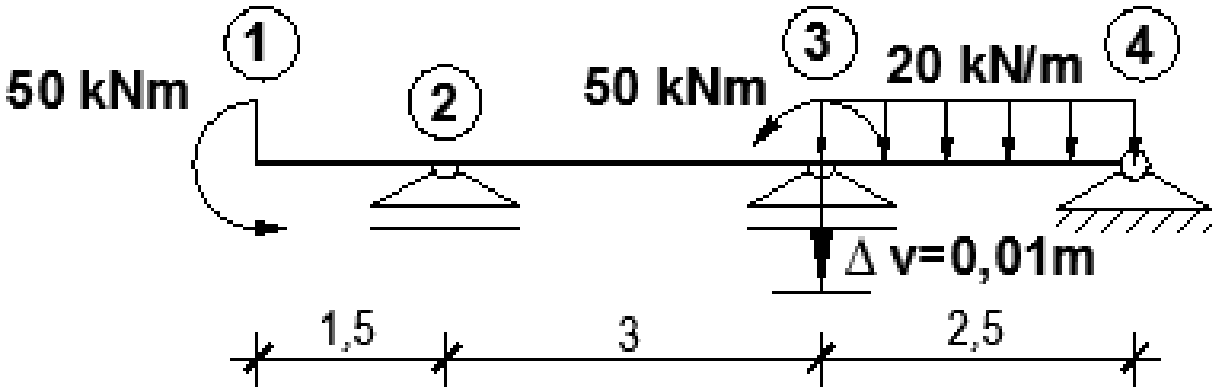
$$k_{ik} = \frac{E_{ik} I_{ik}}{L_{ik}}$$

$$\text{GREDA}: \quad EI = 20\,250 \ \text{kNm}^2$$

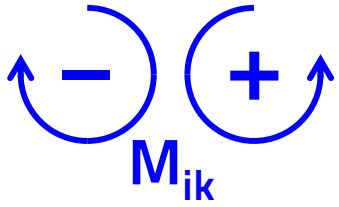
$$k_{12} = \frac{EI}{1.5}$$

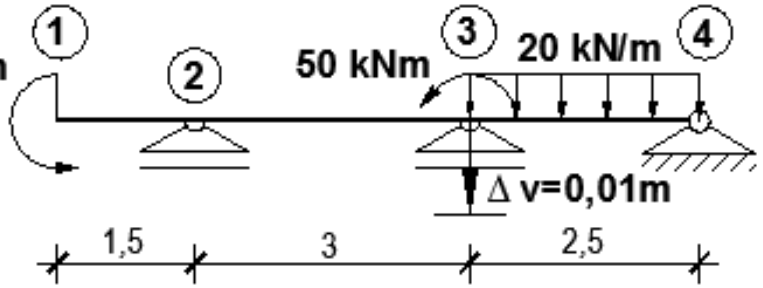
$$k_{23} = \frac{EI}{3}$$

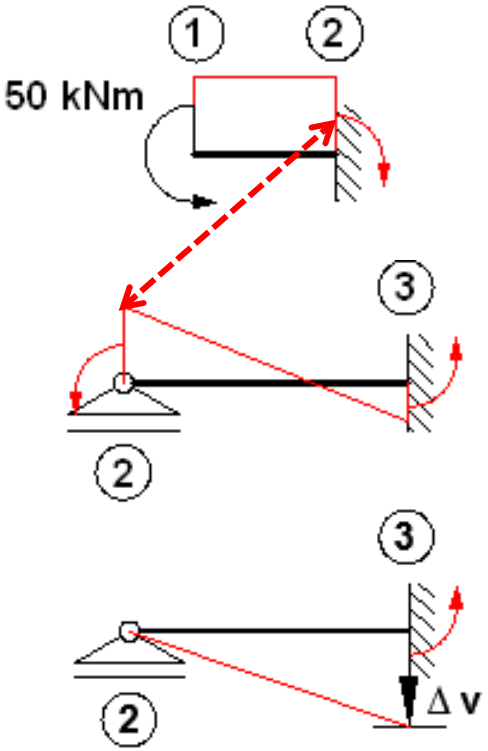
$$k_{34} = \frac{EI}{2.5}$$



3. Momenti upetosti







$$\bar{M}_{21} = -50 \text{ kNm}$$

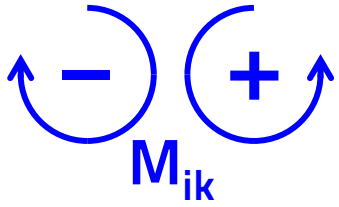
$$\bar{M}_{23}(M) = +50 \text{ kNm}$$

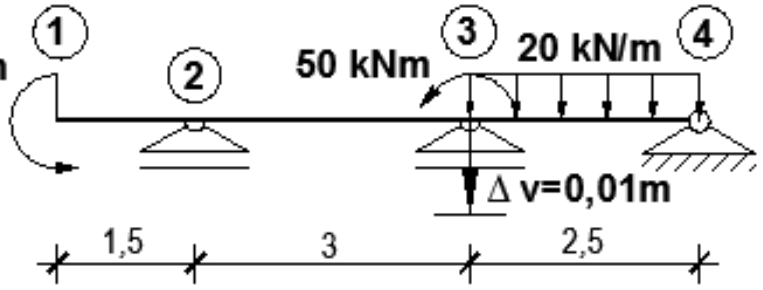
$$\bar{M}_{32}(M) = +\frac{\bar{M}_{23}}{2} = +25 \text{ kNm}$$

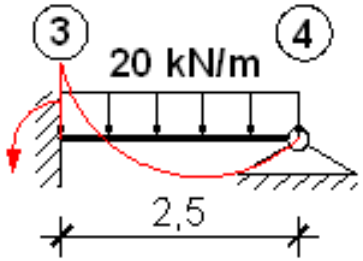
$$\bar{M}_{32}(\Delta v) = \frac{3 \cdot k_{23} \cdot \Delta v}{L_{23}} = 67.5 \text{ kNm}$$

$$\bar{M}_{32} = \underbrace{\bar{M}_{32}(M) + \bar{M}_{32}(\Delta v)}_{\text{Superpozicija momenata upetosti!}} = 92.5 \text{ kNm}$$

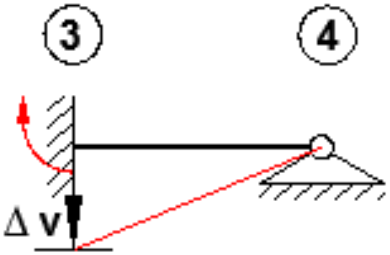
3. Momenti upetosti







$$\bar{M}_{34}(q) = +\frac{qL^2}{8} = +15.625 \text{ kNm}$$



$$\bar{M}_{34}(\Delta v) = -\frac{3 \cdot k_{34} \cdot \Delta v}{L_{34}} = -97.20 \text{ kNm}$$

$$\bar{M}_{34} = \underbrace{\bar{M}_{34}(q) + \bar{M}_{34}(\Delta v)}_{\text{Superpozicija momenata upetosti!}} = -81.57 \text{ kNm}$$

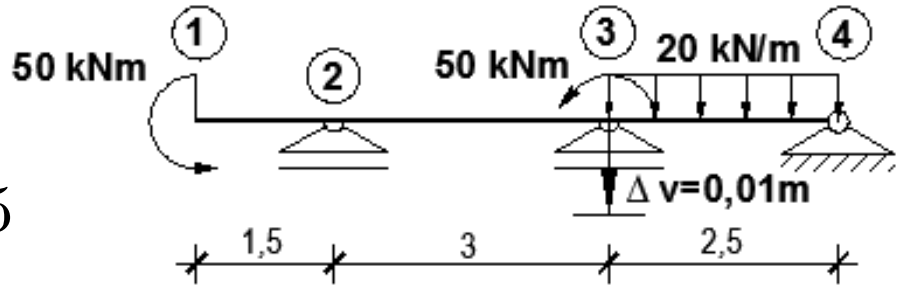
4. Jednadžbe momenata na krajevima štapova

$$M_{32} = k_{23}\cdot\left(3\theta_3 - 3\psi_{23}^{\,0}\cdot u\right) + \bar{M}_{32}$$

$$= \frac{3EI}{3}\theta_3 + 25 = EI\theta_3 + 92.5$$

$$M_{34} = k_{34}\cdot\left(3\theta_3 - 3\psi_{34}^{\,0}\cdot u\right) + \bar{M}_{34}$$

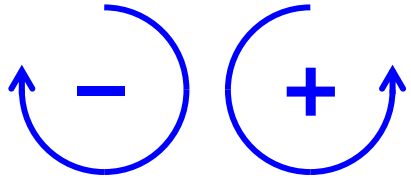
$$= \frac{3EI}{2.5}\theta_3 + 15.625 = 1.2EI\theta_3 - 81.57$$



5. Jednadžba ravnoteže

$$\sum M_3 = +50 \qquad M_{32} + M_{34} = +50$$

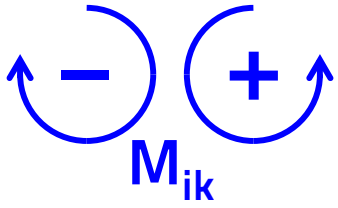
$$\underbrace{EI\theta_3 + 92.5}_{M_{32}} + \underbrace{1.2EI\theta_3 - 81.57}_{M_{34}} = +50$$



$$2.2EI\theta_3 = 39.07 \;\Rightarrow\; \theta_3 = +17.76/EI$$

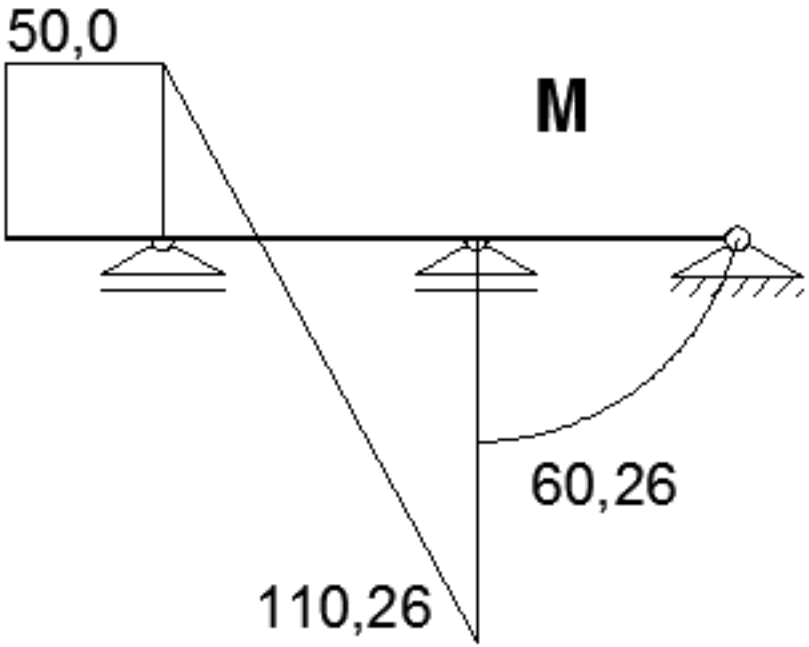
$$\boxed{\theta_3 = +0.000\,877 \text{ rad}}$$

6. Konačni momentni dijagram



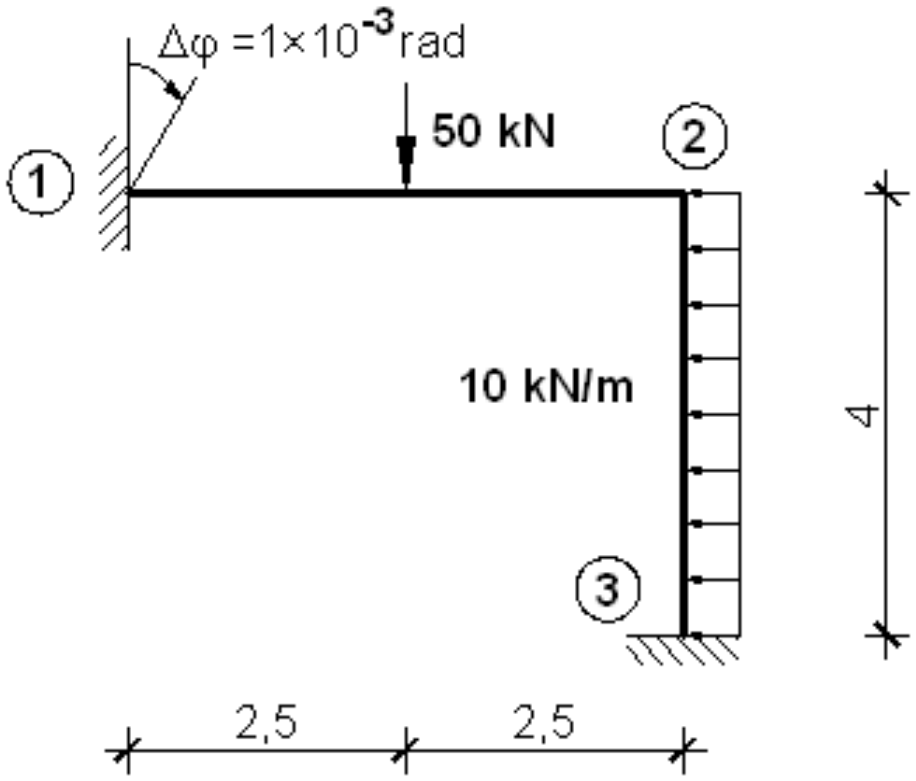
$$M_{32} = EI\left(17.76/EI\right) + 92.5 = +110.26 \text{ kNm}$$

$$M_{21} = 1.2EI\left(17.76/EI\right) - 81.57 = -60.26 \text{ kNm}$$



Zadatak #6

Za prikazani sustav metodom pomaka odrediti dijagram momenata savijanja.

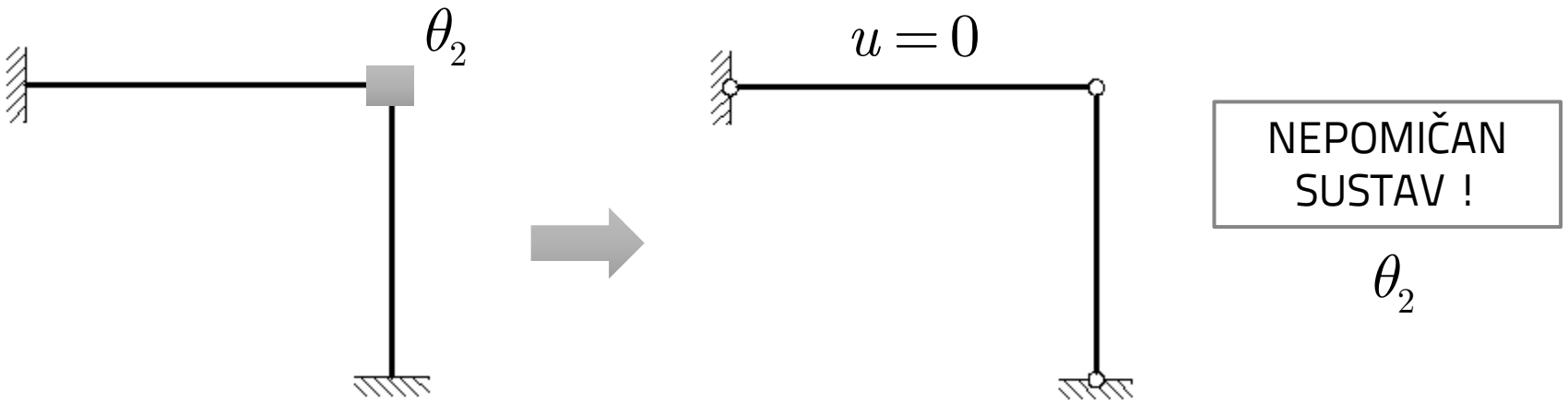


STUP: b/h = 30/30 cm

GREDA: b/h = 30/40 cm

E = 30 GPa

1. Nepoznanice



2. Krutosti štapova

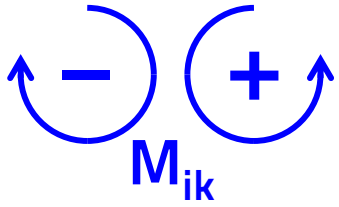
$$k_{ik} = \frac{E_{ik} I_{ik}}{L_{ik}}$$

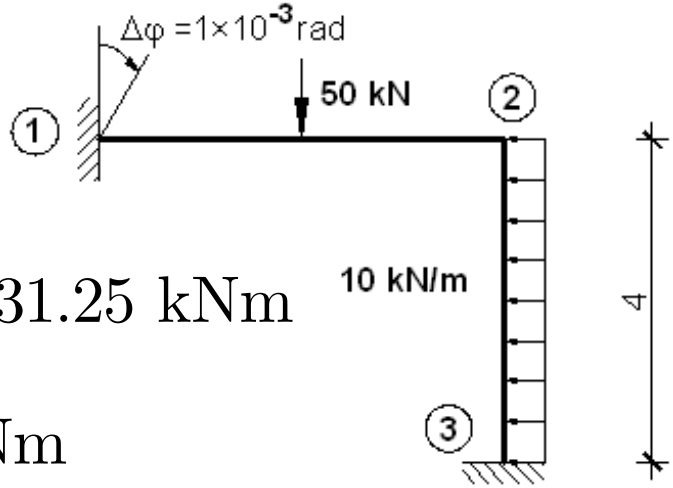
$$\text{STUP:} \quad EI_S = 20\,250 \text{ kNm}^2$$

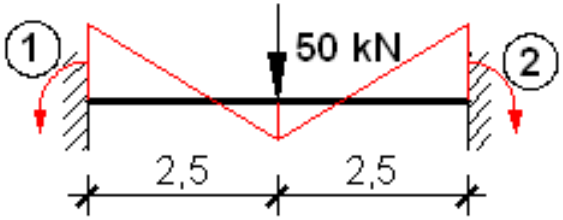
$$\text{GREDA:} \quad EI_G = 48\,000 \text{ kNm}^2$$

$$k_{12} = \frac{EI_G}{5} = 9\,600 \text{ kNm} \qquad k_{23} = \frac{EI_S}{4} = 5\,062.5 \text{ kNm}$$

3. Momenti upetosti

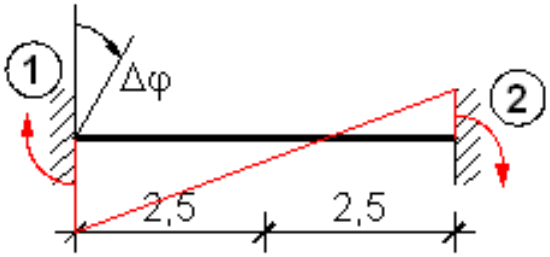


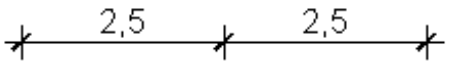




$$\bar{M}_{12}(P) = +\frac{PL}{8} = +31.25 \text{ kNm}$$

$$\bar{M}_{21}(P) = -31.25 \text{ kNm}$$



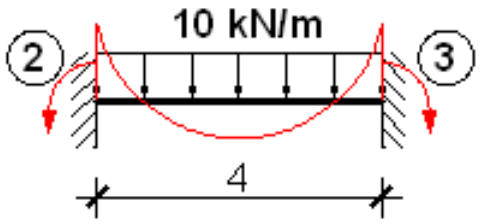


$$\bar{M}_{12}(\Delta\theta) = -4 \cdot k_{12} \cdot \Delta\theta = -38.4 \text{ kNm}$$

$$\bar{M}_{21}(\Delta\theta) = -2 \cdot k_{12} \cdot \Delta\theta = -19.2 \text{ kNm}$$

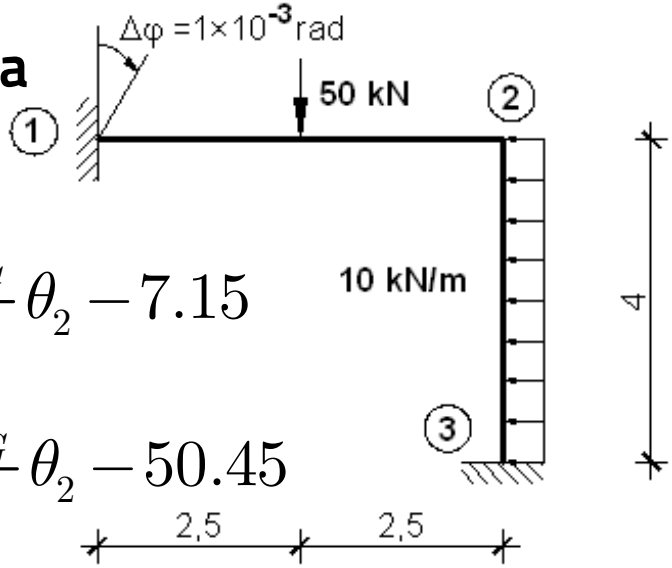
$$\bar{M}_{12} = \bar{M}_{12}(\Delta\theta) + \bar{M}_{12}(P) = -7.15 \text{ kNm}$$

$$\bar{M}_{21} = \bar{M}_{21}(\Delta\theta) + \bar{M}_{21}(P) = -50.45 \text{ kNm}$$



$$\bar{M}_{23} = \frac{qL^2}{12} = +13.33 \text{ kNm} \quad \bar{M}_{32} = -13.33 \text{ kNm}$$

4. Jednadžbe momenata na krajevima štapova



$$M_{12} = k_{12} \cdot (4\theta_1 + 2\theta_2 - 6\psi_{12} \cdot u) + \bar{M}_{12} = \frac{2EI_G}{5}\theta_2 - 7.15$$

$$M_{21} = k_{12} \cdot (4\theta_2 + 2\theta_1 - 6\psi_{12} \cdot u) + \bar{M}_{21} = \frac{4EI_G}{5}\theta_2 - 50.45$$

$$M_{23} = k_{23} \cdot (4\theta_2 + 2\theta_3 - 6\psi_{23} \cdot u) + \bar{M}_{23} = EI_S\theta_2 + 13.33$$

$$M_{32} = k_{23} \cdot (4\theta_3 + 2\theta_2 - 6\psi_{23} \cdot u) + \bar{M}_{32} = \frac{EI_S}{2}\theta_2 - 13.33$$

5. Jednadžba ravnoteže

$$\sum M_2 = 0 \qquad M_{21} + M_{23} = 0$$

$$\underbrace{\frac{4EI_G}{5}\theta_2 - 50.45}_{M_{21}} + \underbrace{EI_S\theta_2 + 13.33}_{M_{23}} = 0$$

$$\theta_2 = +0.000\,636 \text{ rad}$$

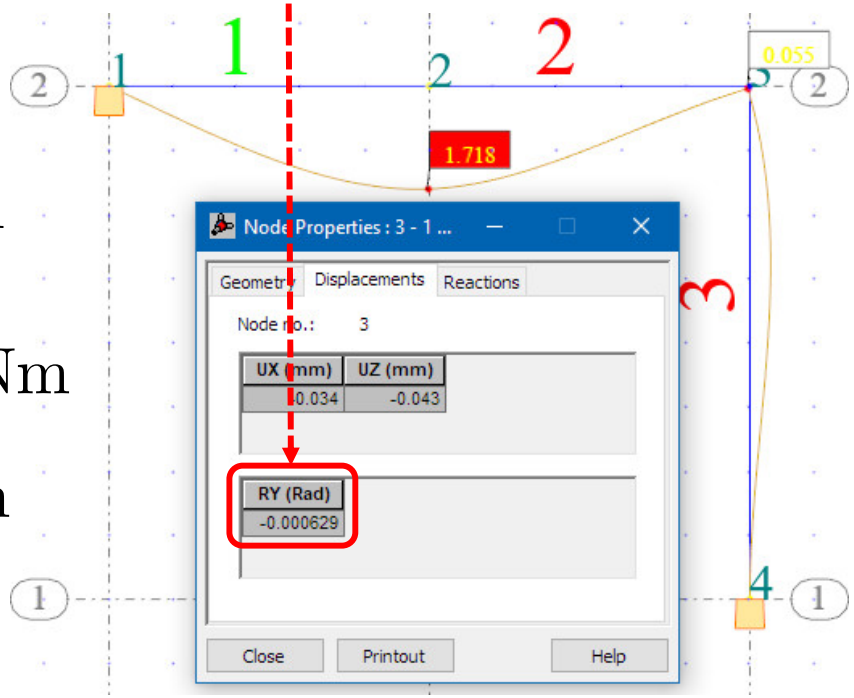
6. Konačni momentni dijagram

$$M_{12} = \frac{2EI_G}{5}\theta_2 - 7.15 = +4.96 \text{ kNm}$$

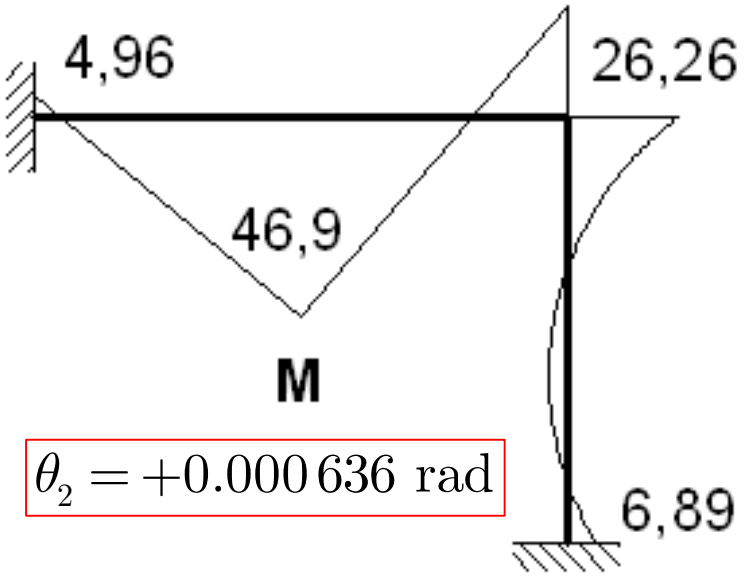
$$M_{21} = \frac{4EI_G}{5}\theta_2 - 50.45 = -26.24 \text{ kNm}$$

$$M_{23} = EI_S\theta_2 + 13.33 = +26.21 \text{ kNm}$$

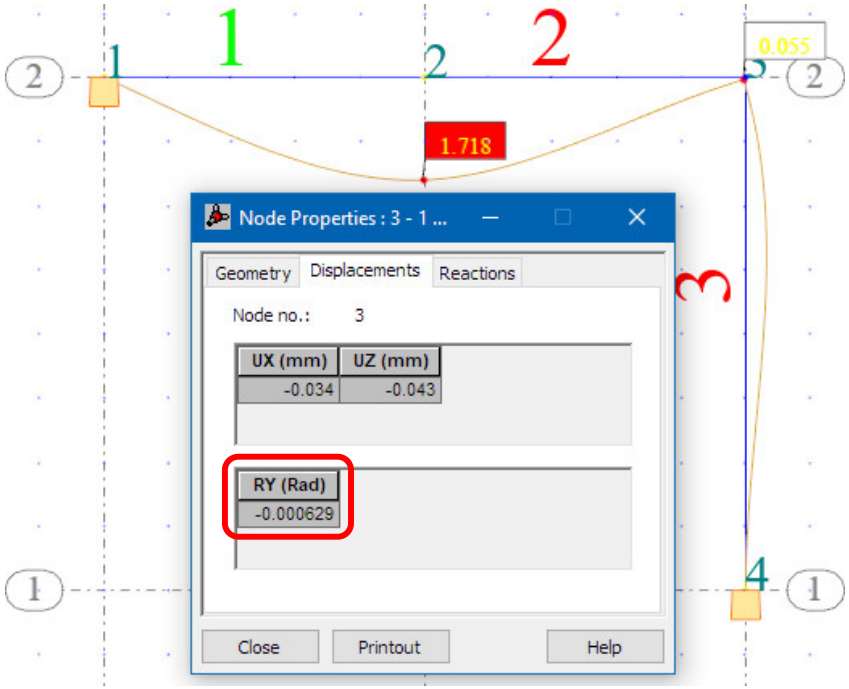
$$M_{32} = \frac{EI_S}{2}\theta_2 - 13.33 = -6.89 \text{ kNm}$$

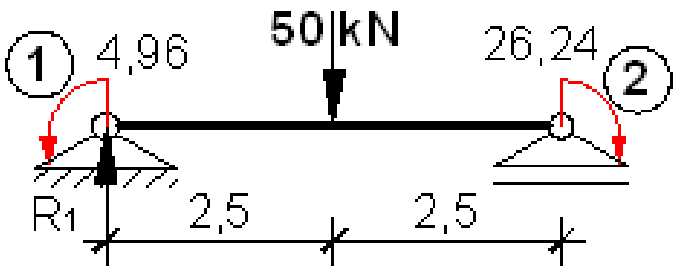


6. Konačni momentni dijagram



$$\theta_2 = +0.000\,636 \text{ rad}$$





$$\sum M_2 = 0$$

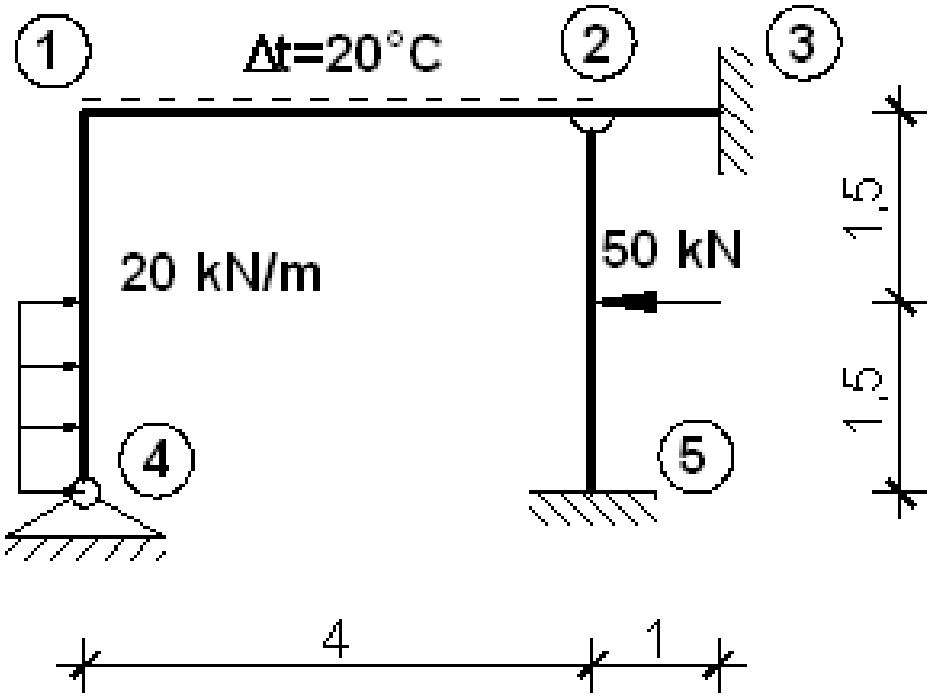
$$4.96 - 26.24 + 50 \cdot 2.5 - R_1 \cdot 5 = 0$$

$$R_1 = 20.74 \text{ kN}$$

$$M_4 = -4.96 + R_1 \cdot 2.5 = 46.9 \text{ kNm}$$

Zadatak #7

Za prikazani sustav metodom pomaka odrediti dijagram momenata savijanja.



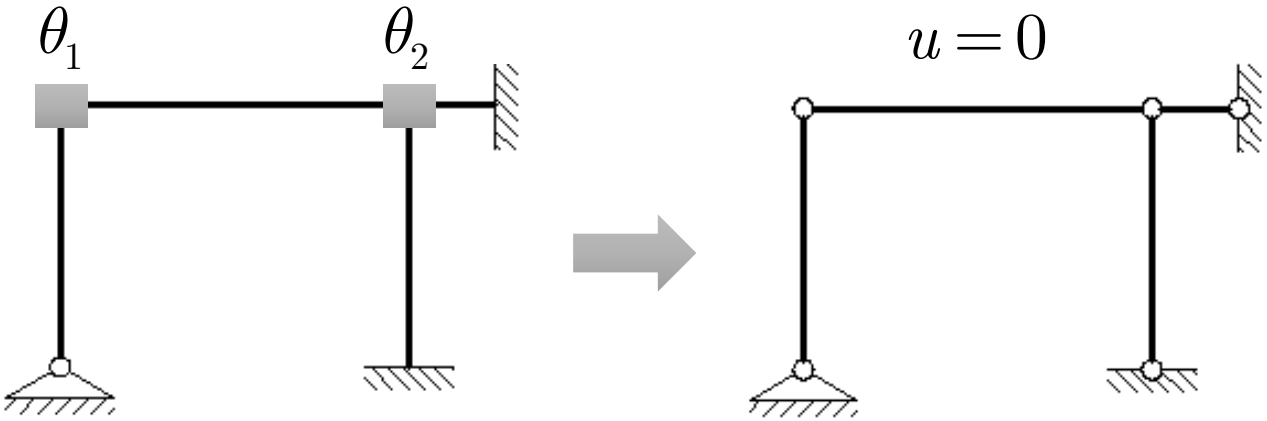
STUP/GREDA

$b/h = 40/40 \text{ cm}$

$E = 30 \text{ GPa}$

$$\alpha_T = \frac{1 \cdot 10^{-5}}{1°\text{C}}$$

1. Nepoznanice



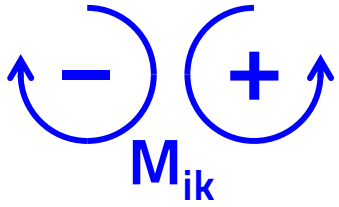
NEPOMIČAN SUSTAV !

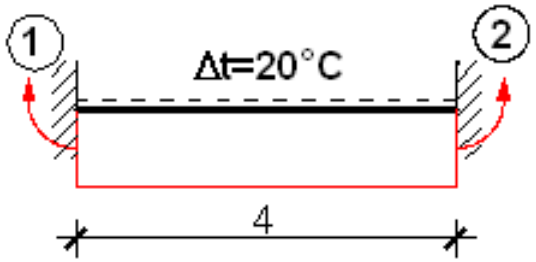
2. Krutosti štapova

$$k_{ik} = \frac{E_{ik} I_{ik}}{L_{ik}} \qquad \text{STUP/GREDA}: \quad EI = 64\,000 \ \text{kNm}^2$$

$$k_{12} = \frac{EI}{4} = 0.25EI \qquad k_{23} = EI \qquad k_{14} = \frac{EI}{3} = 0.333EI = k_{25}$$

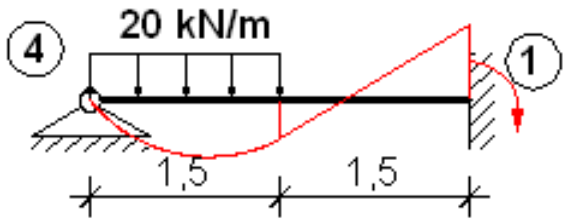
3. Momenti upetosti





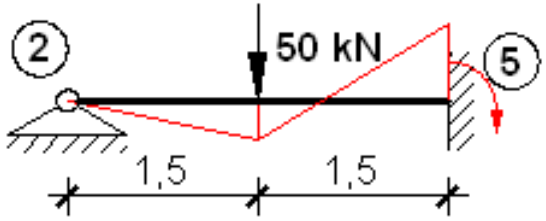
$$\bar{M}_{12} = -\frac{EI \cdot \alpha_T \cdot \Delta t}{h} = -32.0 \text{ kNm}$$

$$\bar{M}_{21} = +32.0 \text{ kNm}$$



$$\bar{M}_{14} = -\frac{qc^2}{8L^2}\left(2L^2 - c^2\right) = -9.84 \text{ kNm}$$

$$c = 1.5 \text{ m}; \quad L = 3 \text{ m}$$



$$\bar{M}_{52} = -\frac{3PL}{16} = -28.125 \text{ kNm}$$

4. Jednadžbe momenata na krajevima štapova

$$M_{12} = k_{12} \cdot (4\theta_1 + 2\theta_2 - 6\psi_{12} \cdot u) + \bar{M}_{12} = \frac{4EI}{4}\theta_1 + \frac{2EI}{4}\theta_2 - 32$$

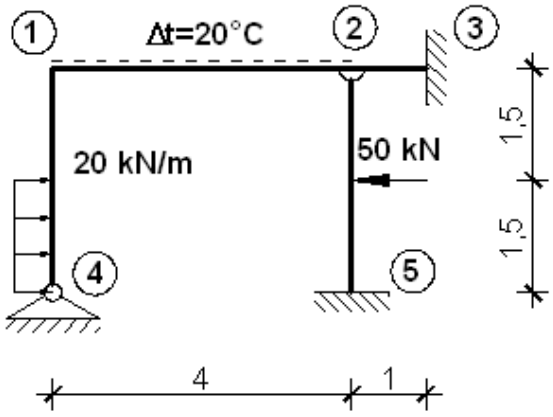
$$M_{21} = k_{12} \cdot (4\theta_2 + 2\theta_1 - 6\psi_{12} \cdot u) + \bar{M}_{21} = \frac{2EI}{4}\theta_1 + \frac{4EI}{4}\theta_2 + 32$$

$$M_{23} = k_{23} \cdot (4\theta_2 + 2\theta_3 - 6\psi_{23} \cdot u) + \bar{M}_{23} = 4EI\theta_2$$

$$M_{32} = k_{23} \cdot (4\theta_3 + 2\theta_2 - 6\psi_{23} \cdot u) + \bar{M}_{32} = 2EI\theta_2$$

$$M_{14} = k_{14} \cdot (3\theta_1 - 3\psi_{14} \cdot u) + \bar{M}_{14} = \frac{3EI}{3}\theta_1 - 9.84$$

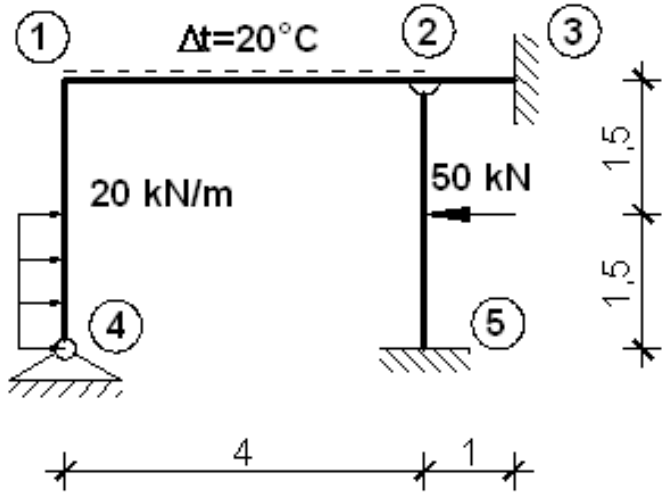
$$M_{52} = k_{25} \cdot (3\theta_5 - 3\psi_{25} \cdot u) + \bar{M}_{52} = -28.125 \text{ kNm}$$



5. Jednadžbe ravnoteže

$$\sum M_1 = 0 \qquad M_{12} + M_{14} = 0$$

$$\underbrace{\frac{\cancel{4}EI}{\cancel{4}}\theta_1 + \frac{2EI}{4}\theta_2 - 32}_{M_{12}} + \underbrace{\frac{\cancel{3}EI}{\cancel{3}}\theta_1 - 9.84}_{M_{14}} = 0$$



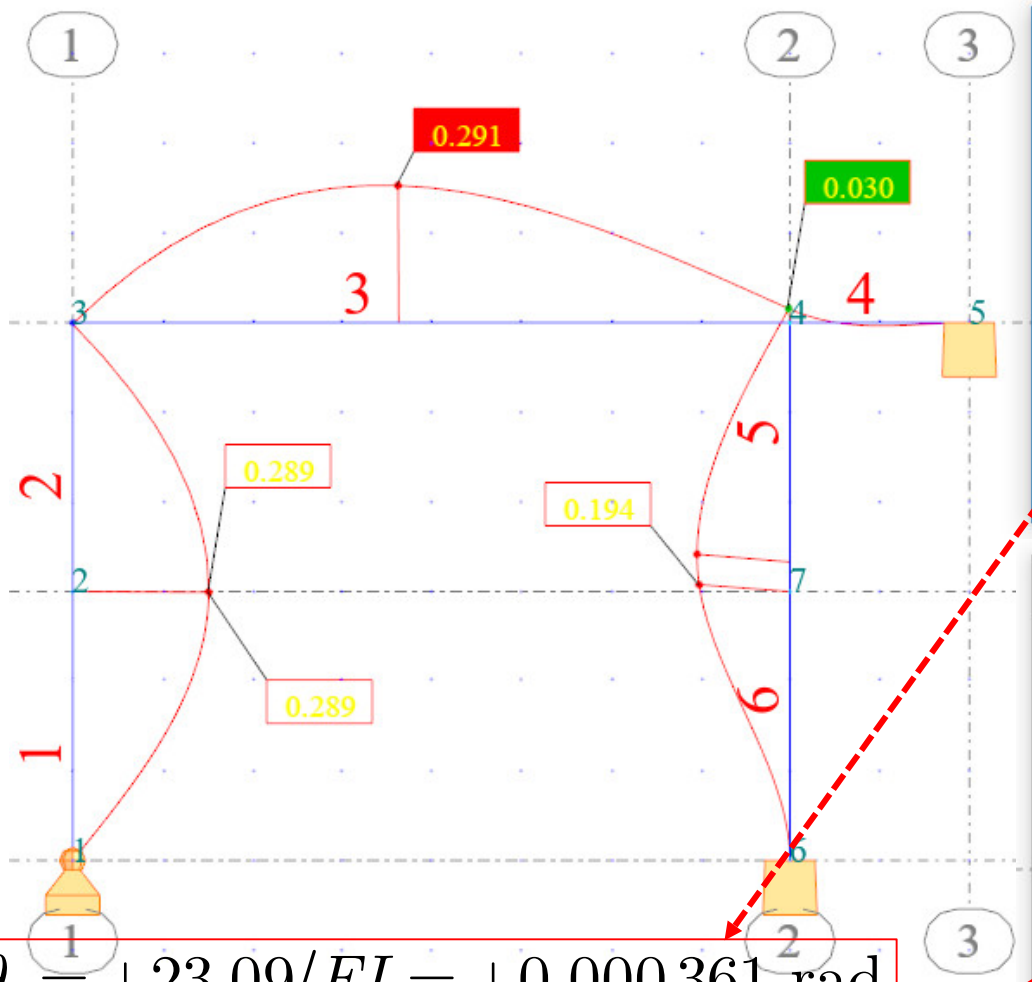
$$\sum M_2 = 0 \qquad M_{21} + M_{23} = 0$$

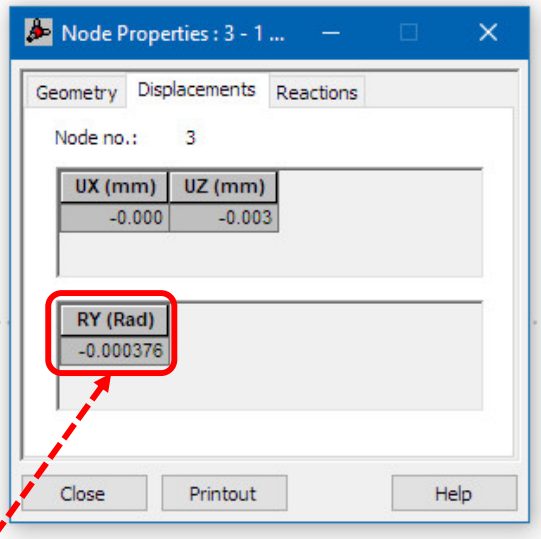
$$\underbrace{\frac{2EI}{4}\theta_1 + \frac{\cancel{4}EI}{\cancel{4}}\theta_2 + 32}_{M_{21}} + \underbrace{4EI\theta_2}_{M_{23}} = 0$$

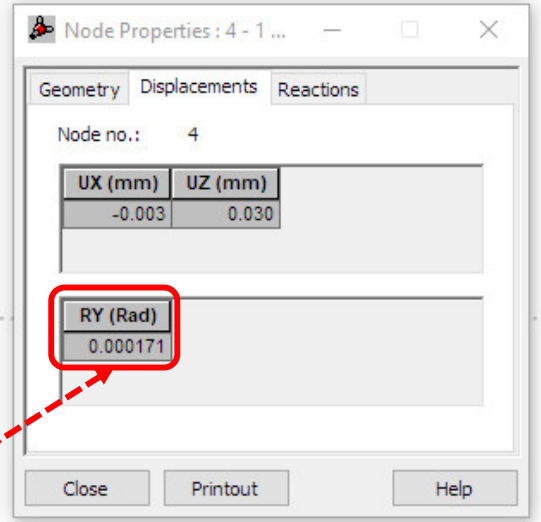
$$2EI\,\theta_1 + \frac{1EI}{2}\theta_2 = +41.48$$

$$\frac{1EI}{2}\theta_1 + 5\,\theta_2 = -32$$

$$\theta_1 = +23.09/EI = +0.000\,361 \text{ rad}$$
$$\theta_2 = -8.71/EI = -0.000\,136 \text{ rad}$$



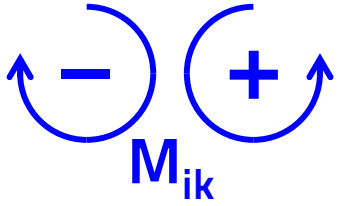




$$\theta_1 = +23.09/EI = +0.000\,361 \text{ rad}$$

$$\theta_2 = -8.71/EI = -0.000\,136 \text{ rad}$$

6. Konačni momentni dijagram



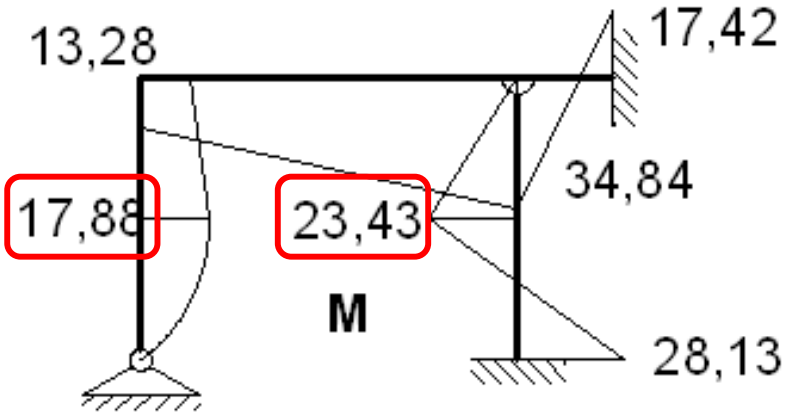
$$M_{12} = EI\left(+23.09/EI\right)+0.5\left(-8.71/EI\right)-32 = -13.26\ \text{kNm}$$

$$M_{21} = 0.5+\left(+23.09/EI\right)+\left(-8.71/EI\right)+32 = +34.84\ \text{kNm}$$

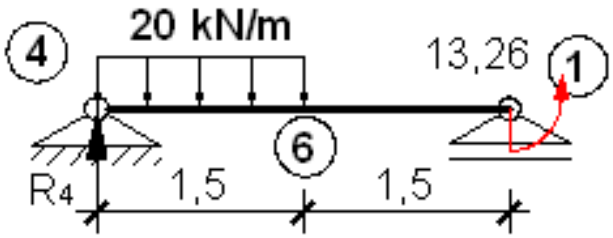
$$M_{23} = 4EI\left(-8.71/EI\right) = -34.84\ \text{kNm}$$

$$M_{32} = 2EI\left(-8.71/EI\right) = -17.42\ \text{kNm}$$

$$M_{14} = EI\left(+23.09/EI\right)-9.84 = +13.26\ \text{kNm}$$

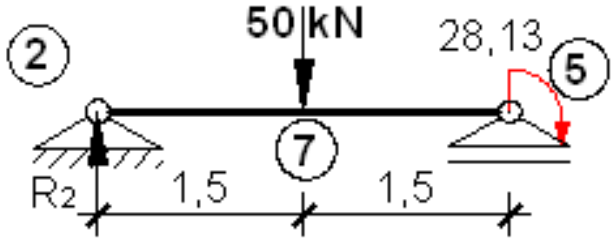


Iznosi unutarnjih sila na elementima



$$\sum M_1 = 0 \quad \Rightarrow \quad R_4 = 26.92 \text{ kN} \uparrow$$

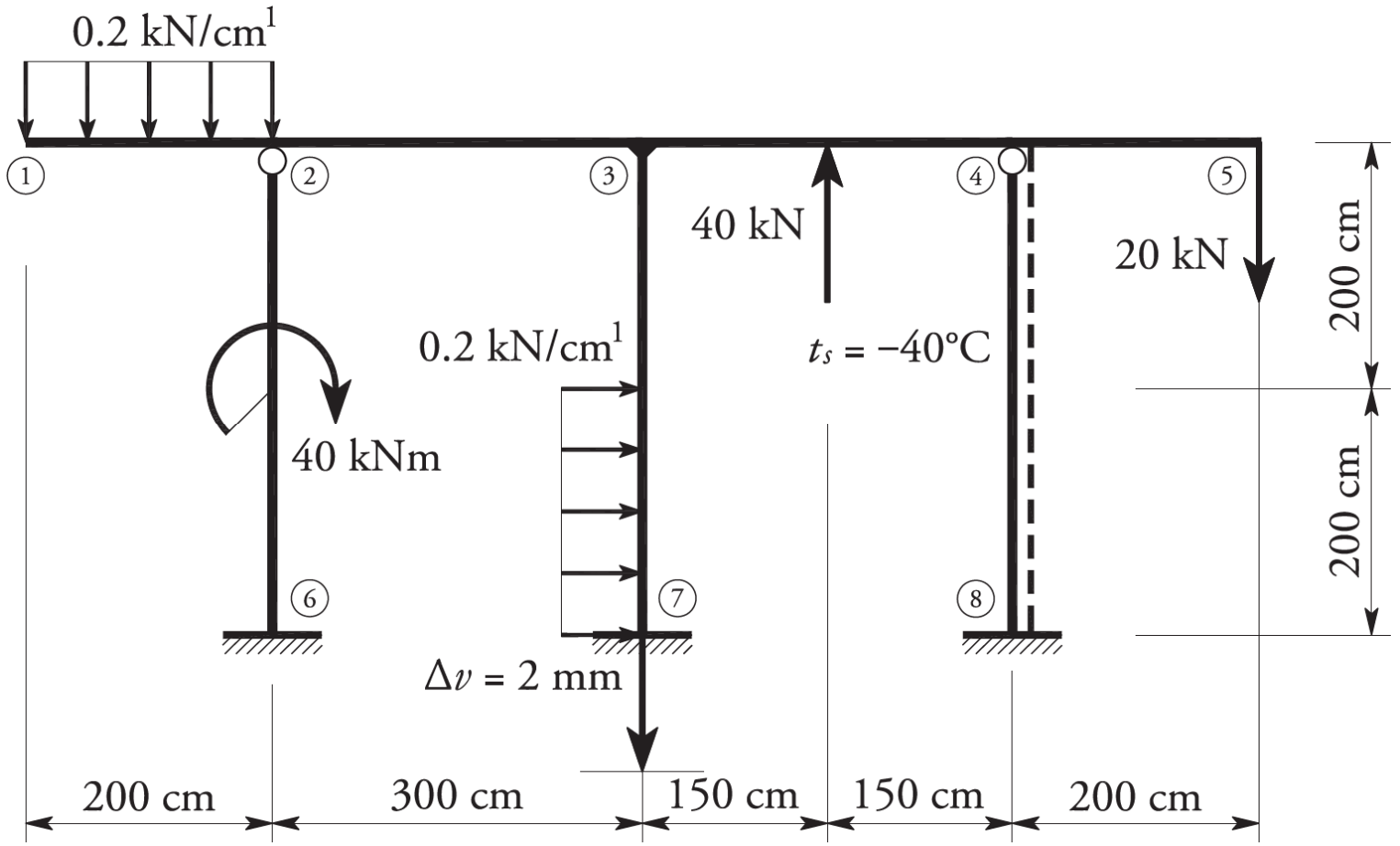
$$M_6 = R_4 \cdot 1.5 - 20 \cdot 1.5 \cdot 0.75 = +17.88 \text{ kNm}$$



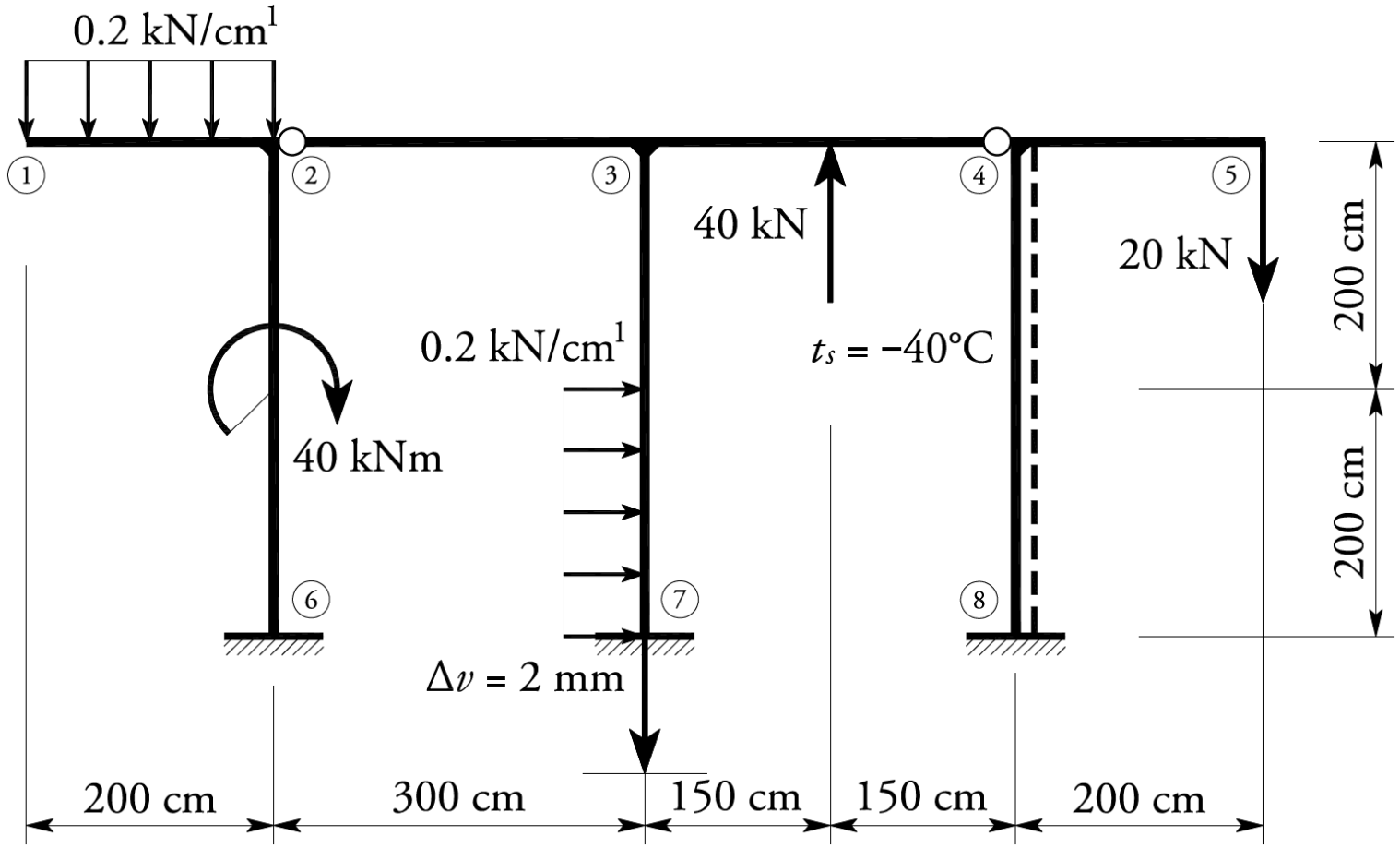
$$\sum M_5 = 0 \quad \Rightarrow \quad R_2 = 15.62 \text{ kN} \uparrow$$

$$M_7 = R_2 \cdot 1.5 = +23.43 \text{ kNm}$$

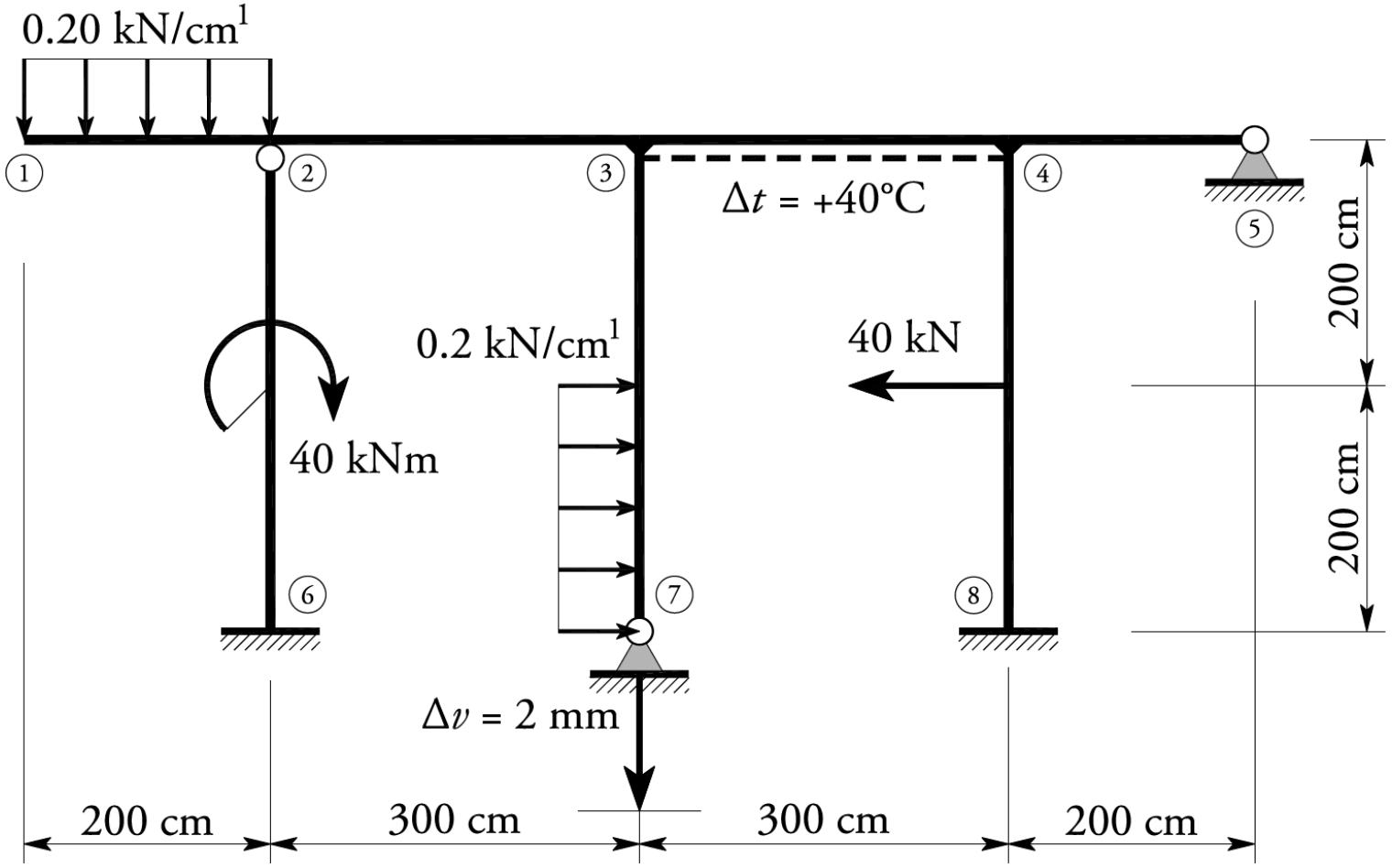
Ispitni rok | 1. listopada 2020.



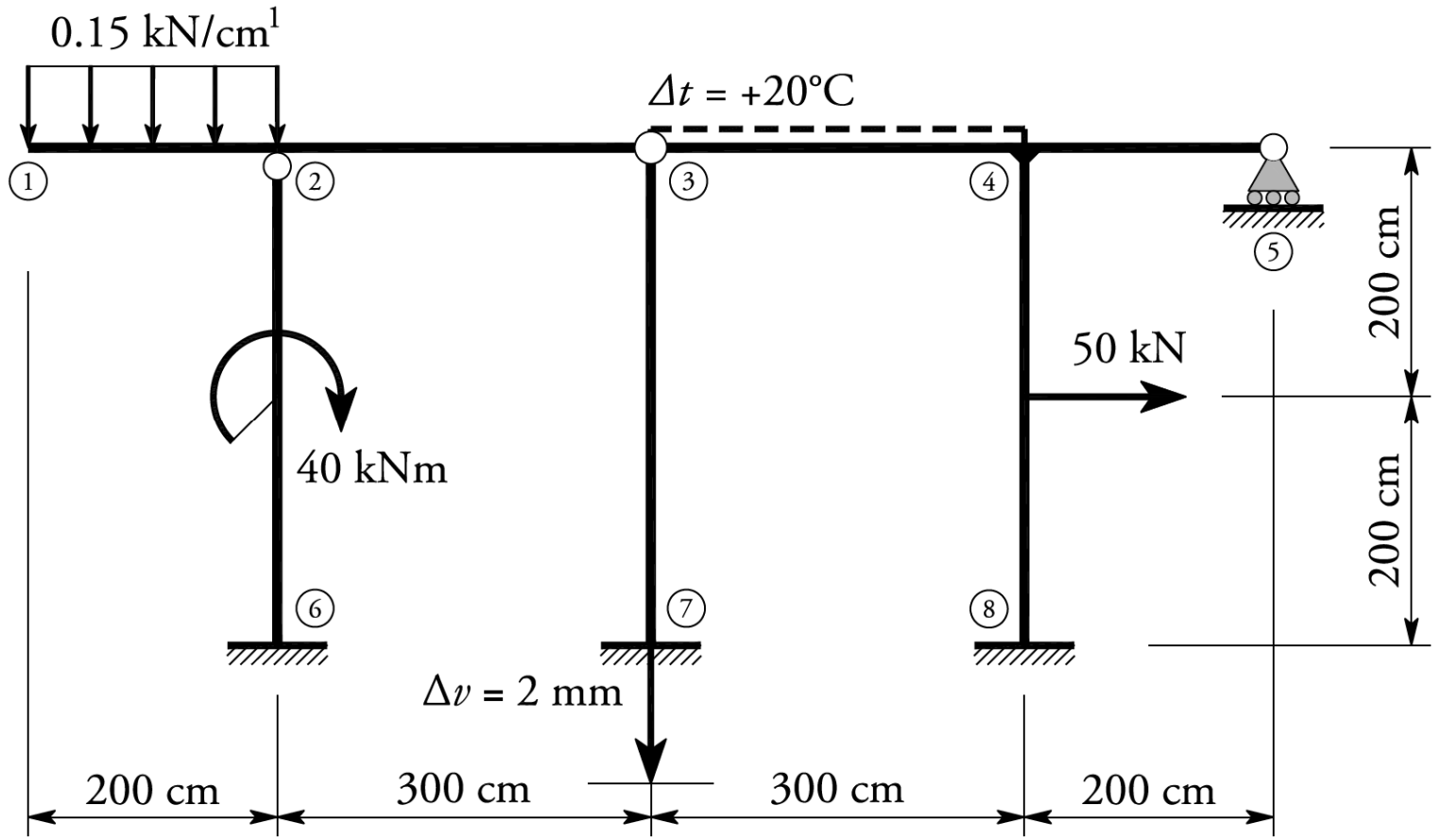
Ispitni rok | 17. rujna 2020.



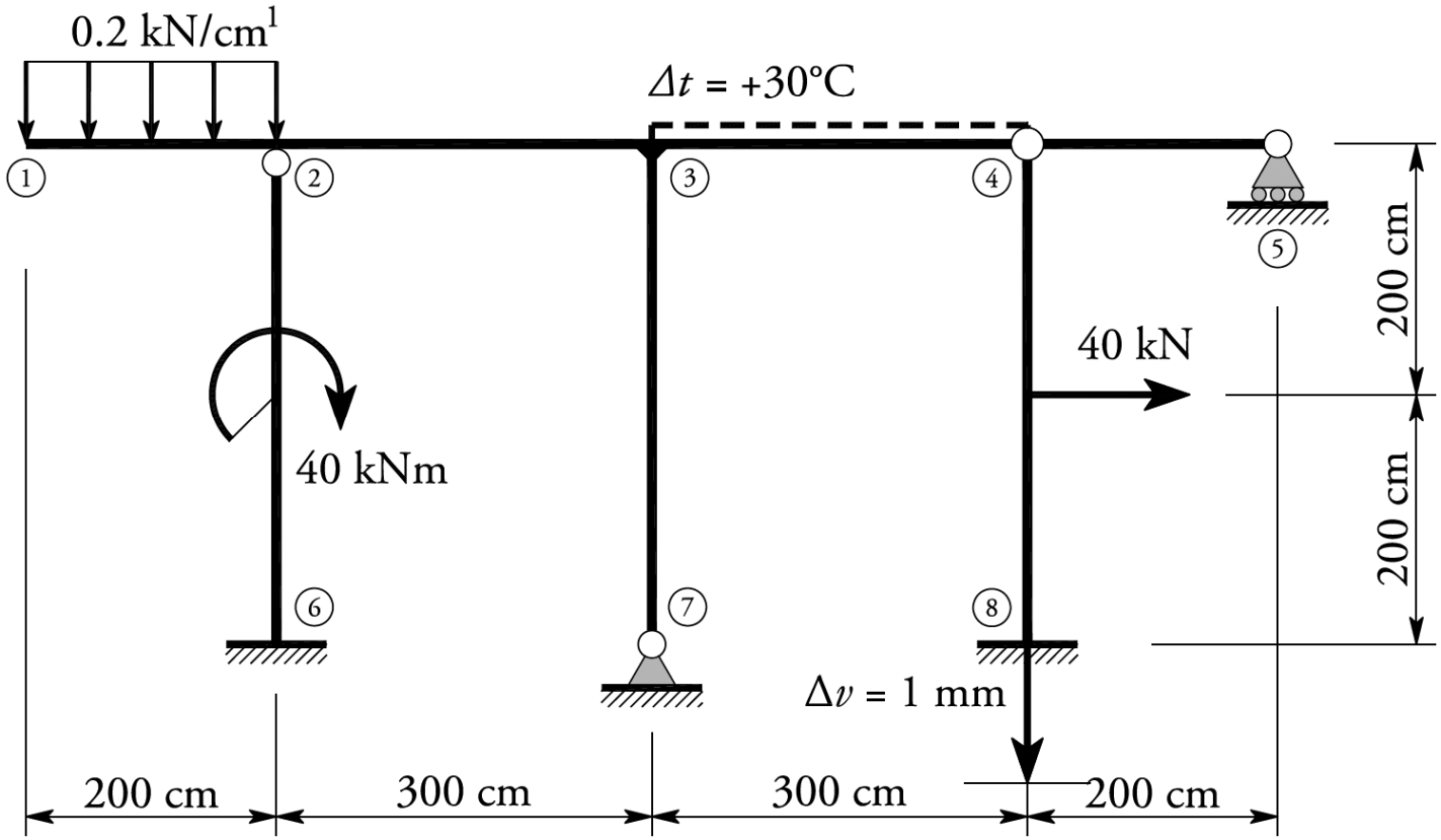
Ispitni rok | 3. rujna 2020.



Ispitni rok | 23. srpnja 2020.



Ispitni rok | 9. srpnja 2020.



Doc. dr. sc. Marin Grubišić, mag. ing. aedif.
Ured II.26, Vladimira Preloga 3, marin.grubisic@gfos.hr | Google Classroom: **qmvjpo6**

Hvala na pažnji!
Pitanja?

Doc. dr. sc. Marin Grubišić, mag. ing. aedif.

Sveučilište u Osijeku (UNIOS)
Građevinski i arhitektonski fakultet Osijek (GrAFOS)
Zavod za tehničku mehaniku (ZTM)
Katedra/Laboratorij za eksperimentalnu mehaniku
Vladimira Preloga 3, **Ured II.26**, HR–31 000 Osijek, Hrvatska

marin.grubisic@gfos.hr

Konzultacije: **srijedom 8:00 — 9:00 sati**
Google Classroom: **qmvjpo6**