

# *Fluidi*

**FIZIKA (RAZ)  
17. studenog 2021.**



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### DEFINICIJA MASENE GUSTOĆE

masena gustoća tvari je masa tvari podijeljena s volumenom tvari

$$\rho = \frac{m}{V}$$

***jedinica SI za masenu gustoću: kg/m<sup>3</sup>***

## 11.1 Masena gustoća

krutine	$\rho$ (kg/m <sup>3</sup> )
aluminij	2700
mjed	8470
beton	2200
bakar	8890
dijamant	3520
zlato	19300
led	917
čelik	7860
olovo	11300
kvarc	2660
srebro	10500
drvo	550

tekućine	$\rho$ (kg/m <sup>3</sup> )
krv (37 °C)	1060
etilni alkohol	806
živa	13600
ulje	800
voda (4 °C)	1000

plinovi	$\rho$ (kg/m <sup>3</sup> )
zrak	1,29
ugljičkov dioksid	1,98
helij	179
vodik	0,0899
dušik	1,25
kisik	1,43

### Primjer 1 Udio krvi u tjelesnoj težini čovjeka

Ljudsko tijelo težine 690 N sadrži oko 5,2 L krvi.

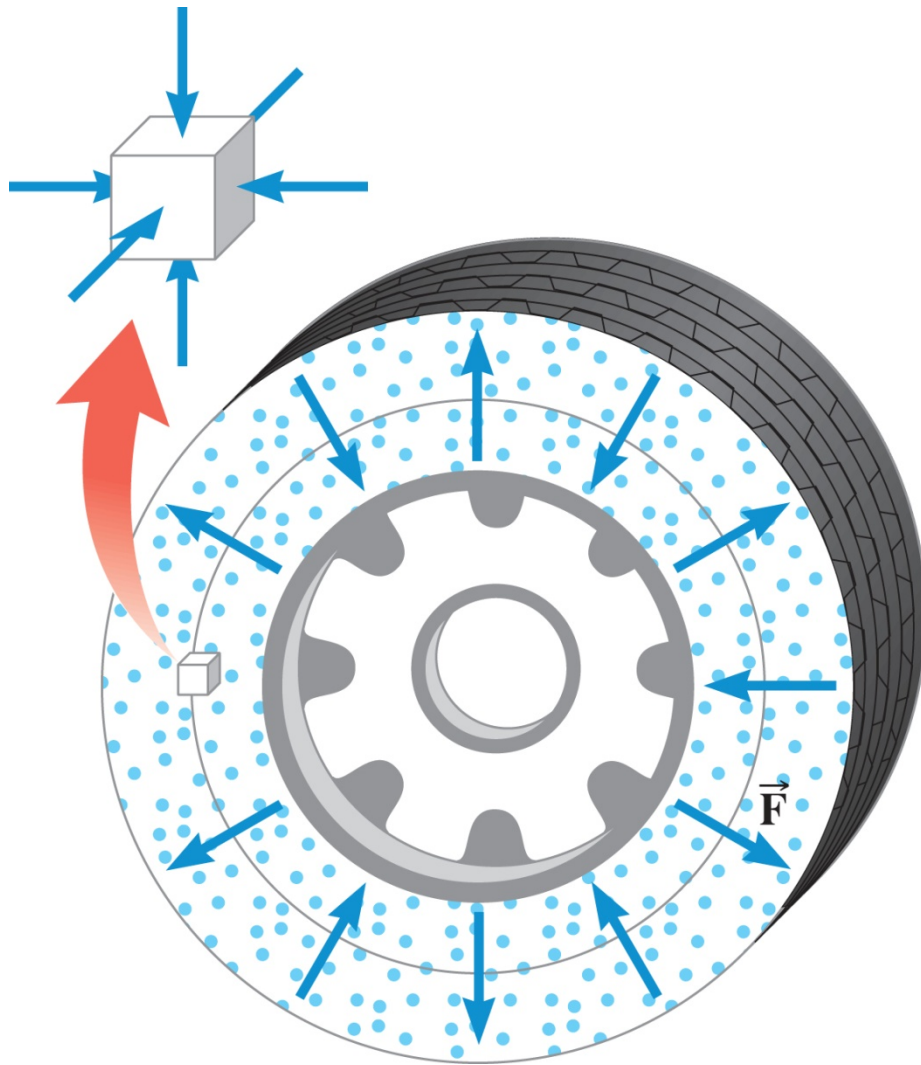
- (a) Odredite težinu krvi.  
(b) Koliki je težinski udio krvi?

$$(a) \quad m = \rho V = 5,2 \cdot 10^{-3} \text{ m}^3 \cdot 1060 \text{ kg m}^{-3} = 5,5 \text{ kg}$$

$$G = m g = 5,5 \text{ kg} \cdot 9,8 \text{ m s}^{-2} = 54 \text{ N}$$

$$(b) \quad w = \frac{G}{G_{\text{ukupno}}} = \frac{54 \text{ N}}{690 \text{ N}} = 7,8 \%$$

## 11.2 Tlak



$$p = \frac{F}{S}$$

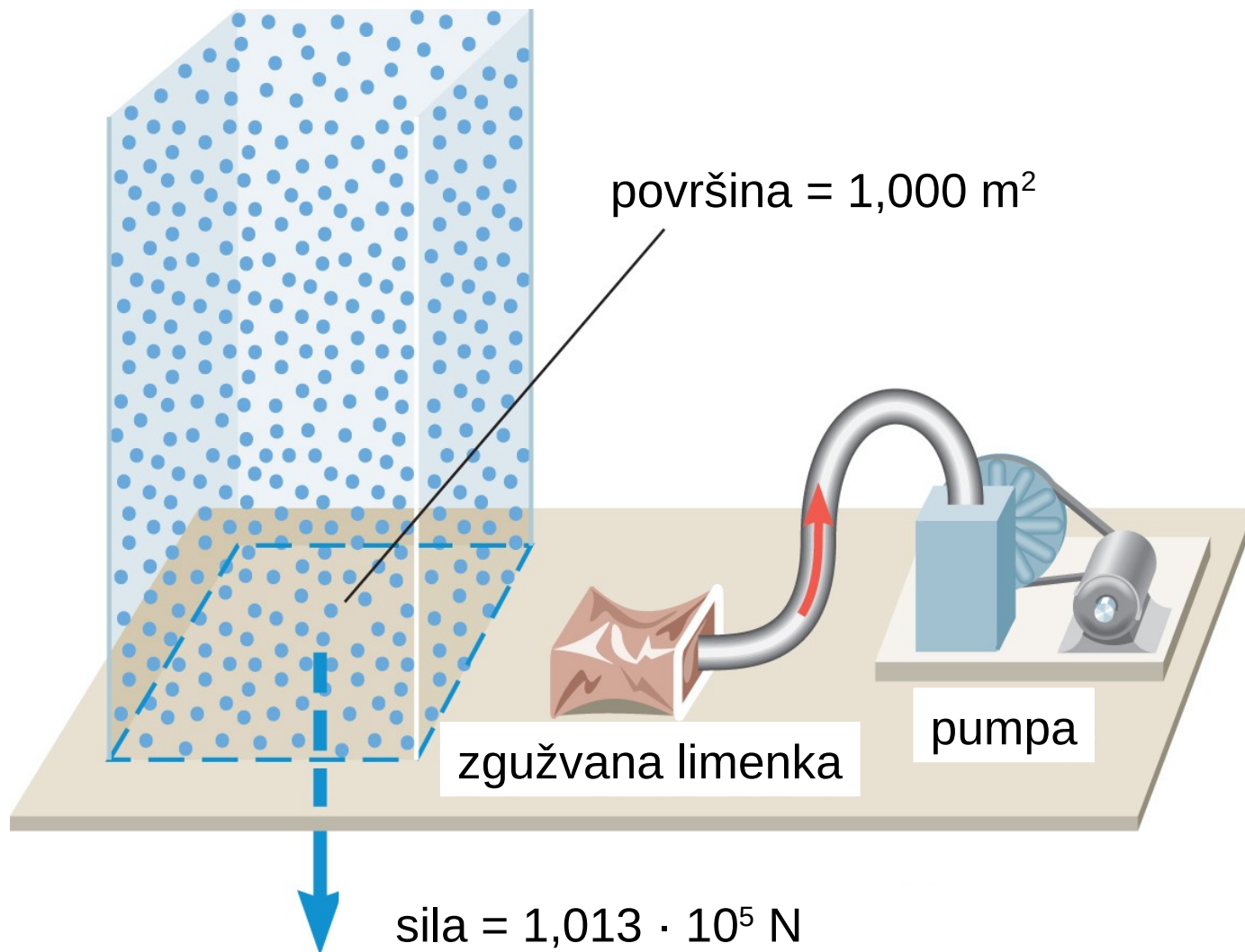
*jedinica SI za tlak:*  $1 \text{ N/m}^2 = 1 \text{ Pa}$

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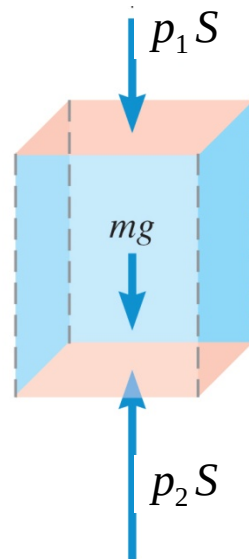
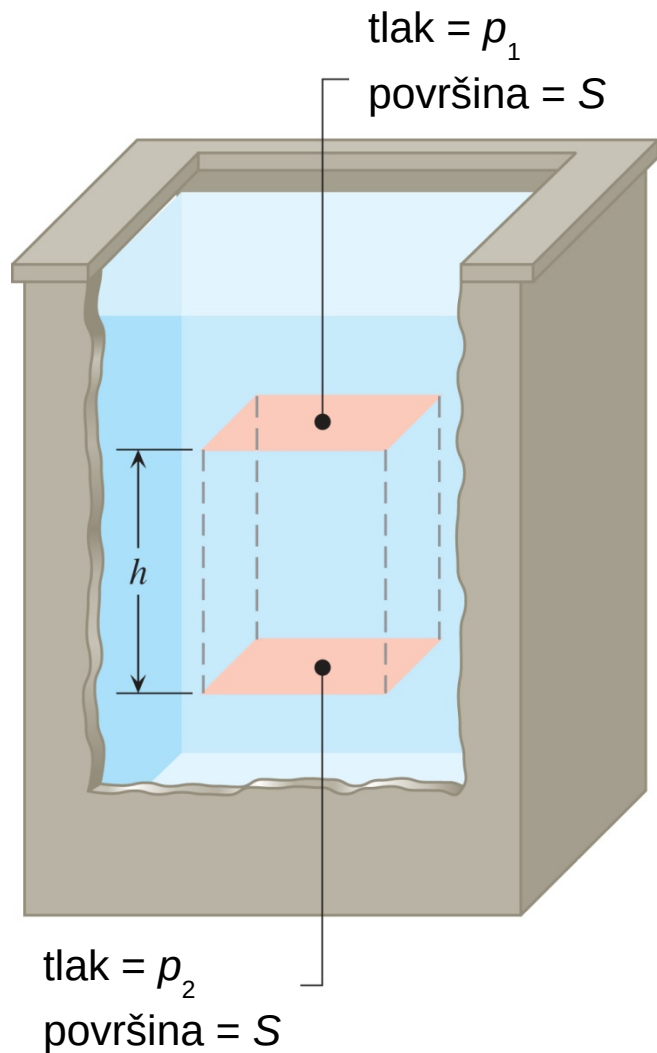
## 11.2 Tlak

**Atmosferski tlak na nultoj nadmorskoj visini:**  $1,013 \cdot 10^5 \text{ Pa}$



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### 11.3 Odnos tlaka i dubine u mirnom fluidu



$$\Sigma F = p_2 S - p_1 S - mg = 0$$

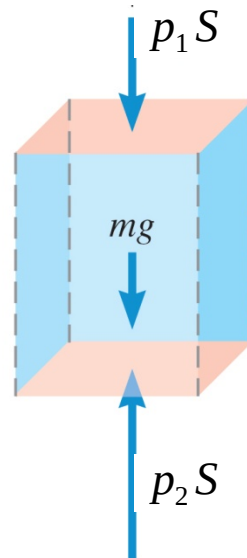
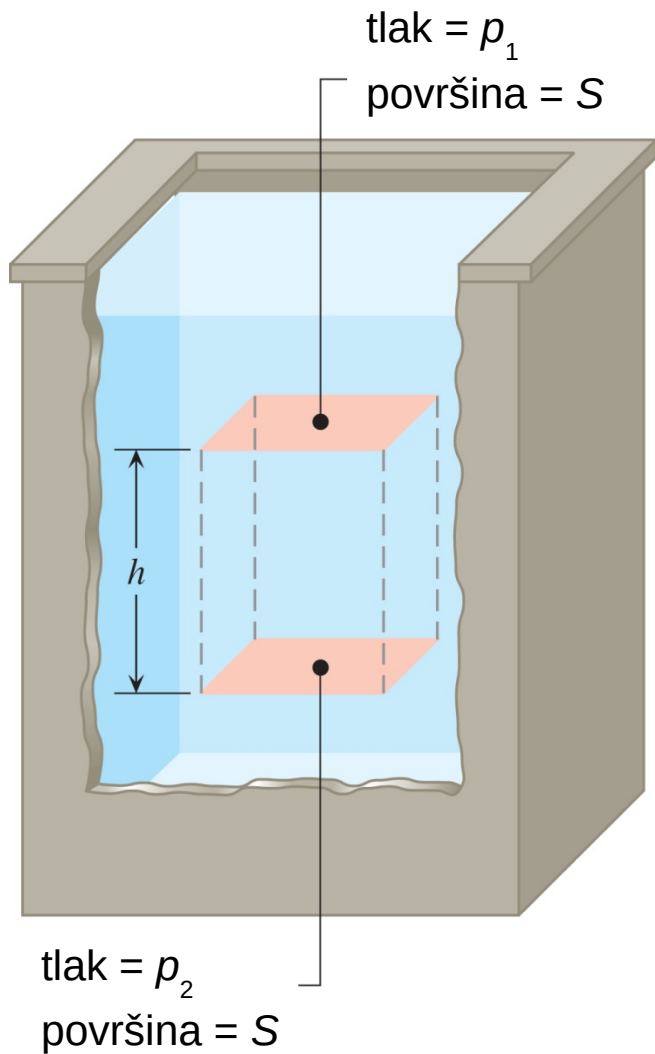


$$p_2 S = p_1 S + mg$$

$$m = \rho V$$

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### 11.3 Odnos tlaka i dubine u mirnom fluidu



$$p_2 S = p_1 S + \rho V g$$

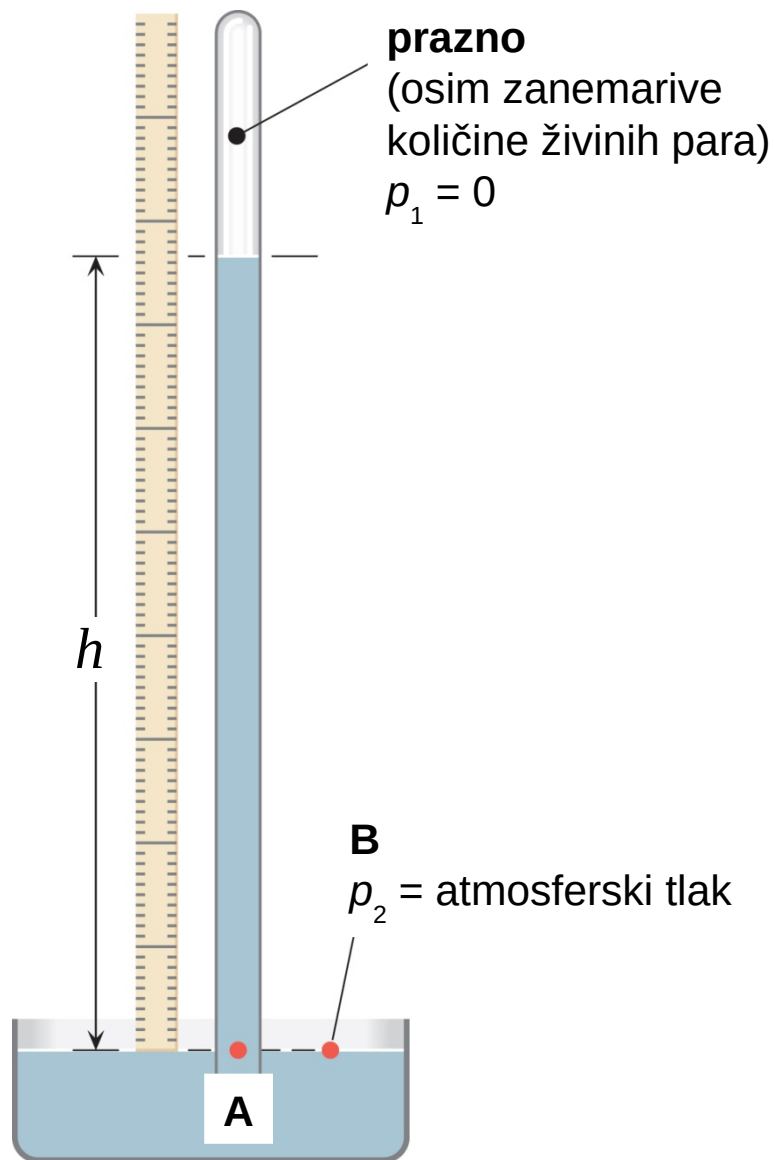
$V = Sh$

$$p_2 S = p_1 S + \rho S h g$$

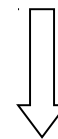
$$p_2 = p_1 + \rho h g$$



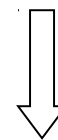
## 11.4 Mjerenje tlaka



$$p_2 = p_1 + \rho h g$$



$$p_{\text{atm}} = \rho h g$$



$$h = \frac{p_{\text{atm}}}{\rho g} = \frac{1,01 \cdot 10^5 \text{ Pa}}{13600 \text{ kg m}^{-3} \cdot 9,8 \text{ m s}^{-2}}$$

$$h = 760 \text{ mm}$$

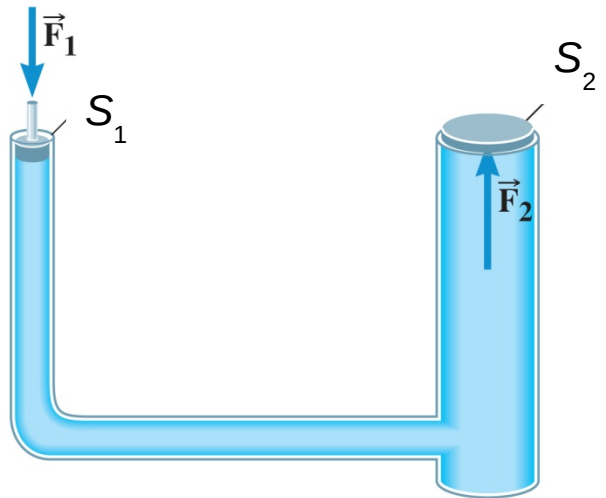
atmosferski tlak odgovara  
stupcu žive od 760 mm

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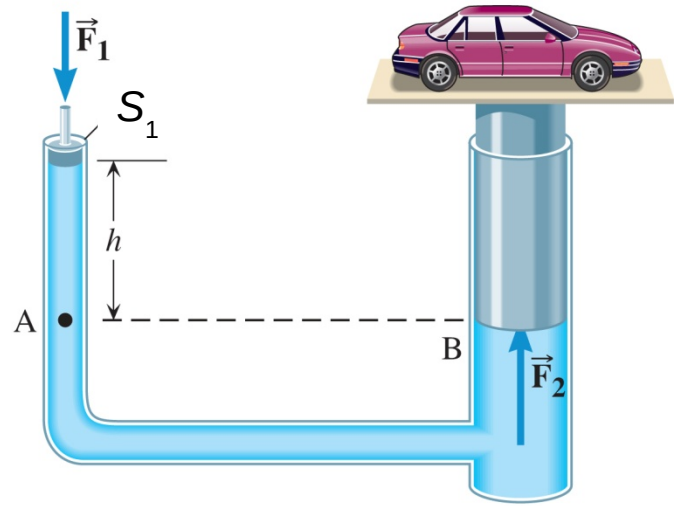
# 11.5 Pascalovo načelo

## PASCALOVO NAČELO

Svaka promjena tlaka u zatvorenom fluidu prenosi se, u istom iznosu, na sve dijelove fluida i stijenke posude.

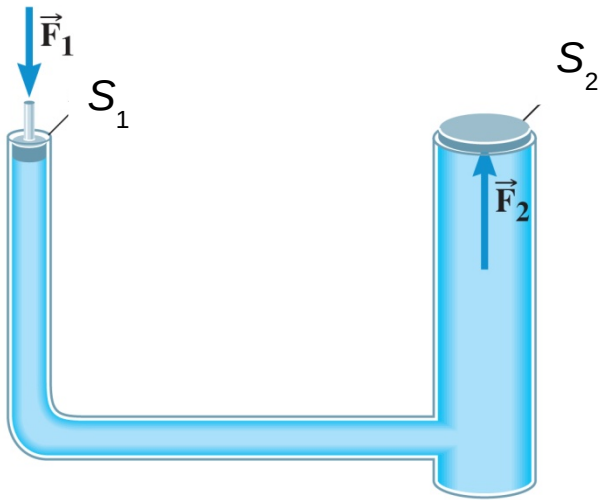


(a)



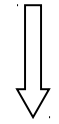
(b)

# 11.5 Pascalovo načelo

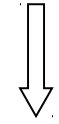


(a)

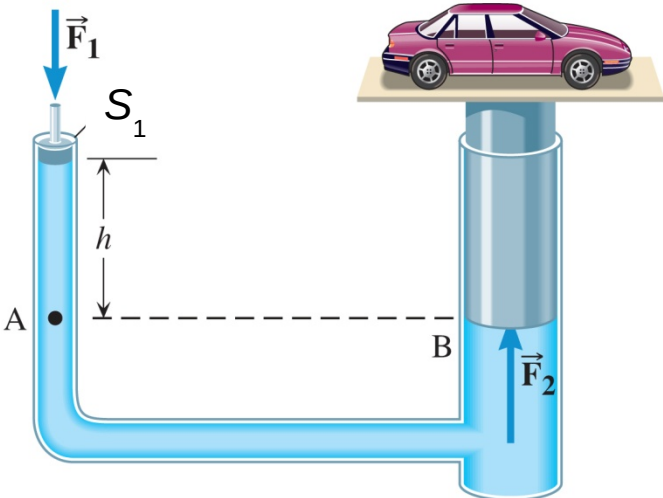
$$p_1 = p_2$$



$$\frac{F_1}{S_1} = \frac{F_2}{S_2}$$



$$F_2 = F_1 \frac{S_2}{S_1}$$



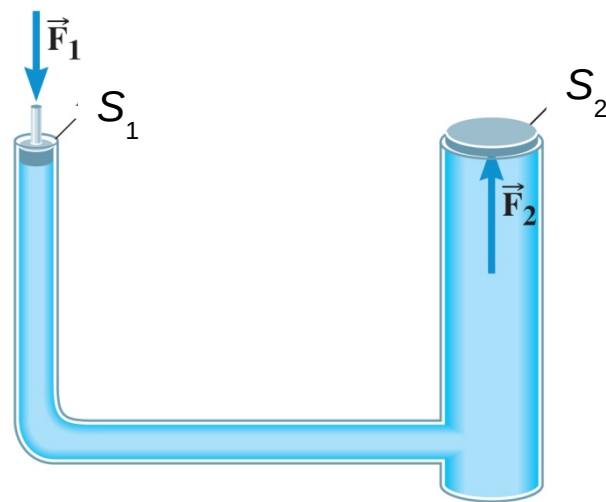
(b)

## 11.5 Pascalovo načelo

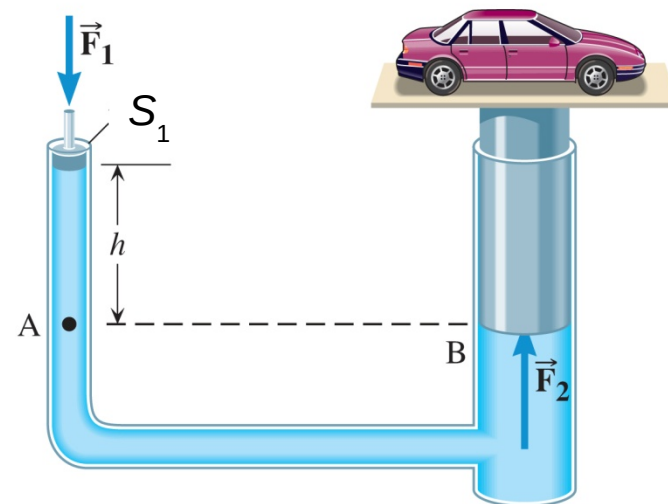
### Primjer 7 Podizanje automobila

Ulazni klip ima polumjer od 0,0120 m  
a izlazni klip polumjer od 0,150 m.

Težina automobila (zajedno s izlaznim klipom)  
je 20500 N. Težinu ulaznog klipa zanemarite.  
Odredite ulaznu silu.



(a)



(b)

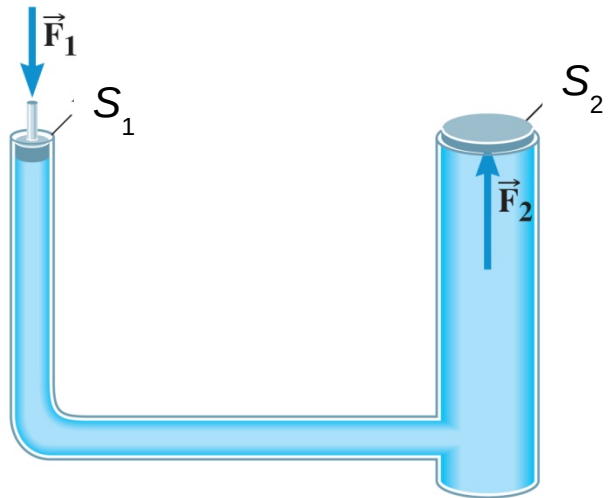
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11.5 Pascalovo načelo

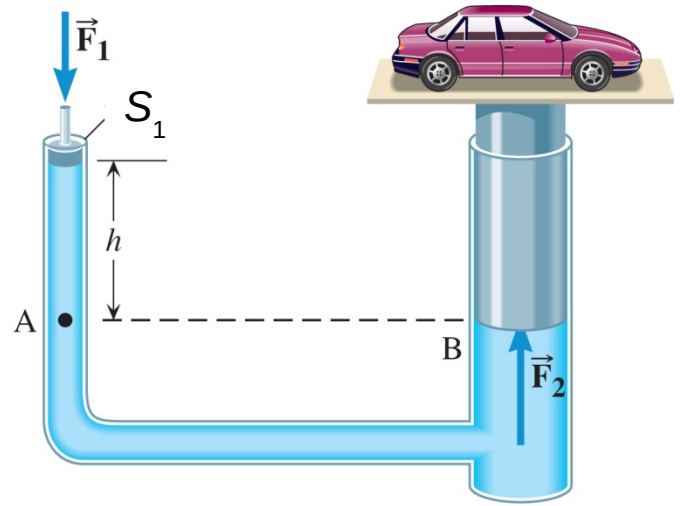
$$F_2 = F_1 \frac{S_2}{S_1}$$

$$F_2 = 20500 \text{ N} \frac{(0,0120 \text{ m})^2 \pi}{(0,150 \text{ m})^2 \pi}$$

$$F_2 = 131 \text{ N}$$

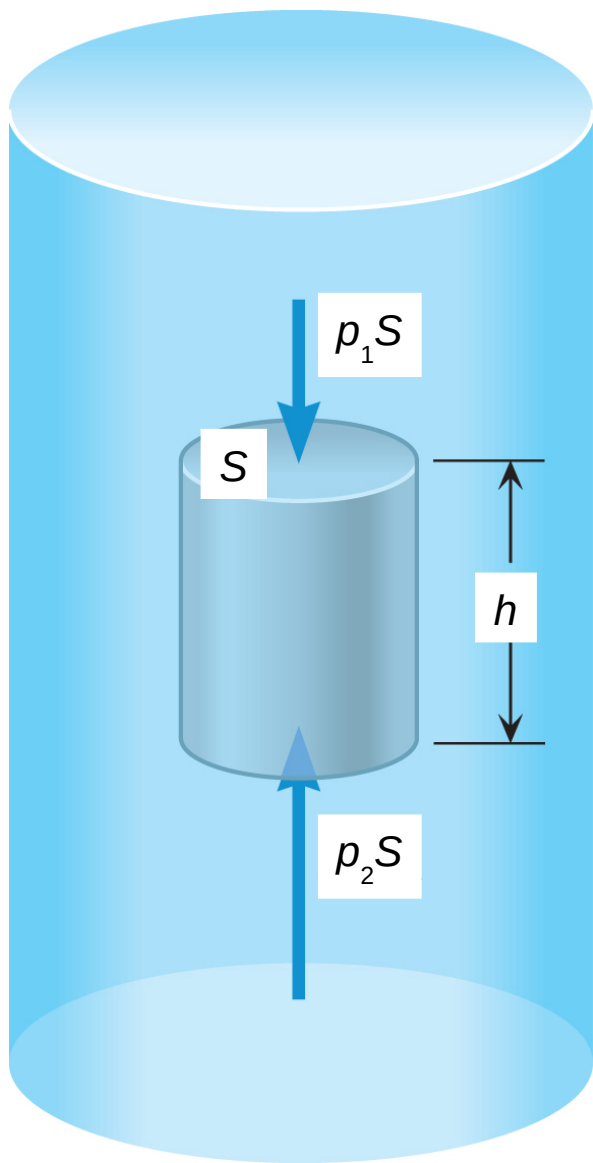


(a)



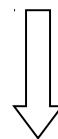
(b)

## 11.6 Arhimedov zakon



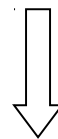
$$p_2 - p_1 = \rho h g$$

$$U = p_2 S - p_1 S = (p_2 - p_1) S$$



$$V = h S$$

$$U = \rho g h S$$



$$U = \rho V g$$

masa istisnutog fluida

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## ARHIMEDOV ZAKON

Tijelo uronjeno u fluid izgubi na težini onoliko koliko teži istisnuti fluid.

$$U = G_{\text{fluid}}$$

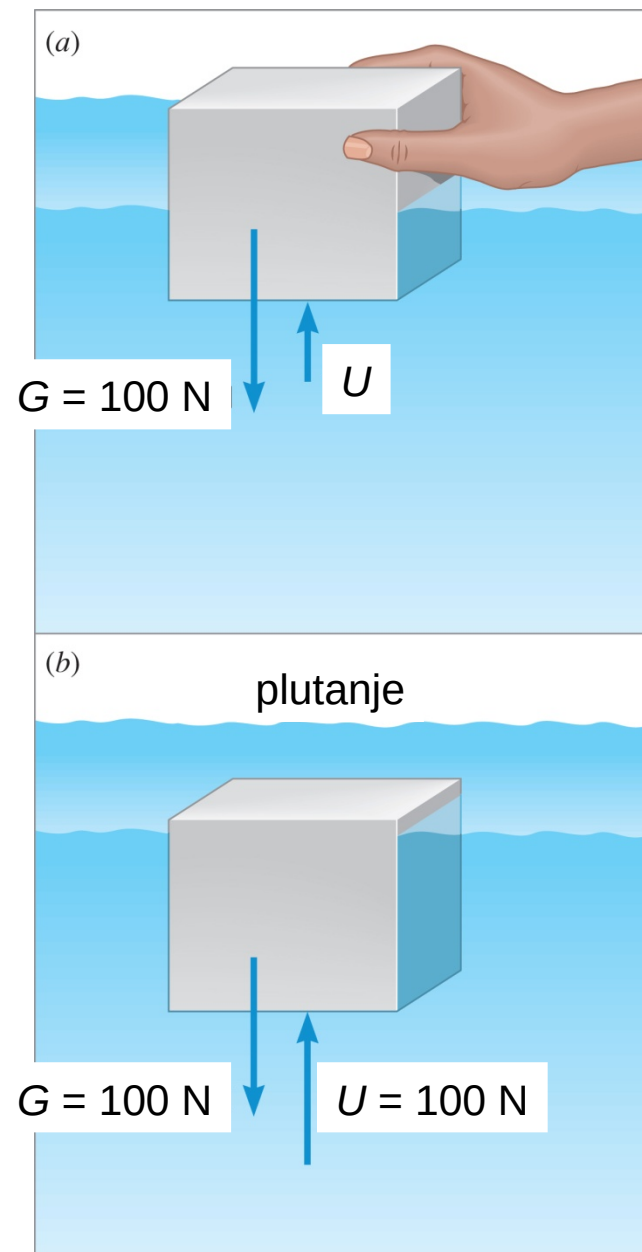
uzgon

težina istisnutog fluida

## 11.6 Arhimedov zakon

Ako je uzgon jednak težini  
onda tijelo u fluidu pluta.

$$U = \rho_{\text{fluid}} g V_{\text{uronjen}}$$



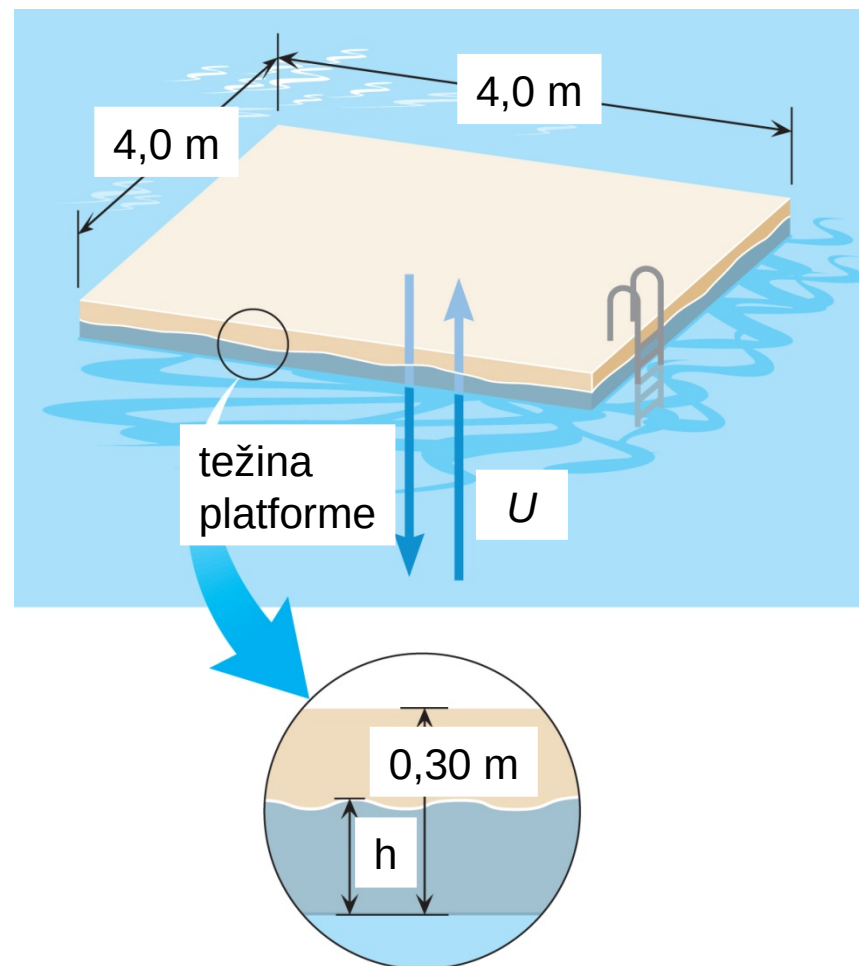
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## 11.6 Arhimedov zakon

### Primjer 9 Plutajuća platforma

Pluta li drvena platforma na vodi?  
Ako pluta, koliko je duboko uronjena u vodu?



## 11.6 Arhimedov zakon

$$V_{\text{ukupni}} = 4,0 \text{ m} \cdot 4,0 \text{ m} \cdot 0,30 \text{ m} = 4,8 \text{ m}^3$$

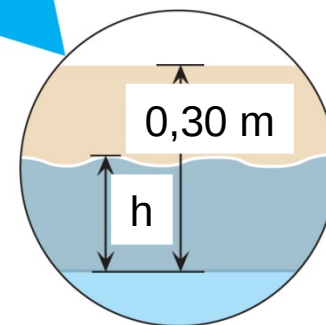
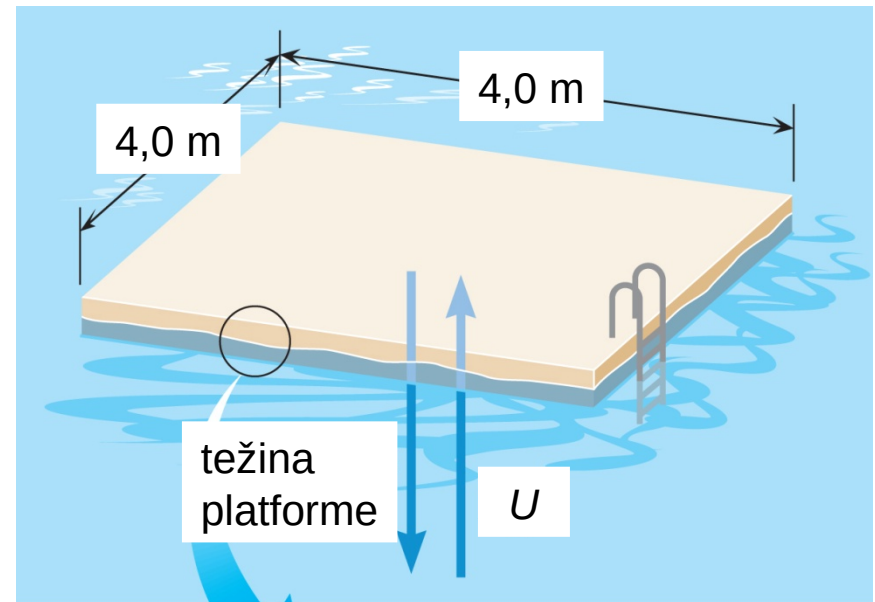
$$U = \rho_{\text{fluid}} g V_{\text{uronjen}}$$

$$U_{\text{max}} = \rho_{\text{fluid}} g V_{\text{ukupni}}$$

$$U_{\text{max}} = 1000 \text{ kg m}^{-3} \cdot 9,8 \text{ m s}^{-2} \cdot 4,8 \text{ m}^3$$

$$U_{\text{max}} = 47000 \text{ N}$$

kad bi cijela platforma bila uronjena  
tad bi uzgon bio 47000 N



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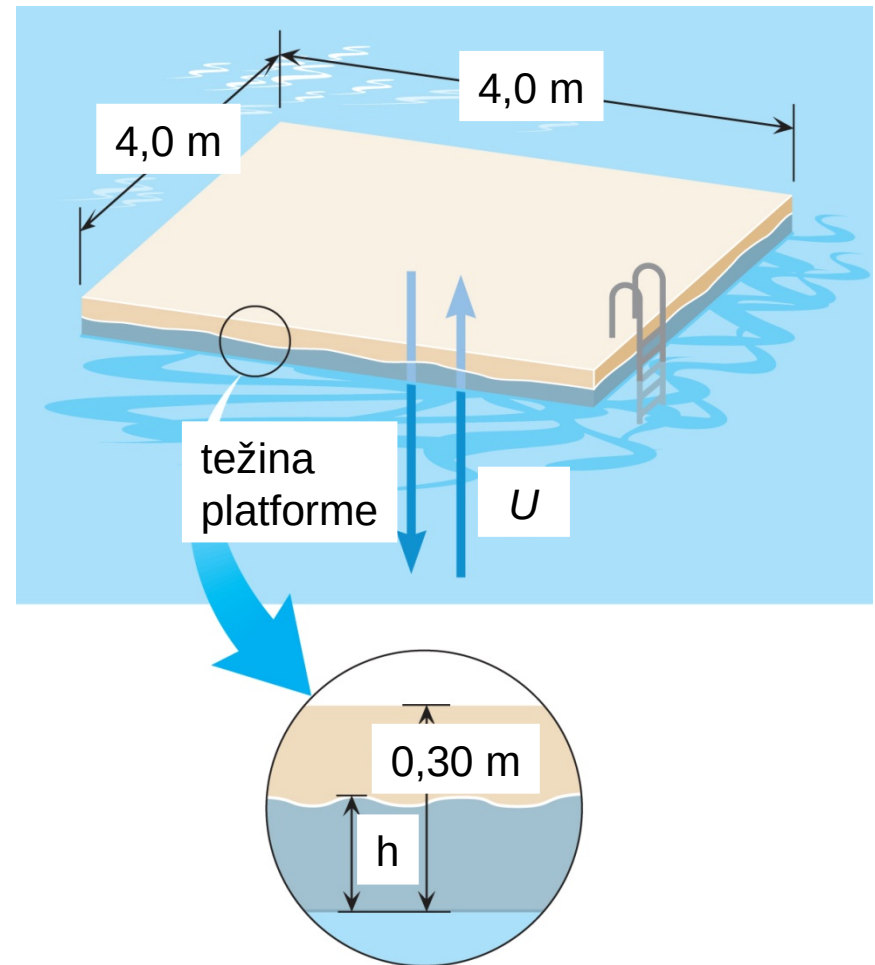
## 11.6 Arhimedov zakon

$$G = m g = \rho_{\text{drvo}} V_{\text{ukupni}} g$$

$$G = 550 \text{ kg m}^{-3} \cdot 4,8 \text{ m}^3 \cdot 9,8 \text{ m s}^{-2}$$

$$G = 26000 \text{ N} < 47000 \text{ N}$$

⇒ platforma pluta

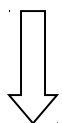


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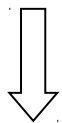
## 11.6 Arhimedov zakon

$$G = U$$

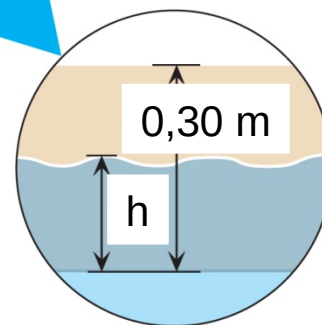
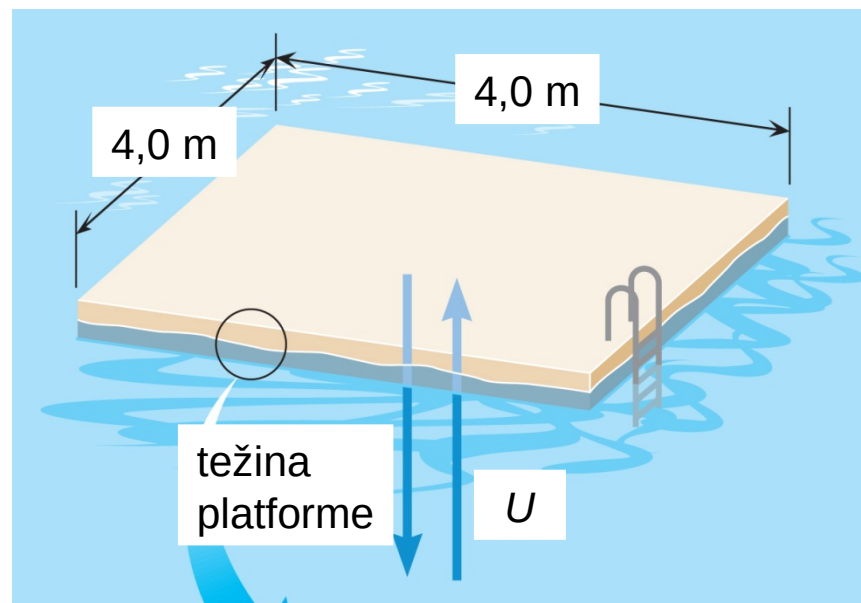


$$26000 \text{ N} = 1000 \text{ kg m}^{-3} \cdot 9,8 \text{ m s}^{-2} \cdot 4,0 \text{ m} \cdot 4,0 \text{ m} \cdot h$$

$$h = \frac{26000 \text{ N}}{1000 \text{ kg m}^{-3} \cdot 9,8 \text{ m s}^{-2} \cdot 4,0 \text{ m} \cdot 4,0 \text{ m}}$$



$$h = 0,17 \text{ m}$$

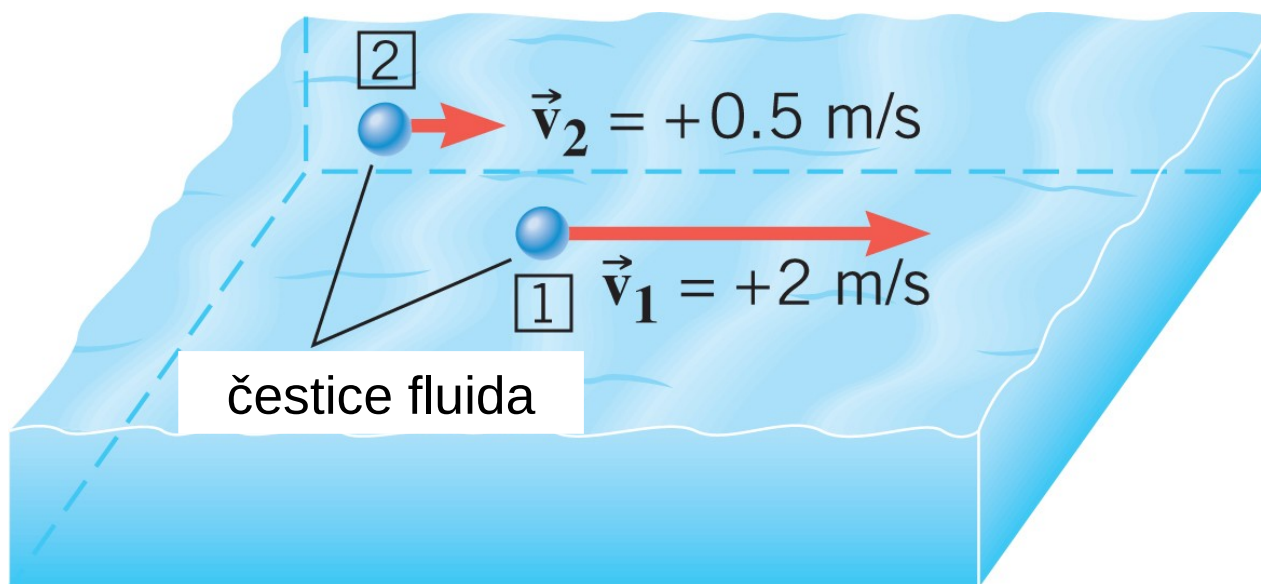


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## 11.7 Gibanje fluida

Pri **stalnom toku** u svakoj je točki brzina čestica konstantna.

**Nestalni tok** znači da se brzina čestica, u svakoj točki, s vremenom mijenja.

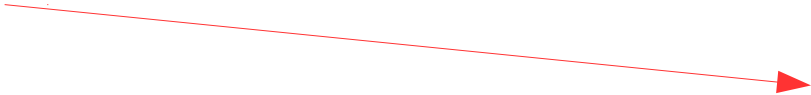


**Turbulentni tok** je ekstremni oblik nestalnog toka u kojem se brzina čestica u svakoj točki fluida nepredvidljivo mijenja i po iznosu i po smjeru.

## 11.7 Gibanje fluida

Fluidi mogu biti **stlačivi** ili **nestlačivi**. Većina tekućina su gotovo nestlačive.

Fluidi mogu biti **viskozni** ili **neviskozni**.

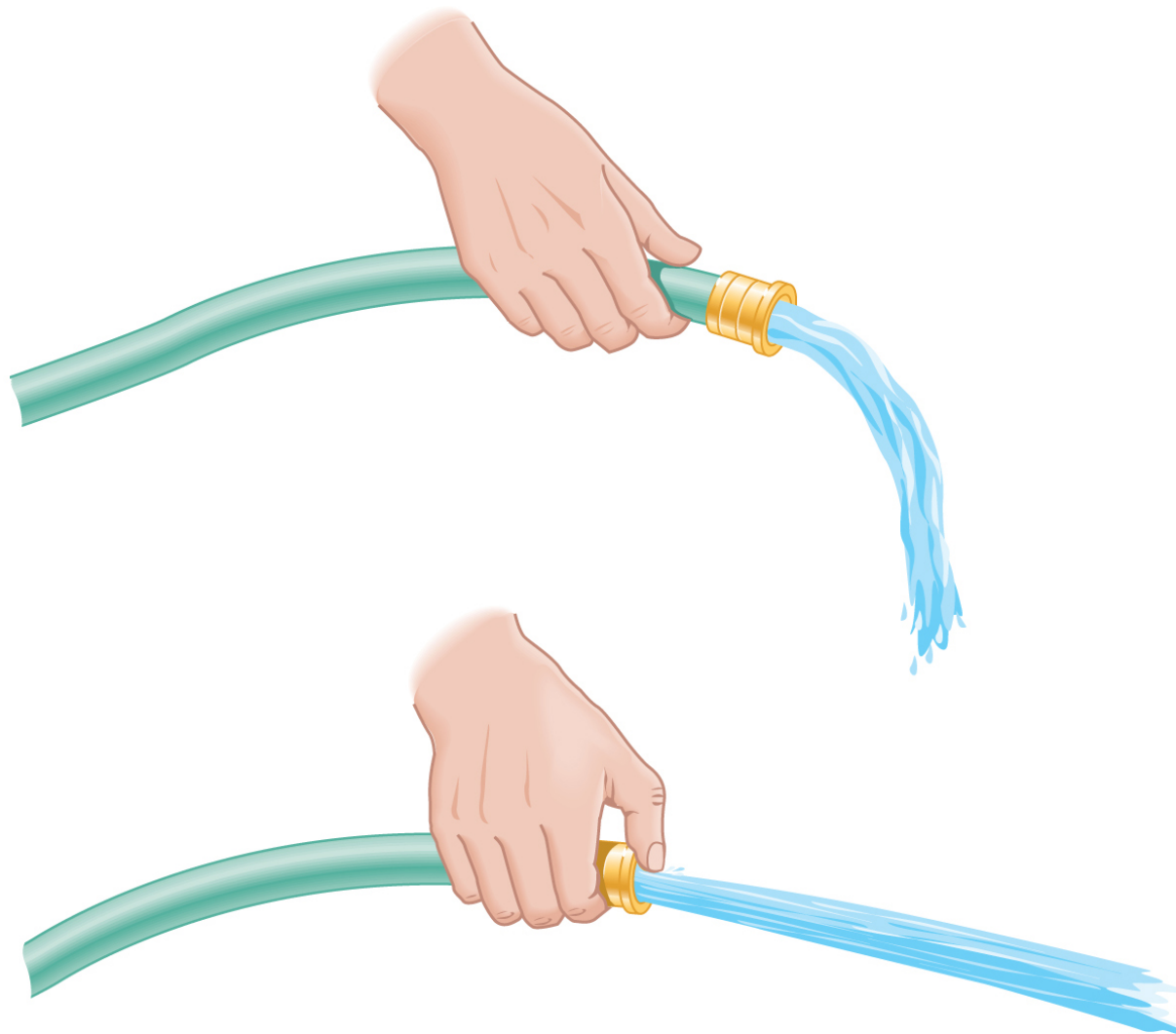


ljepljiv,  
gusto tekući

Nestlačiv i neviskozni fluid nazivamo **idealnim fluidom**.

## 11.8 Jednadžba kontinuiteta

Maseni protok je masa fluida koja, u jedinici vremena, prođe kroz cijev.

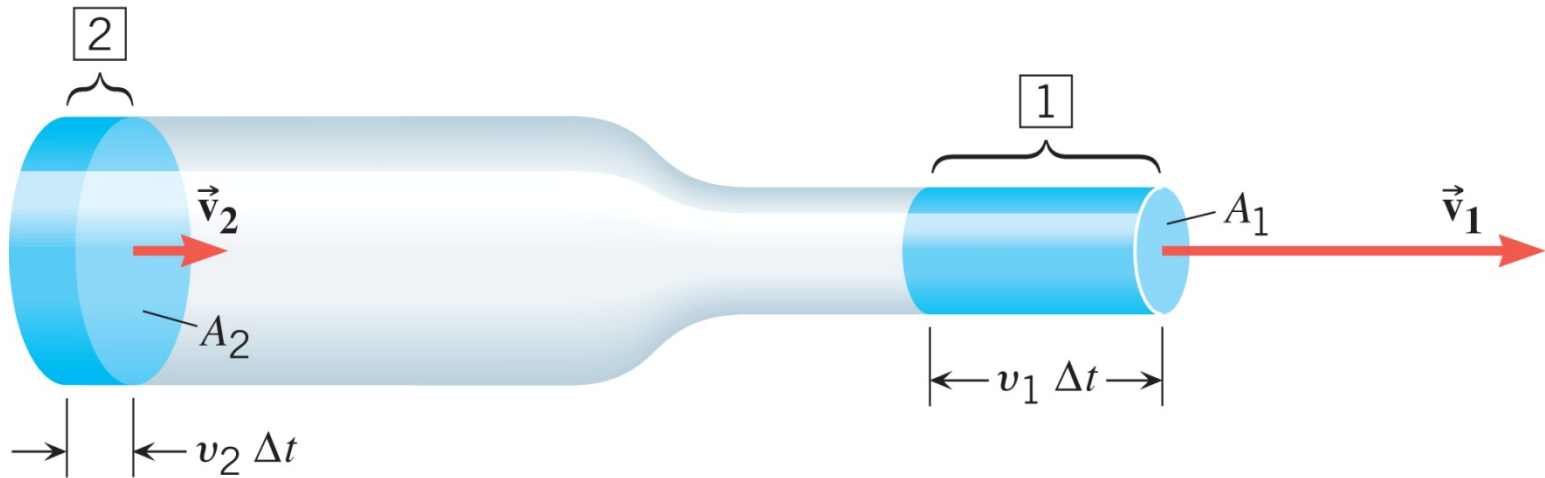


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## 11.8 Jednadžba kontinuiteta

udaljenost

$$\Delta m = \rho V = \rho A v \Delta t$$



$$\frac{\Delta m_2}{\Delta t} = \rho_2 A_2 v_2$$

$$\frac{\Delta m_1}{\Delta t} = \rho_1 A_1 v_1$$

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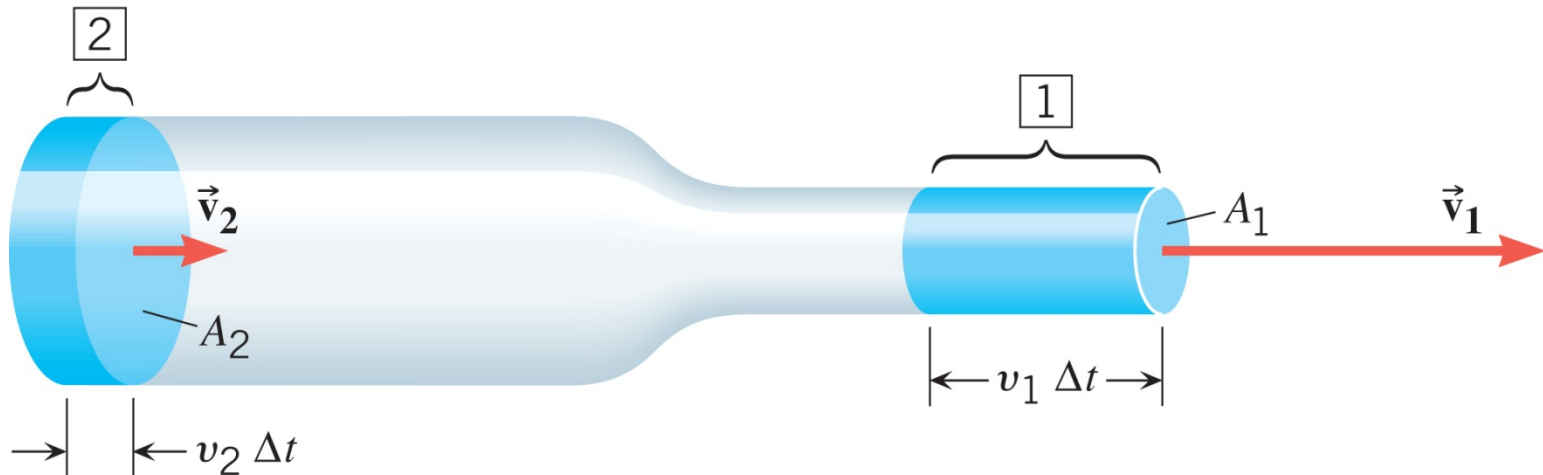


### JEDNADŽBA KONTINUITETA

Maseni protok isti je na svakom položaju uzduž cijevi koja ima jedan ulaz i jedan izlaz za tok fluida.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

*jedinica SI za maseni protok: kg/s*



## 11.8 Jednadžba kontinuiteta

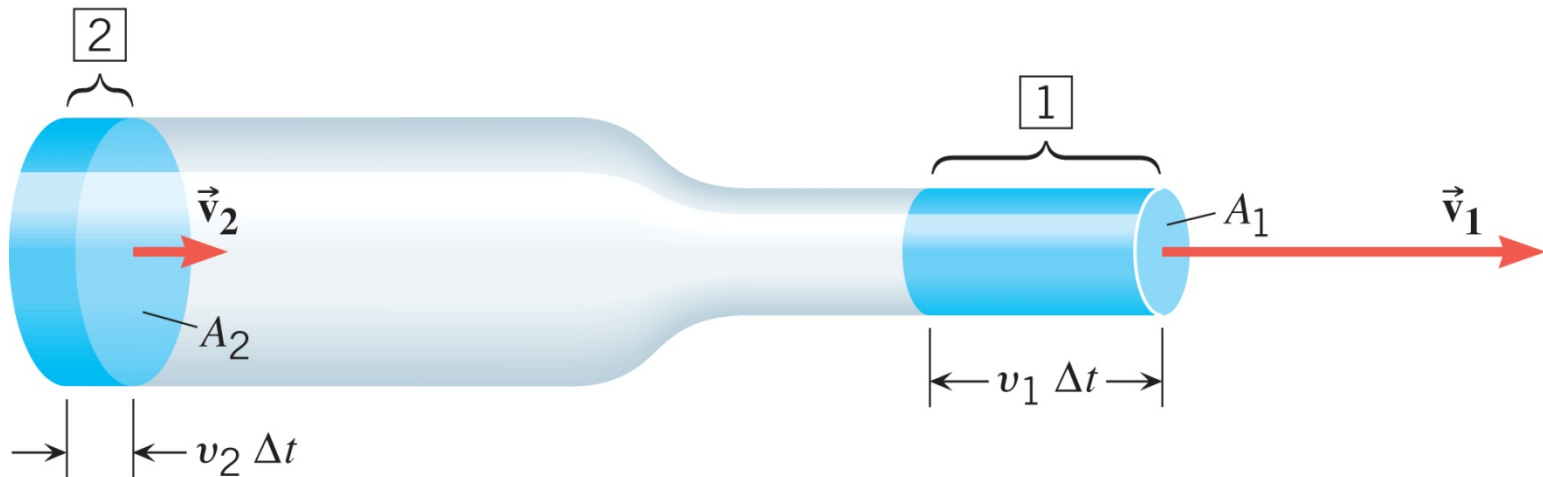
Za nestlačivi fluid:

$$\cancel{\rho}_1 A_1 v_1 = \cancel{\rho}_2 A_2 v_2$$

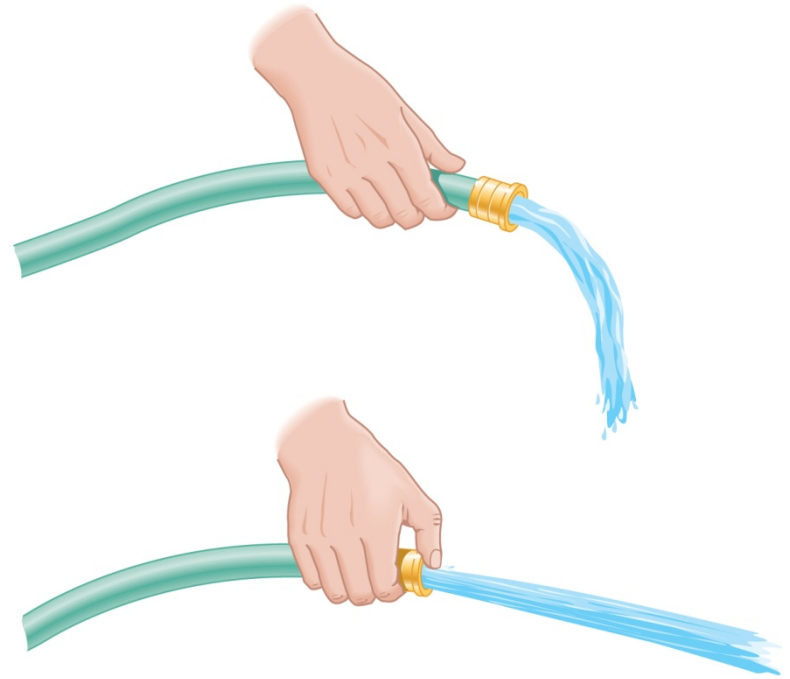
$$A_1 v_1 = A_2 v_2$$

Volumni protok  $Q$ :

$$Q = A v$$



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### **Primjer 12 Crijevo za polijevanje**

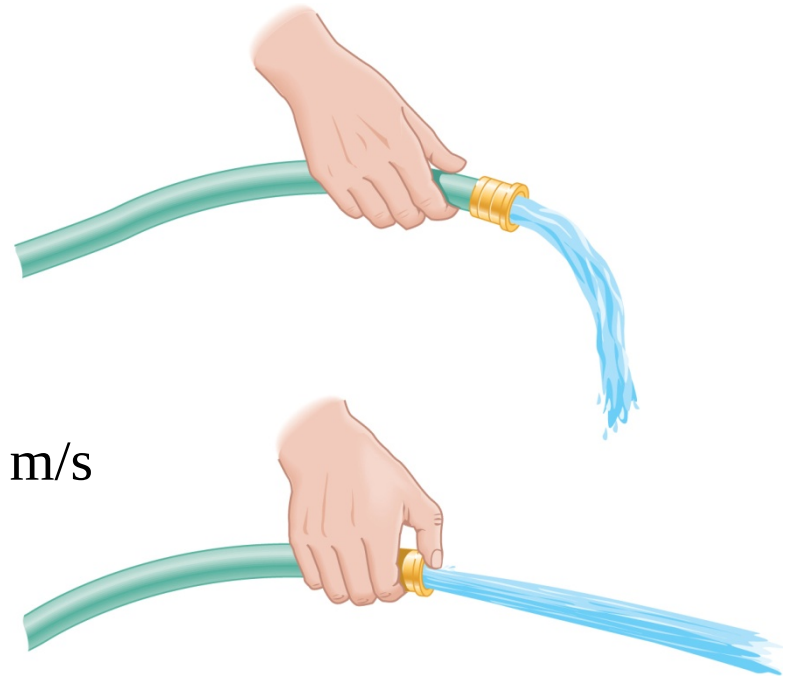
Crijevo za polijevanje ima otvor poprečnog presjeka  $2,85 \cdot 10^{-4} \text{ m}^2$ . Tim se crijevom kanta volumena 8,00 L napuni za 30 s.

Kolikom brzinom voda: (a) nesmetano izlazi kroz crijevo?  
(b) izlazi kroz crijevo čiji je izlazni poprečni presjek smanjen na polovicu?

## 11.8 Jednadžba kontinuiteta

$$(a) \quad Q = A v$$

$$v = \frac{Q}{A} = \frac{(8,00 \cdot 10^{-3} \text{ m}^3) / (30,0 \text{ s})}{2,85 \cdot 10^{-4} \text{ m}^2} = 0,936 \text{ m/s}$$



$$(b) \quad A_1 v_1 = A_2 v_2$$

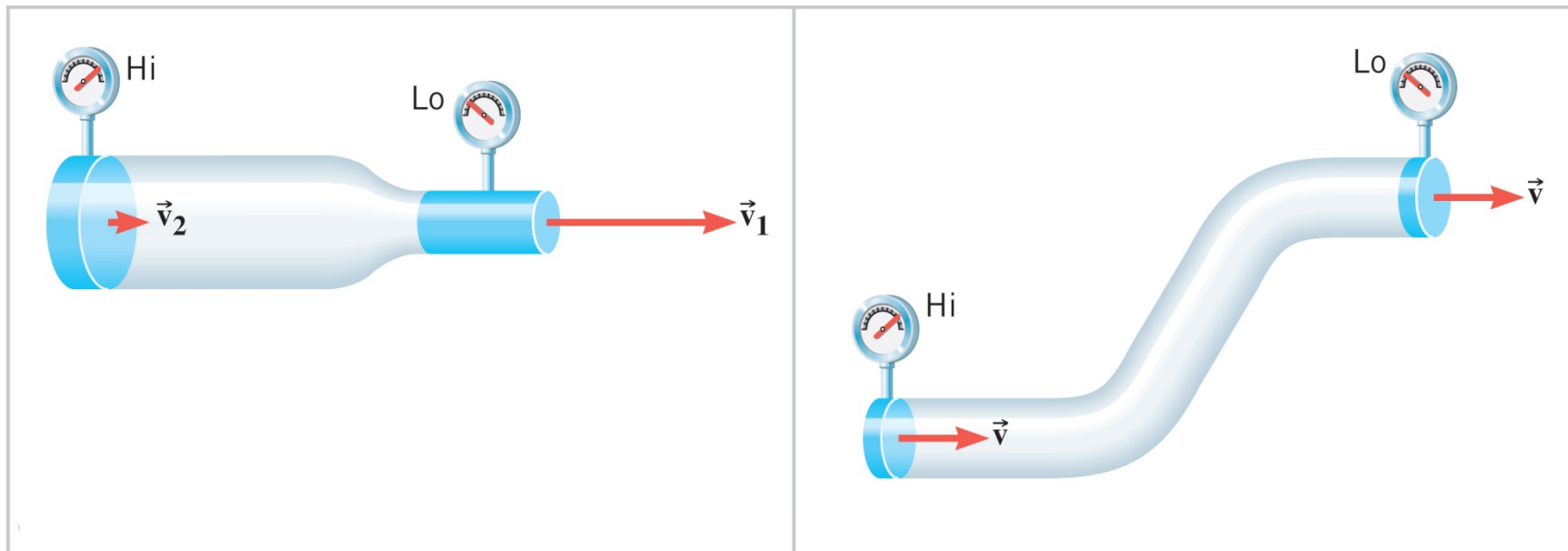
$$v_2 = v_1 \frac{A_1}{A_2} = 0,936 \text{ m/s} \cdot \frac{A_1}{0,5 \cdot A_2} = 0,936 \text{ m/s} \cdot 2 = 1,87 \text{ m/s}$$

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## 11.9 Bernoullijeva jednadžba

Fluid ubrzava prema području nižeg tlaka.

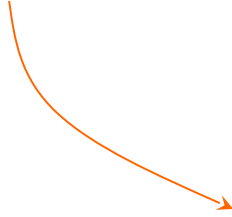
Iz odnosa tlaka i dubine, tlak je niži na višem nivou (ako se presjek cijevi ne mijenja).



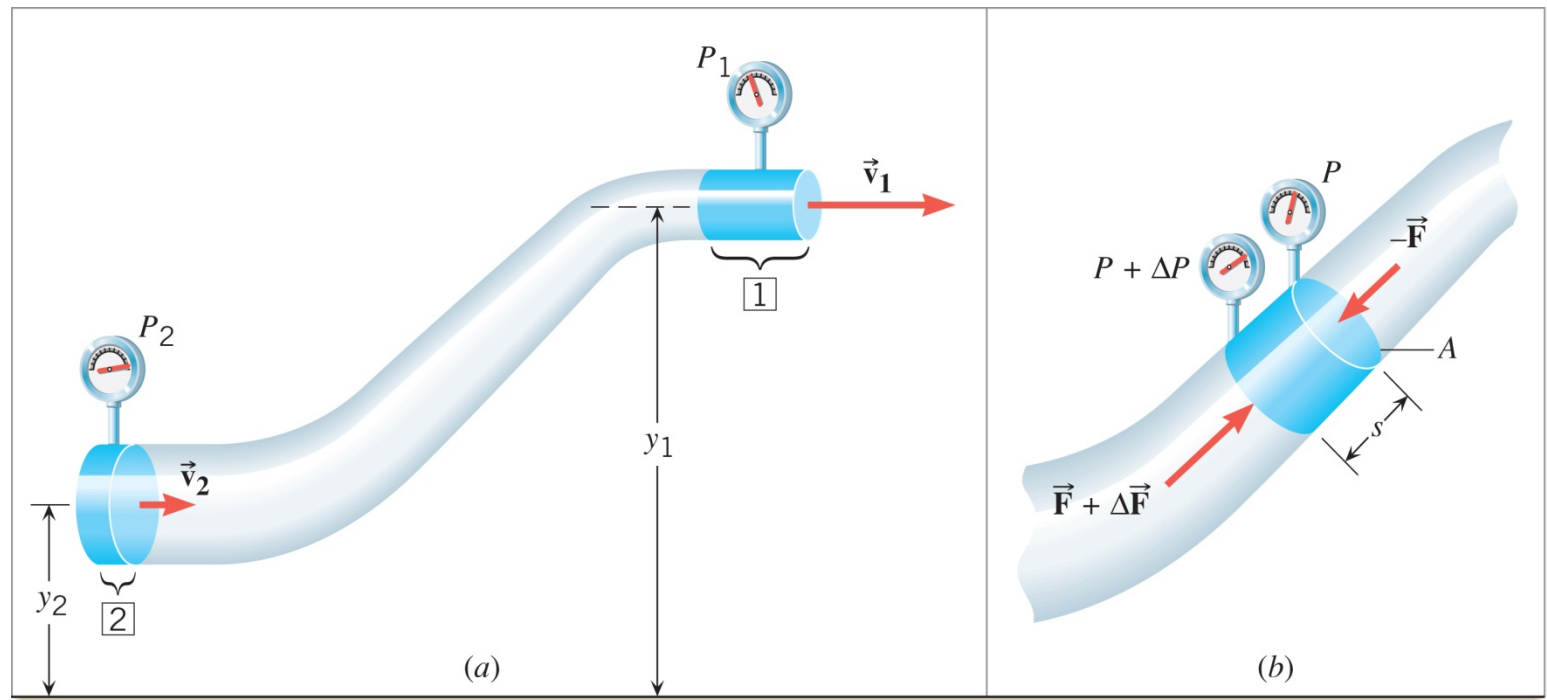
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# 11.9 Bernoulli's equation

$$\Delta W = \Delta F \cdot s = \Delta p \cdot A \cdot s = (p_2 - p_1) \cdot V$$

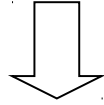


$$\Delta W = \left(\frac{1}{2} m v_1^2 + m g y_1\right) - \left(\frac{1}{2} m v_2^2 + m g y_2\right)$$



## 11.9 Bernoullijeva jednažba

$$(p_2 - p_1) \cdot V = \left( \frac{1}{2} m v_1^2 + m g y_1 \right) - \left( \frac{1}{2} m v_2^2 + m g y_2 \right)$$



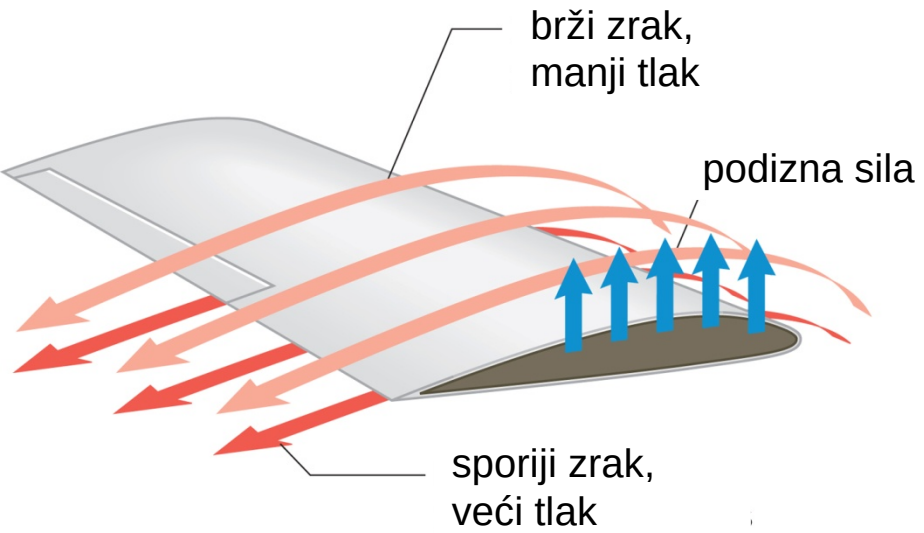
$$p_2 - p_1 = \frac{1}{2} \rho v_1^2 + \rho g y_1 - \frac{1}{2} \rho v_2^2 - \rho g y_2$$

### BERNOULLI'S EQUATION

Pri stalnom toku neviskoznog, nestlačivog fluida, tlak, brzina i visina povezani su ovako:

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

# 11.10 Primjene Bernoullijeve jednadžbe



(a)



(b)

Joe McBride/Getty Images

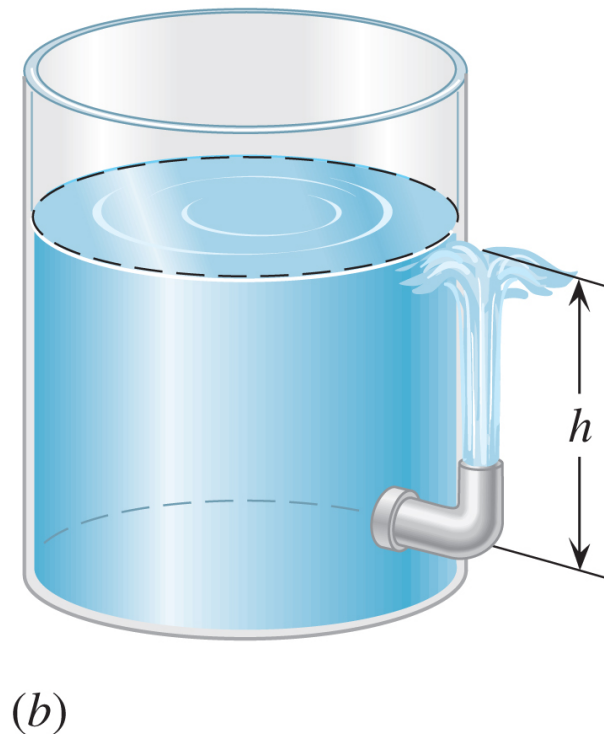
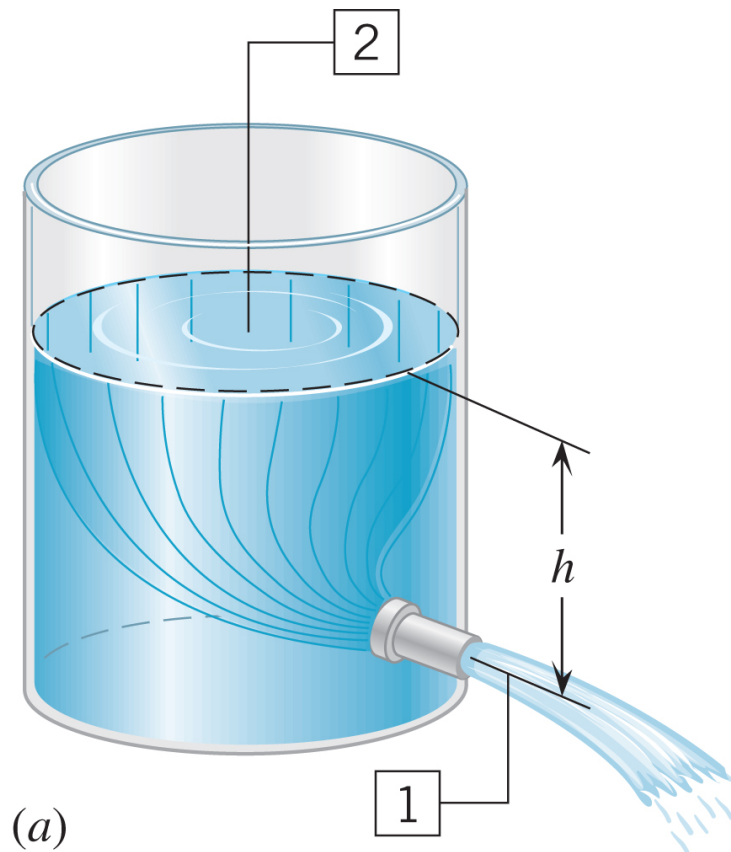


## 11.10 Primjene Bernoullijeve jednadžbe

### Primjer 16 Brzina istjecanja

Rezervoar je s gornje strane otvoren.

Izvedite izraz za brzinu kojom tekućina izlazi iz cijevi na dnu.



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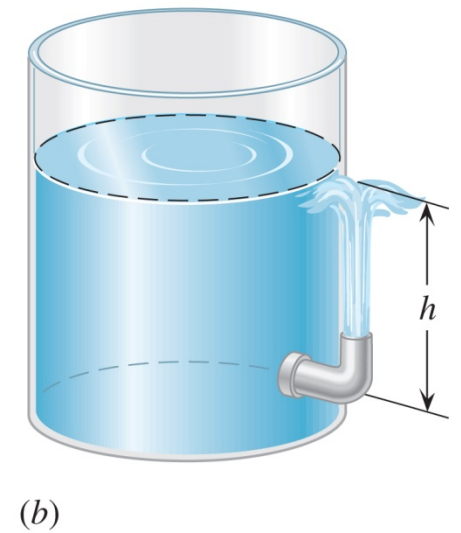
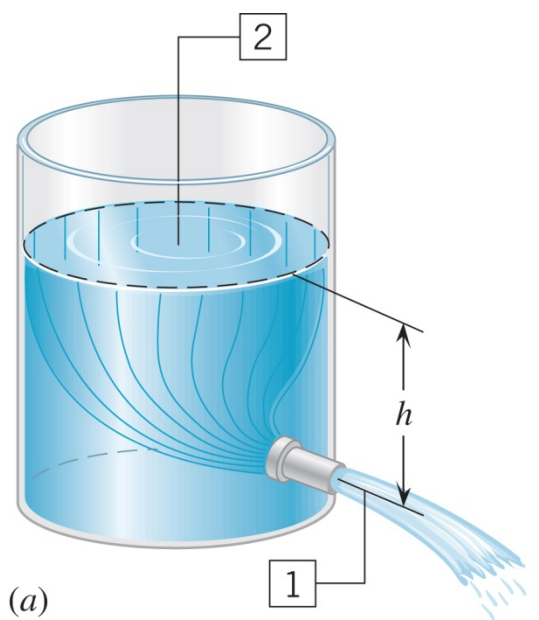
**11.10 Primjene Bernoullijeve jednadžbe**

$$\begin{aligned}
 & p_{\text{atm}} \quad p_{\text{atm}} \quad v_2 \approx 0 \\
 & \swarrow \quad \swarrow \quad \swarrow \\
 & \cancel{p_1} + \frac{1}{2} \rho v_1^2 + \rho g y_1 = \cancel{p_2} + \frac{1}{2} \rho \cancel{v_2^2} + \rho g y_2
 \end{aligned}$$

$$\begin{aligned}
 & \Downarrow \quad y_2 - y_1 = h \\
 & \frac{1}{2} \rho v_1^2 + \rho g y_1 = \rho g y_2
 \end{aligned}$$

$$\frac{1}{2} \rho v_1^2 = \rho g h$$

$$\Downarrow \\
 v_1 = \sqrt{2gh}$$



# ZADACI ZA VJEŽBU

1. Jedan kraj žice pričvršćen je za strop, a na drugom kraju visi puna mjedena kugla. Napetost žice je 120 N. Odredite polumjer kugle.

**RJEŠENJE: 7,0 cm**

2. Hipotetski sferni planet građen je samo od željeza. Odredite period satelita koji oko tog planeta orbitira uz samu površinu.

**RJEŠENJE: 71 min**

3. Osoba težine 625 N vozi bicikl težine 98 N. Pretpostavite da je težina jednako raspoređena na oba kotača. Kolika je dodirna površina svake gume s tlom ako je tlak u svakoj gumi  $7,60 \cdot 10^5$  Pa?

**RJEŠENJE:  $4,76 \cdot 10^{-4} \text{ m}^2$**

4. Skijaš mase 58 kg spušta se niz padinu nagiba  $35^\circ$ . Dodirna površina svake skije sa snijegom je  $0,13 \text{ m}^2$ . Odredite tlak kojim svaka skija pritišće snijeg.

**RJEŠENJE: 1800 Pa**

## ZADACI ZA VJEŽBU

5. Pliva patka preko Save. Izračunajte prosječnu gustoću patke ako je 25% njezinog obujma stalno pod vodom?

**RJEŠENJE:  $250 \text{ kg/m}^3$**

6. Gustoća leda je  $917 \text{ kg/m}^3$ , a gustoća morske vode  $1025 \text{ kg/m}^3$ . Polarni medvjed sjedi na santi leda obujma  $5,2 \text{ m}^3$ . Odredite masu najtežeg medvjeda koji može sjediti na santi, a da santa ne bude potpuno pod vodom.

**RJEŠENJE:  $560 \text{ kg}$**

7. Pacijent koji se oporavlja od operacije dobiva infuziju. Svakih šest sati primi  $9,5 \cdot 10^{-4} \text{ m}^3$  tekućine gustoće  $1030 \text{ kg/m}^3$ . Odredite maseni tok u  $\text{kg/s}$ .

**RJEŠENJE:  $4,5 \cdot 10^{-5} \text{ kg/s}$**

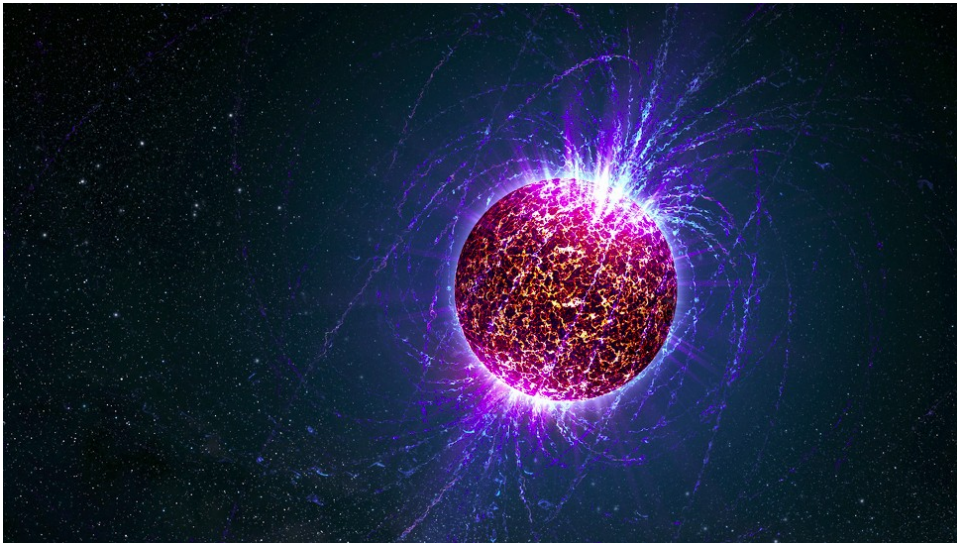
8. Avionsko krilo dizajnirano je tako da brzina zraka iznad krila bude  $251 \text{ m/s}$  onda kad je brzina zraka ispod krila  $225 \text{ m/s}$ . Gustoća zraka je  $1,29 \text{ kg/m}^3$ . Kolika podizna sila djeluje na krilo površine  $24,0 \text{ m}^2$ ?

**RJEŠENJE:  $1,92 \cdot 10^5 \text{ N}$**

# ZADACI ZA VJEŽBU

9. Neutronske zvijezde sastoje se samo od neutrona i nevjerojatno su guste. Tipična masa i polumjer neutronske zvijezde mogu biti  $2,7 \cdot 10^{28}$  kg i  $1,2 \cdot 10^3$  m.
- Odredite gustoću te zvijezde.
  - Kolika bi bila masa novčića (obujma  $2,0 \cdot 10^{-7}$  m<sup>3</sup>) kad bi bio napravljen od istog materijala?

**RJEŠENJE:  $3,7 \cdot 10^{18}$  kg/m<sup>3</sup>;  $7,5 \cdot 10^{11}$  kg (750 milijuna tona)**



10. Tok vode kroz cijev je  $1,50$  m<sup>3</sup>/s. Odredite brzinu vode kroz onaj dio cijevi koji ima polumjer  $0,500$  m.

**RJEŠENJE:  $1,91$  m/s**

**WILEY**

# PITANJA ZA PONAVLJANJE

1. Fluid
2. Gustoća
3. Tlak
4. Atmosferski tlak
5. Pacalovo načelo
6. Uzgon
7. Arhimedov zakon
8. Idealni fluid
9. Jednadžba kontinuiteta
10. Bernoullijeva jednadžba