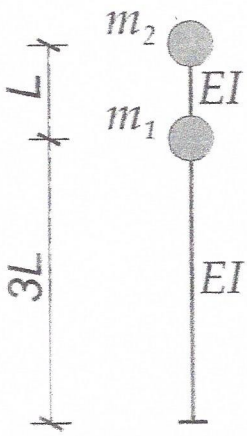
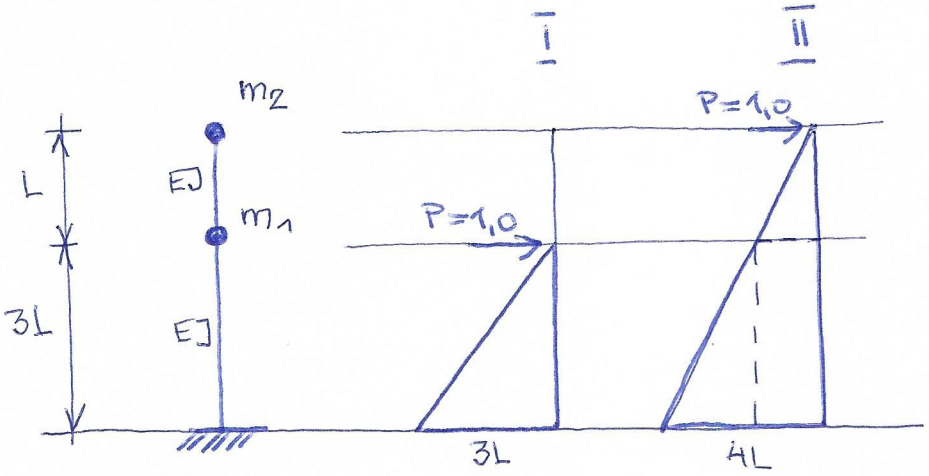
3. Vodotoranj, shematski prikazan na slici, sastoji se od dva rezervoara od kojih svaki teži ***10t*** prazan a ***50t*** kada je ispunjen vodom. Stup vodotornja ukupne visine ***4L=12m*** izveden je kao šuplja armiranobetonska cijev vanjskog promjera ***200cm*** a unutarnjeg ***180cm*** (modul elastičnosti betona ***30GPa***). Modeliranjem vodotornja pomoću zamjenjujućeg sustava s dva stupnja slobode (s obzirom na horizontalne pomake), odredite njegove periode te skicirajte njegove vlastite oblike za slučaj kada je donji rezervoar pun vode a gornji prazan. ***[40]***





$L = 3m$

$L_{uk} = 12m$

$$\delta_{11} = \frac{1}{EJ}\left[\frac{1}{2}\cdot 3L\cdot 3L\cdot \frac{2}{3}\cdot 3L\right] = \frac{9L^3}{EJ}$$

$$\delta_{22} = \frac{1}{EJ}\left[\frac{1}{2}\cdot 4L\cdot 4L\cdot \frac{2}{3}\cdot 4L\right] = \frac{64L^3}{3EJ}$$

$$\delta_{12} = \frac{1}{EJ}\left[\frac{1}{2}\cdot 3L\cdot 3L\cdot \left(\frac{2}{3}\cdot 3L + L\right)\right] = \frac{9L^2}{2EJ}\cdot 3L = \frac{27L^3}{2EJ} = \delta_{21}$$

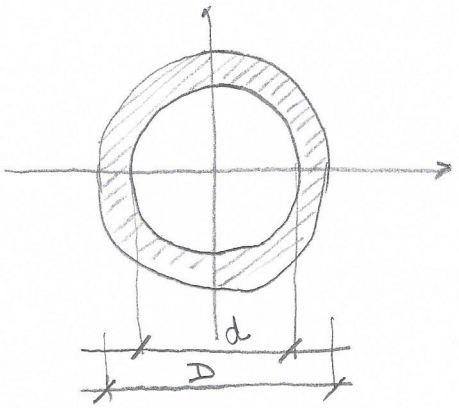
$$[a] = \frac{L^3}{EJ}\begin{bmatrix} 9{,}000 & 13{,}500 \\ 13{,}500 & 21{,}333 \end{bmatrix}; \quad [m] = \begin{bmatrix} m_1 & \emptyset \\ \emptyset & m_2 \end{bmatrix}.$$

Aksijalni moment tromosti cijevi

$$J_o = \frac{(D^4 - d^4)\pi}{64}$$

$$J_o = \frac{(2{,}0^4 - 1{,}8^4)\pi}{64}$$

$$J_o = 0{,}27\ m^4$$



$$EJ = 30\cdot 10^6 \cdot 0{,}27 = 8{,}103\cdot 10^6\ kNm^2$$

$$L^3/EJ = 3{,}0^3 / 8{,}103\cdot 10^6 = 3{,}332\cdot 10^{-6}\ m\,kN^{-1}$$

$$[a] = \begin{bmatrix} 3{,}00\cdot 10^{-5} & 4{,}50\cdot 10^{-5} \\ 4{,}50\cdot 10^{-5} & 7{,}11\cdot 10^{-5} \end{bmatrix} [m]; \qquad [m] = \begin{bmatrix} 50 & \phi \\ \phi & 10 \end{bmatrix} [t]$$

$$[D] = [a][m] = \begin{bmatrix} 3{,}00\cdot 10^{-5} & 4{,}50\cdot 10^{-5} \\ 4{,}50\cdot 10^{-5} & 7{,}11\cdot 10^{-5} \end{bmatrix} \begin{bmatrix} 50 & \phi \\ \phi & 10 \end{bmatrix} = \begin{bmatrix} 15\cdot 10^{-4} & 4{,}50\cdot 10^{-4} \\ 22{,}5\cdot 10^{-4} & 7{,}11\cdot 10^{-4} \end{bmatrix}$$

$$[D] - \lambda[I] = \begin{bmatrix} 15\cdot 10^{-4} - \lambda & 4{,}50\cdot 10^{-4} \\ 22{,}5\cdot 10^{-4} & 7{,}11\cdot 10^{-4} - \lambda \end{bmatrix}$$

$$\det\left[[D] - \lambda[I]\right] = 1{,}067\cdot 10^{-6} - 0{,}221\cdot 10^{-2}\lambda + \lambda^2 - 1{,}013\cdot 10^{-6} =$$

$$= \lambda^2 - 0{,}221\cdot 10^{-2}\lambda + 0{,}054\cdot 10^{-6} = \phi$$

$$\lambda_{1,2} = \frac{0{,}221\cdot 10^{-2} \pm \sqrt{(-0{,}221\cdot 10^{-2})^2 - 4\cdot 0{,}054\cdot 10^{-6}}}{2} = \frac{0{,}221\cdot 10^{-2} \pm 0{,}215\cdot 10^{-2}}{2}$$

$\lambda_1 = 0{,}00218 \Rightarrow \omega_1^2 = 1/\lambda_1 \Rightarrow \omega_1 = 21{,}418\ rad/s \Rightarrow T_1 = 0{,}293\ s$

$\lambda_2 = 0{,}00003 \Rightarrow \omega_2 = 182{,}574\ rad/s \Rightarrow T_2 = 0{,}0344\ s$

I za $\lambda = \lambda_1$:

$$(15\cdot 10^{-4} - 0{,}00218)u_1 + 4{,}50\cdot 10^{-4}u_2 = \phi$$

$$-0{,}00068\,u_1 + 0{,}00045\,u_2 = \phi$$

$$u_1 = 0{,}662\,u_2$$

$$\{\Phi\}_1 = \begin{Bmatrix} 0{,}662 \\ 1{,}000 \end{Bmatrix}$$

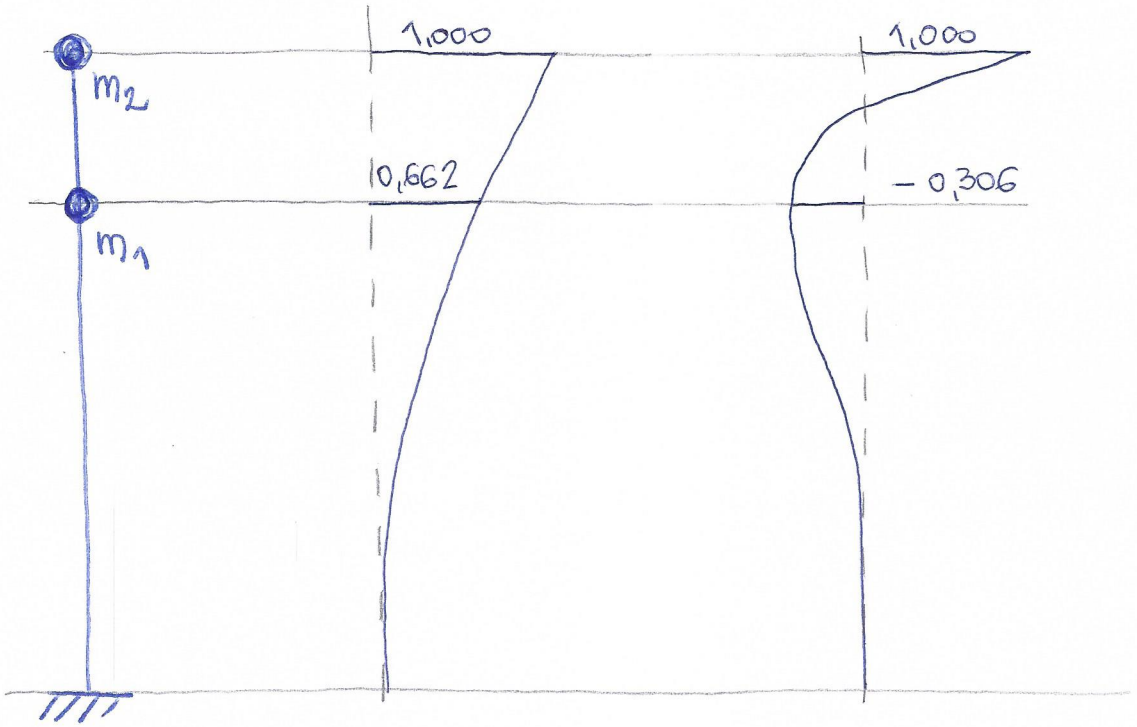
II za $\lambda = \lambda_2$:

$$(15\cdot 10^{-4} - 0{,}00003)u_1 + 4{,}50\cdot 10^{-4}u_2 = \phi$$

$$0{,}00147\,u_1 + 0{,}00045\,u_2 = \phi$$

$$u_1 = -0{,}306\,u_2$$

$$\{\Phi\}_2 = \begin{Bmatrix} -0{,}306 \\ 1{,}000 \end{Bmatrix}$$



I. vlastní oblik

ω_1

II. vlastní oblik

ω_2