

Fluidi

**FIZIKA
PSS-GRAD
25. studenog 2025.**



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DEFINICIJA MASENE GUSTOĆE

masena gustoća tvari je masa tvari
podijeljena s volumenom tvari

$$\rho = \frac{m}{V}$$

jedinica SI za masenu gustoću: kg/m³

11.1 Masena gustoća

krutine	ρ (kg/m ³)
aluminij	2700
mjed	8470
beton	2200
bakar	8890
dijamant	3520
zlato	19300
led	917
čelik	7860
olovo	11300
kvarc	2660
srebro	10500
drvo	550

tekućine	ρ (kg/m ³)
krv (37 °C)	1060
etilni alkohol	806
živa	13600
ulje	800
voda (4 °C)	1000

plinovi	ρ (kg/m ³)
zrak	1,29
ugljičkov dioksid	1,98
helij	179
vodik	0,0899
dušik	1,25
kisik	1,43

Primjer 1 Udio krvi u tjelesnoj težini čovjeka

Ljudsko tijelo težine 690 N sadrži oko 5,2 L krvi.

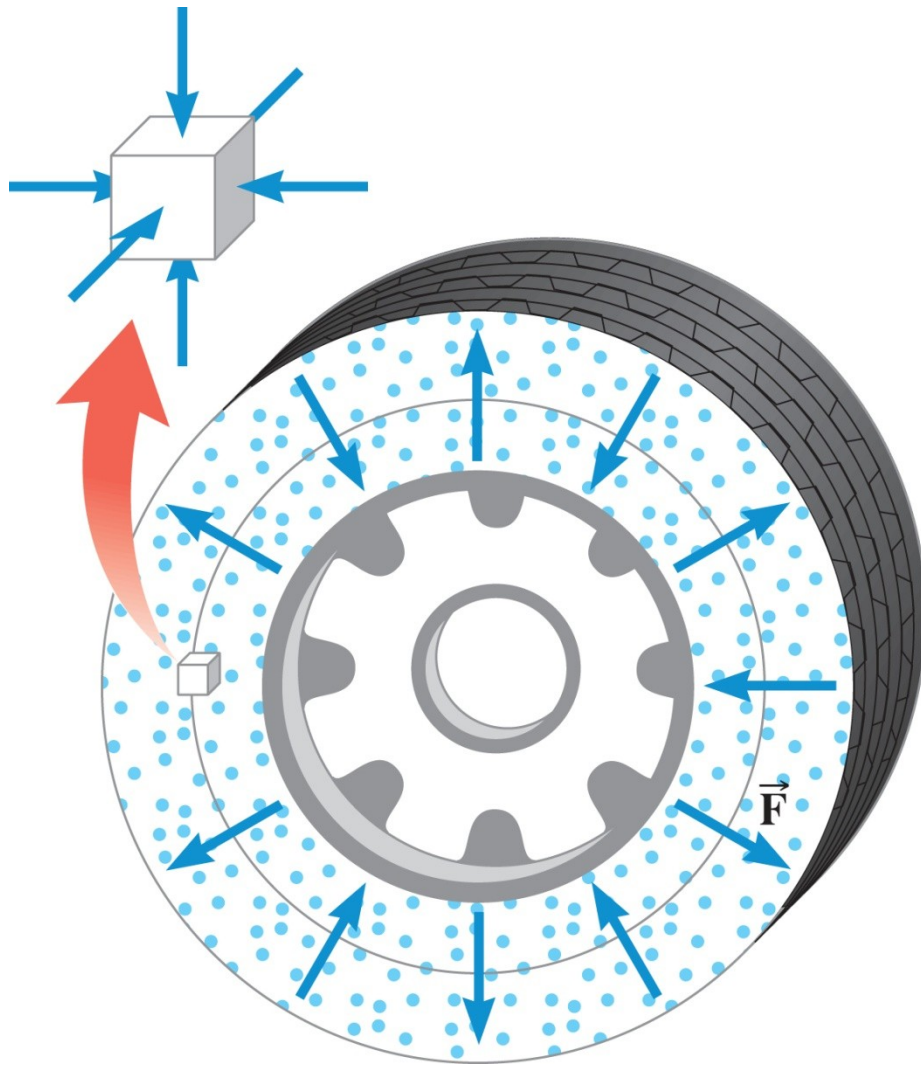
- (a) Odredite težinu krvi.
- (b) Koliki je težinski udio krvi?

$$(a) \quad m = \rho V = 5,2 \cdot 10^{-3} \text{ m}^3 \cdot 1060 \text{ kg m}^{-3} = 5,5 \text{ kg}$$

$$G = m g = 5,5 \text{ kg} \cdot 9,8 \text{ m s}^{-2} = 54 \text{ N}$$

$$(b) \quad w = \frac{G}{G_{\text{ukupno}}} = \frac{54 \text{ N}}{690 \text{ N}} = 7,8 \%$$

11.2 Tlak



$$p = \frac{F}{S}$$

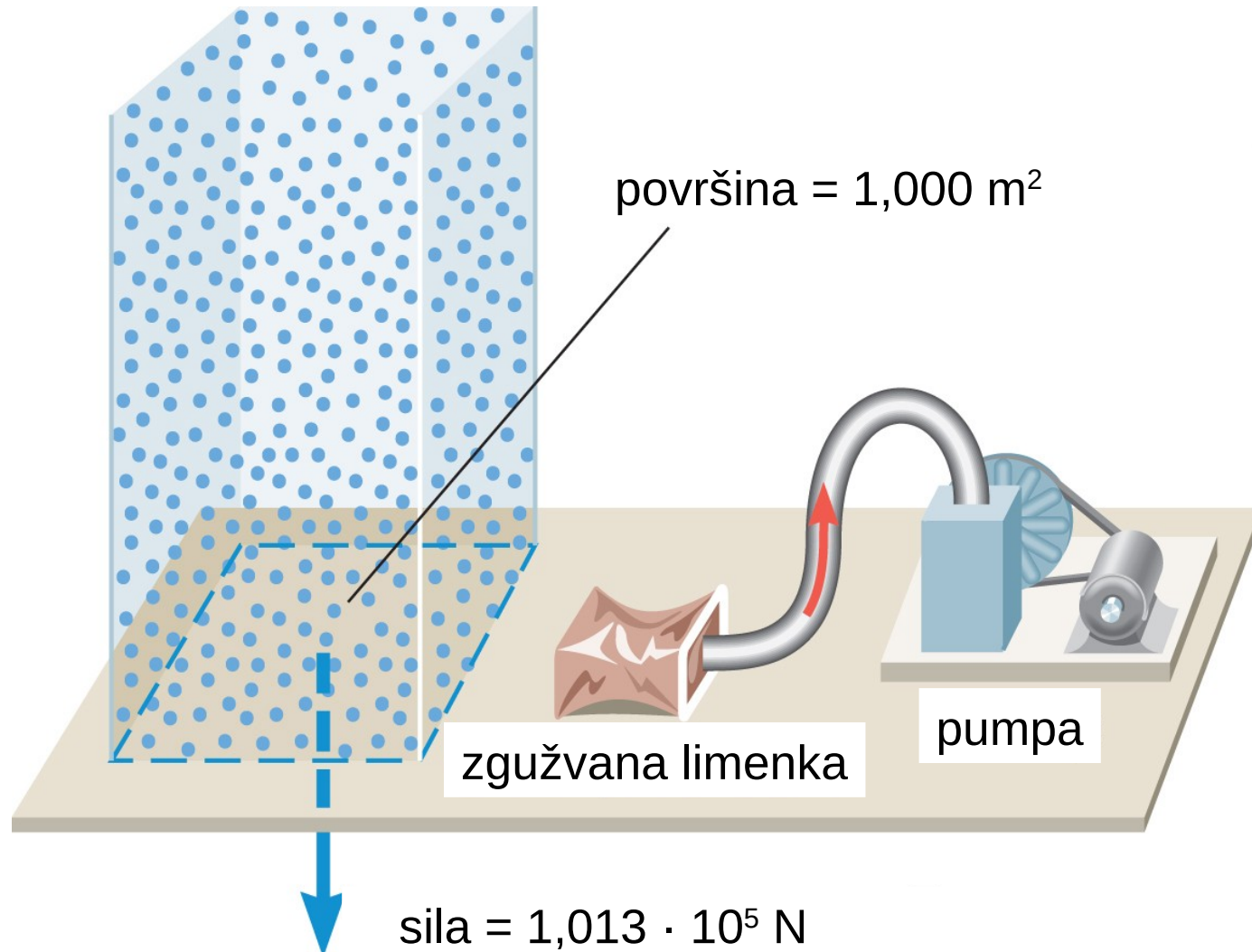
jedinica SI za tlak: $1 \text{ N/m}^2 = 1 \text{ Pa}$

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11.2 Tlak

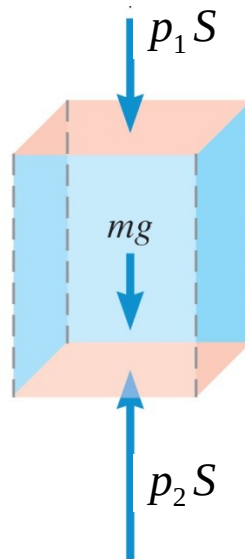
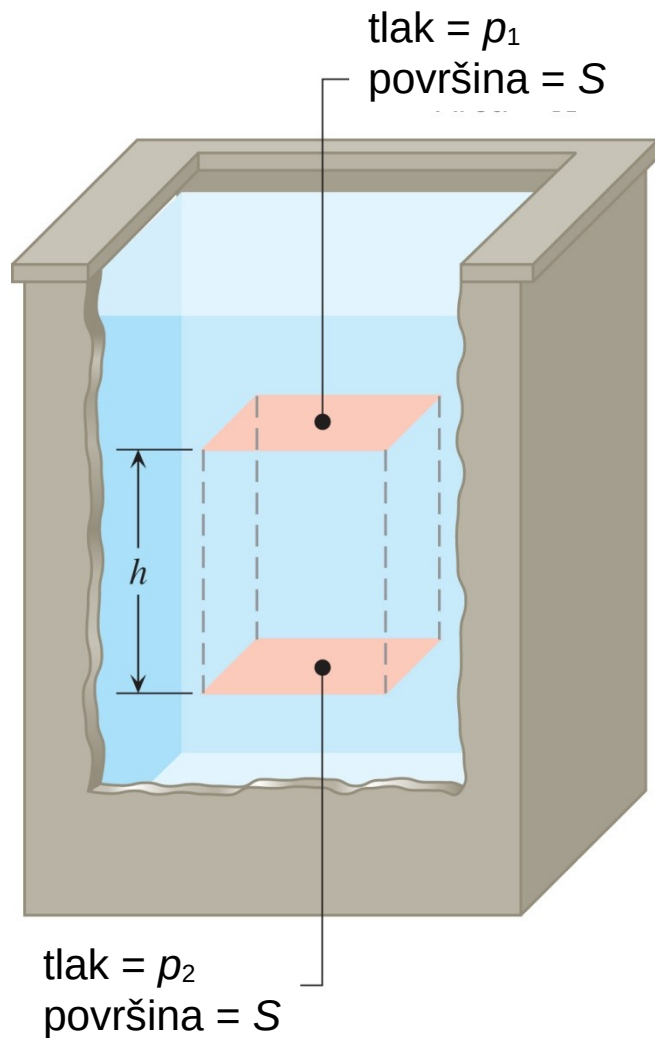
Atmosferski tlak na nultoj nadmorskoj visini: $1,013 \cdot 10^5 \text{ Pa}$



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11.3 Odnos tlaka i dubine u mirnom fluidu



$$\Sigma F = p_2 S - p_1 S - mg = 0$$

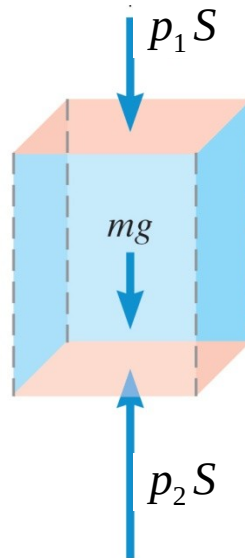
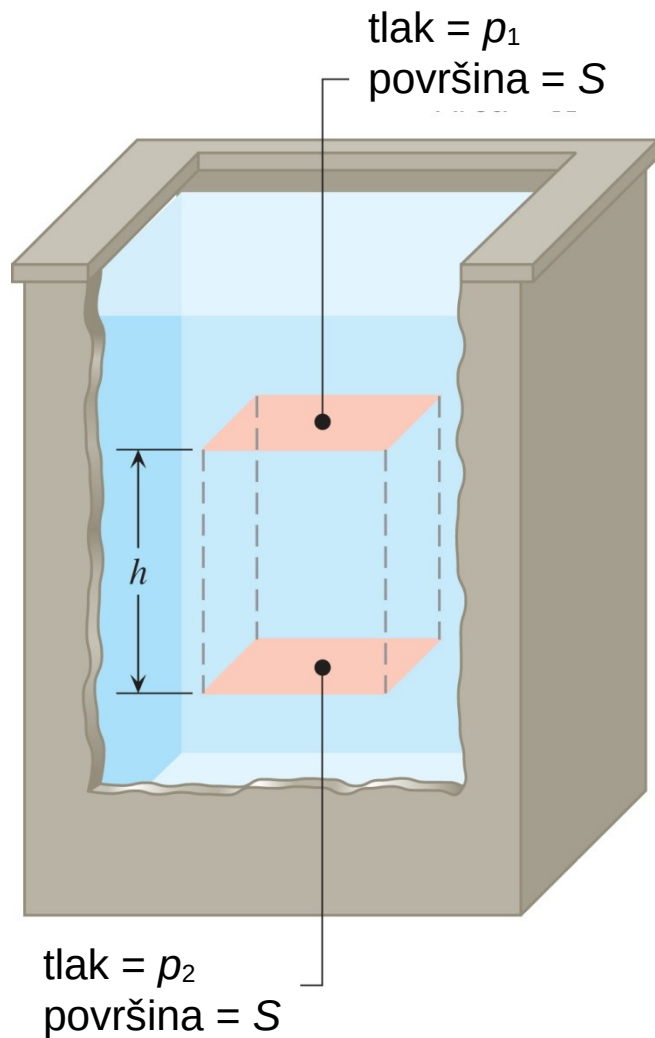


$$p_2 S = p_1 S + mg$$

$m = \rho V$

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11.3 Odnos tlaka i dubine u mirnom fluidu



$$V = Sh$$

$$p_2 S = p_1 S + \rho V g$$



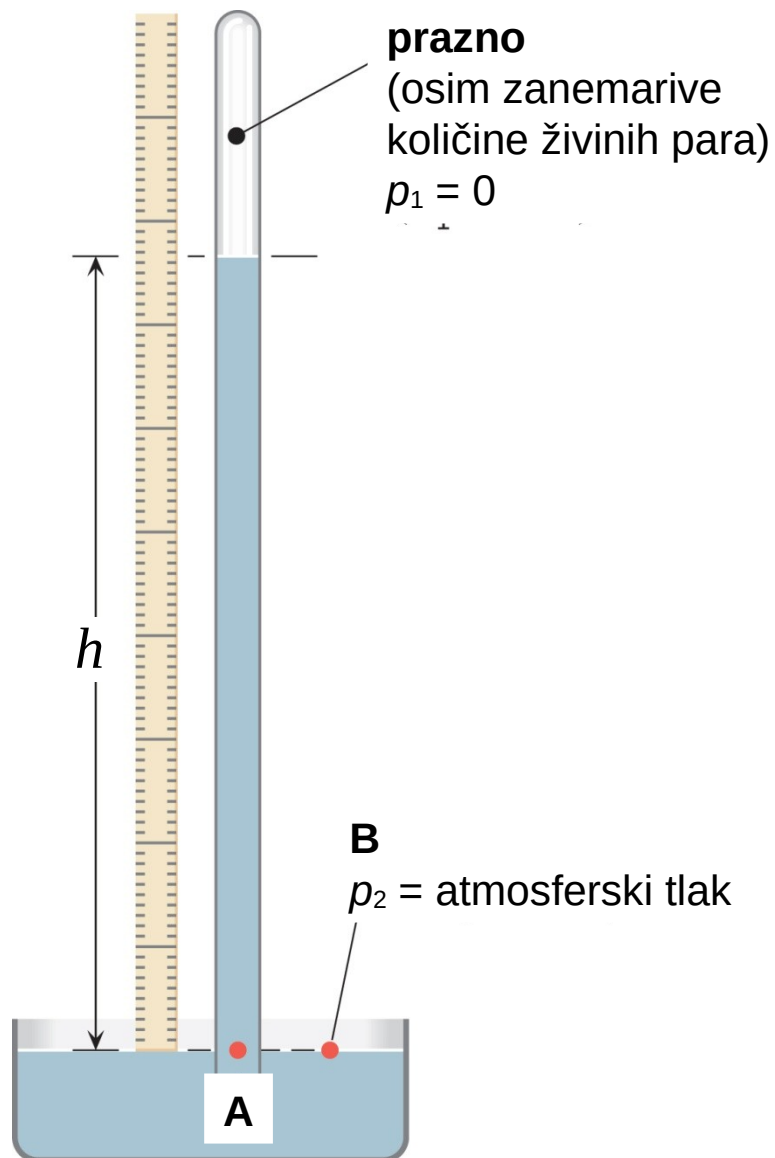
$$p_2 \cancel{S} = p_1 \cancel{S} + \rho \cancel{S} h g$$



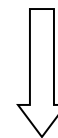
$$p_2 = p_1 + \rho h g$$

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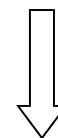
11.4 Mjerenje tlaka



$$p_2 = p_1 + \rho h g$$



$$p_{\text{atm}} = \rho h g$$



$$h = \frac{p_{\text{atm}}}{\rho g} = \frac{1,01 \cdot 10^5 \text{ Pa}}{13600 \text{ kg m}^{-3} \cdot 9,8 \text{ m s}^{-2}}$$

$$h = 760 \text{ mm}$$

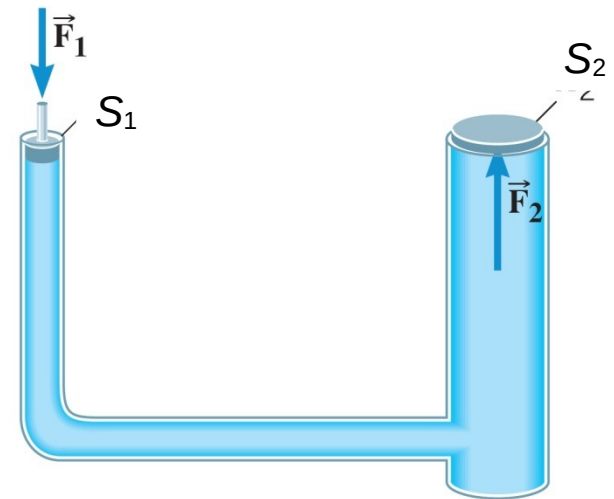
atmosferski tlak odgovara
stupcu žive od 760 mm

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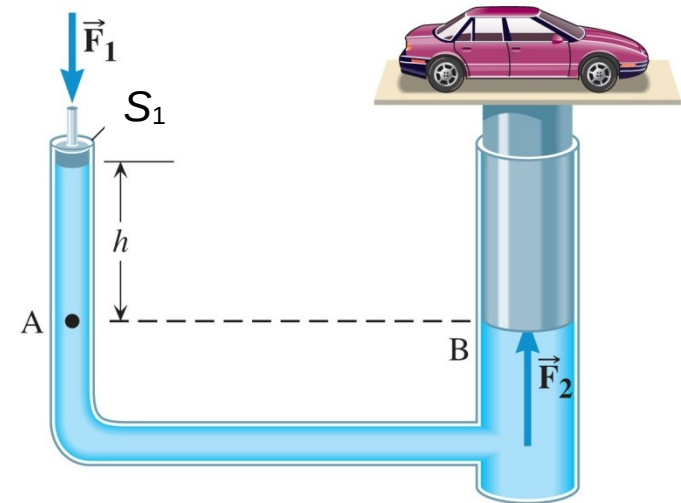
11.5 Pascalovo načelo

PASCALOVO NAČELO

Svaka promjena tlaka u zatvorenom fluidu prenosi se, u istom iznosu, na sve dijelove fluida i stijenke posude.



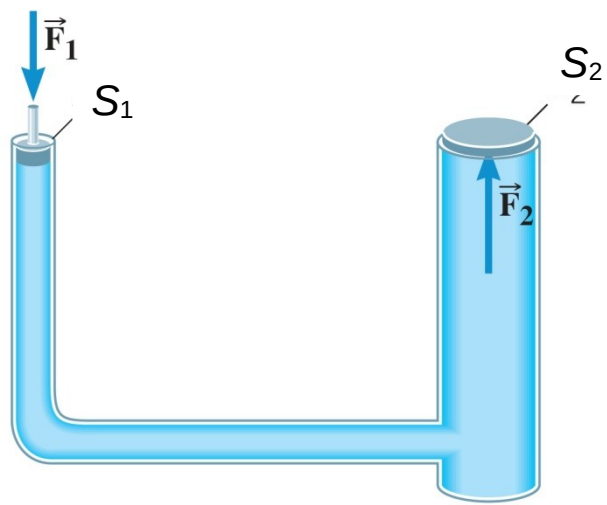
(a)



(b)

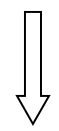
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11.5 *Pascalovo načelo*

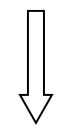


(a)

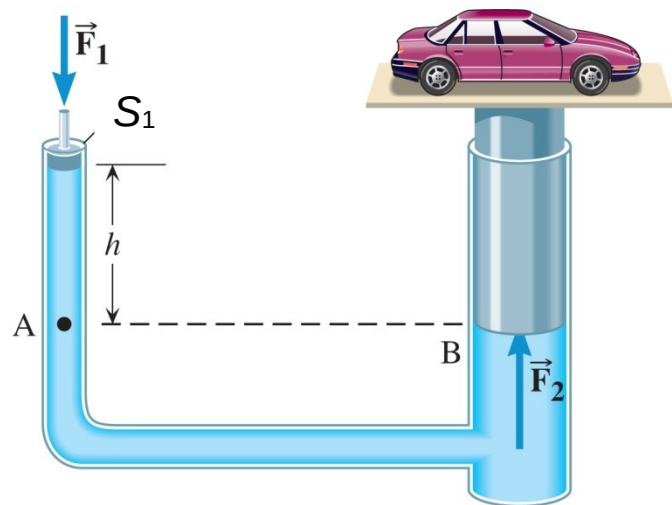
$$p_1 = p_2$$



$$\frac{F_1}{S_1} = \frac{F_2}{S_2}$$



$$F_2 = F_1 \frac{S_2}{S_1}$$



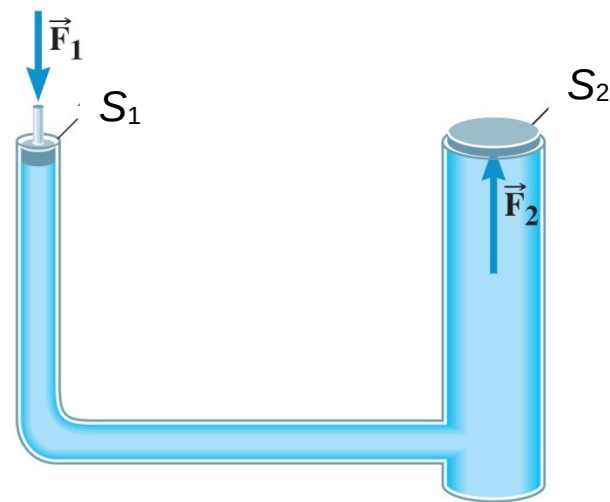
(b)

11.5 Pascalovo načelo

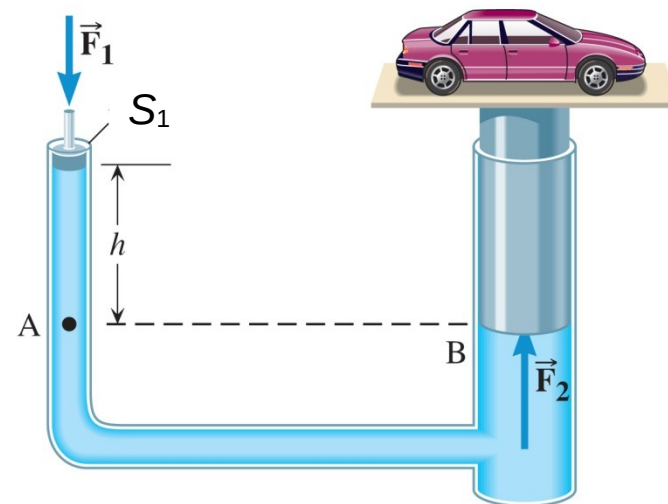
Primjer 7 Podizanje automobila

Ulazni klip ima polumjer od 0,0120 m
a izlazni klip polumjer od 0,150 m.

Težina automobila (zajedno s izlaznim klipom)
je 20500 N. Težinu ulaznog klipa zanemarite.
Odredite ulaznu silu.



(a)



(b)

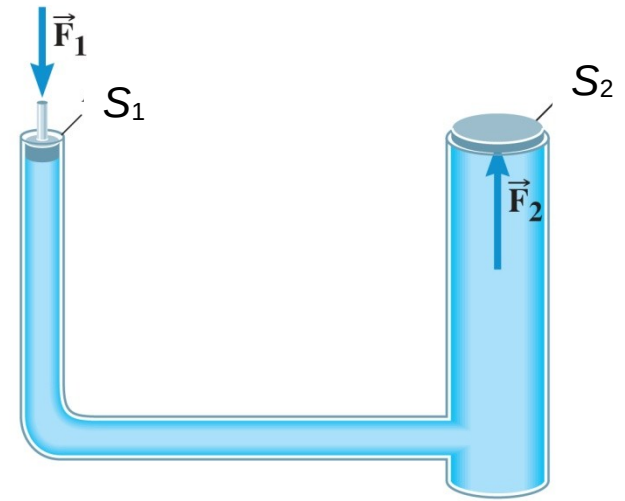
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11.5 Pascalovo načelo

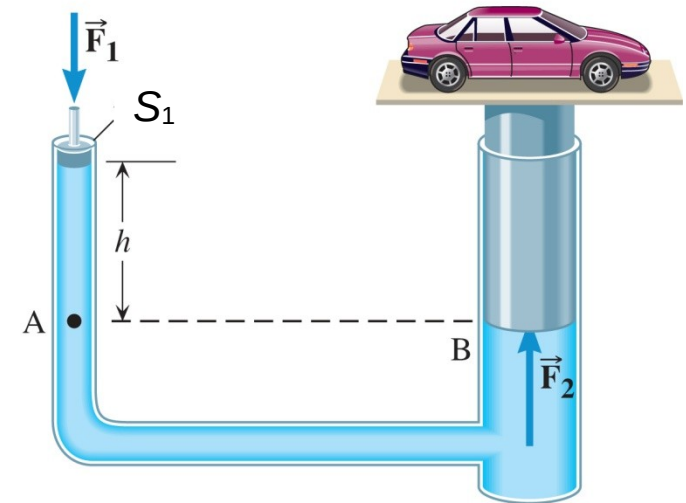
$$F_2 = F_1 \frac{S_2}{S_1}$$

$$F_2 = 20500 \text{ N} \frac{(0,0120 \text{ m})^2 \pi}{(0,150 \text{ m})^2 \pi}$$

$$F_2 = 131 \text{ N}$$



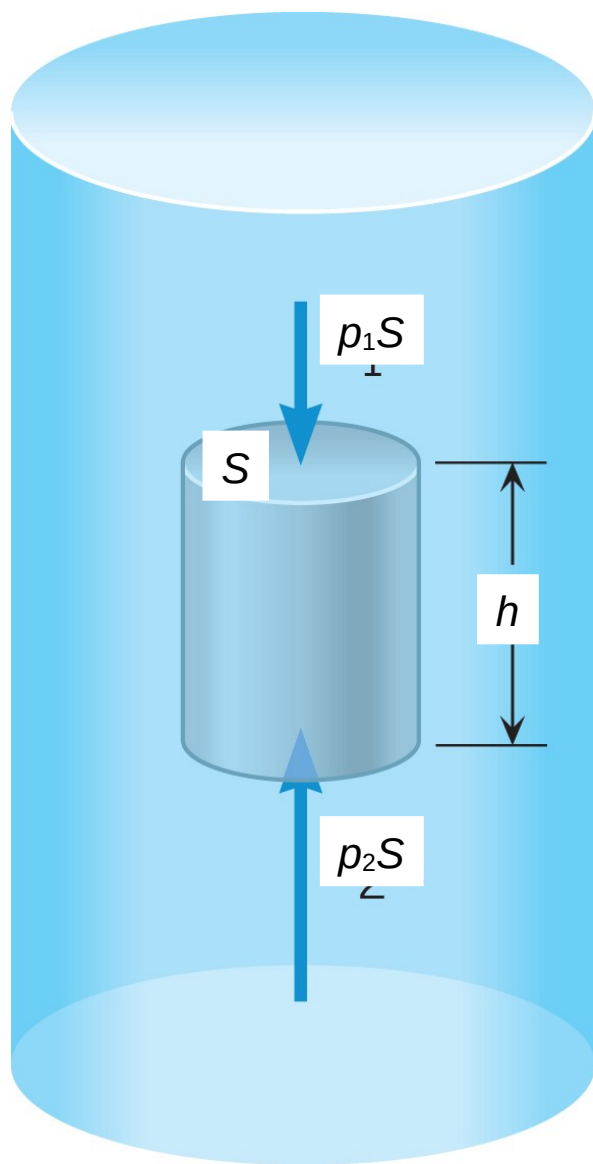
(a)



(b)

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11.6 Arhimedov zakon



$$p_2 - p_1 = \rho h g$$

$$U = p_2 S - p_1 S = (p_2 - p_1) S$$



$$V = h S$$

$$U = \rho g h S$$



$$U = \rho V g$$

masa istisnutog fluida

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ARHIMEDOV ZAKON

Tijelo uronjeno u fluid izgubi na težini onoliko koliko teži istisnuti fluid.

$$U = G_{\text{fluid}}$$

uzgon

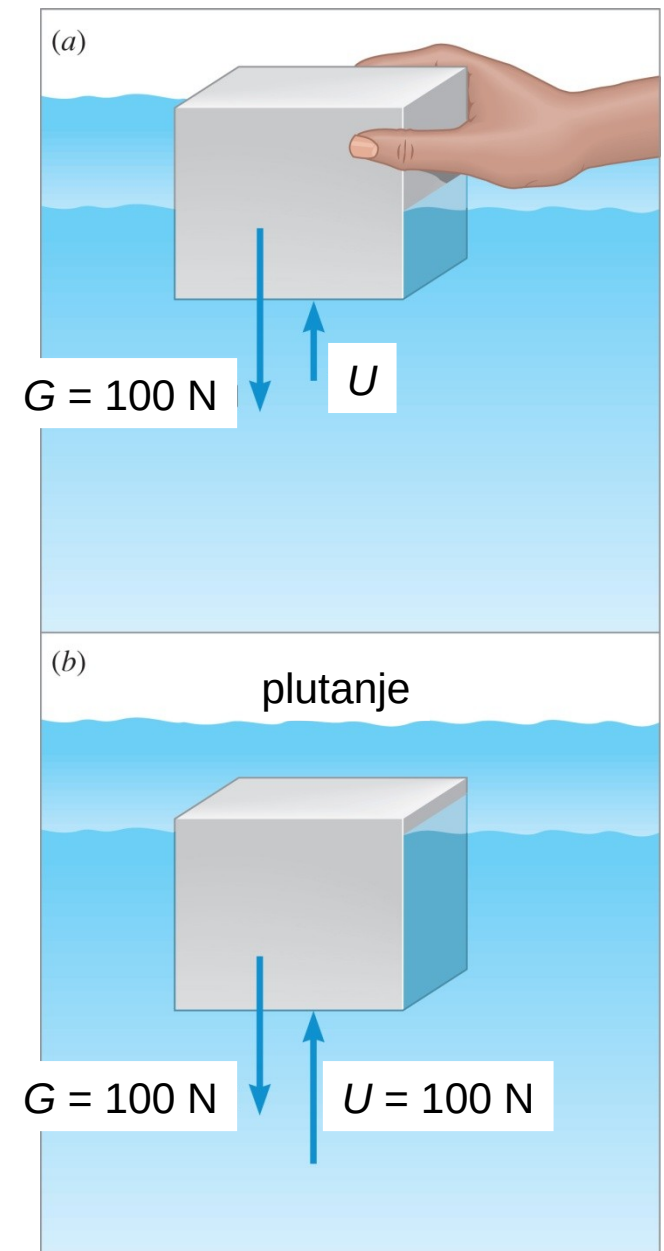
težina istisnutog fluida

The diagram illustrates the equation $U = G_{\text{fluid}}$ from Archimedes' principle. A yellow rectangular box contains the equation. A blue arrow points from the variable U to the word 'uzgon' (buoyancy) in blue text. A red arrow points from the term G_{fluid} to the phrase 'težina istisnutog fluida' (weight of the displaced fluid) in red text.

11.6 Arhimedov zakon

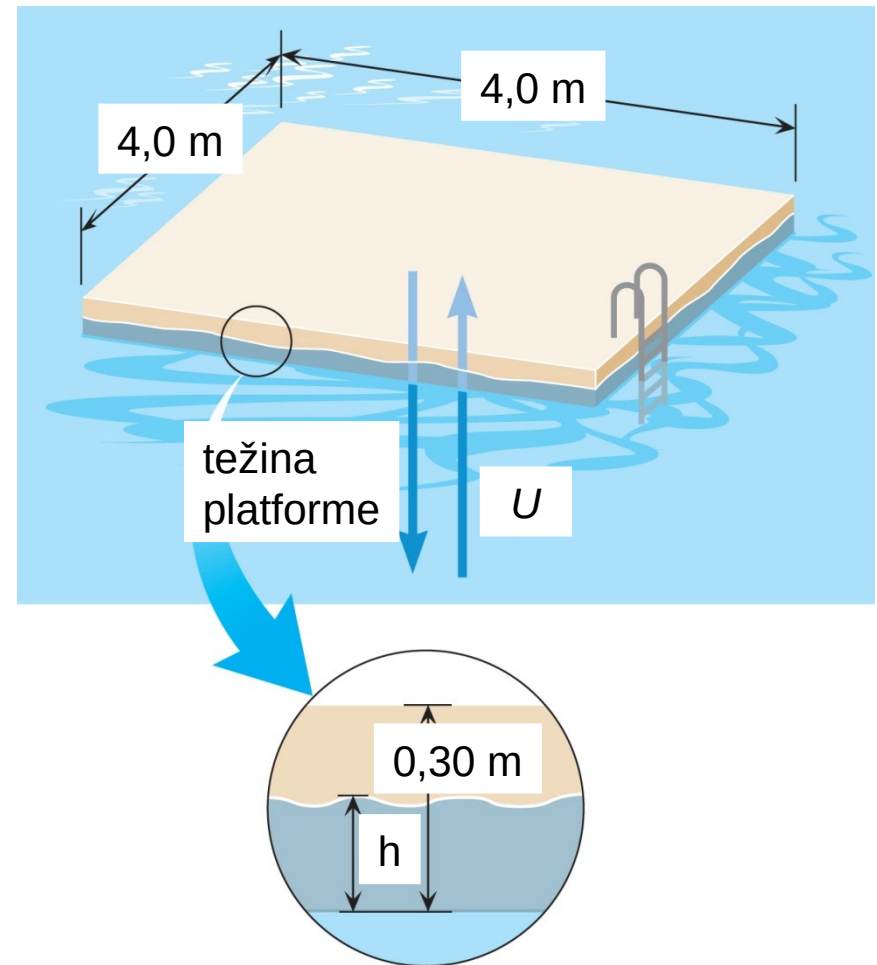
Ako je uzgon jednak težini
onda tijelo u fluidu pluta.

$$U = \rho_{\text{fluid}} g V_{\text{uronjen}}$$



Primjer 9 Plutajuća platforma

Pluta li drvena platforma na vodi?
Ako pluta, koliko je duboko
uronjena u vodu?



11.6 Arhimedov zakon

$$V_{\text{ukupni}} = 4,0 \text{ m} \cdot 4,0 \text{ m} \cdot 0,30 \text{ m} = 4,8 \text{ m}^3$$

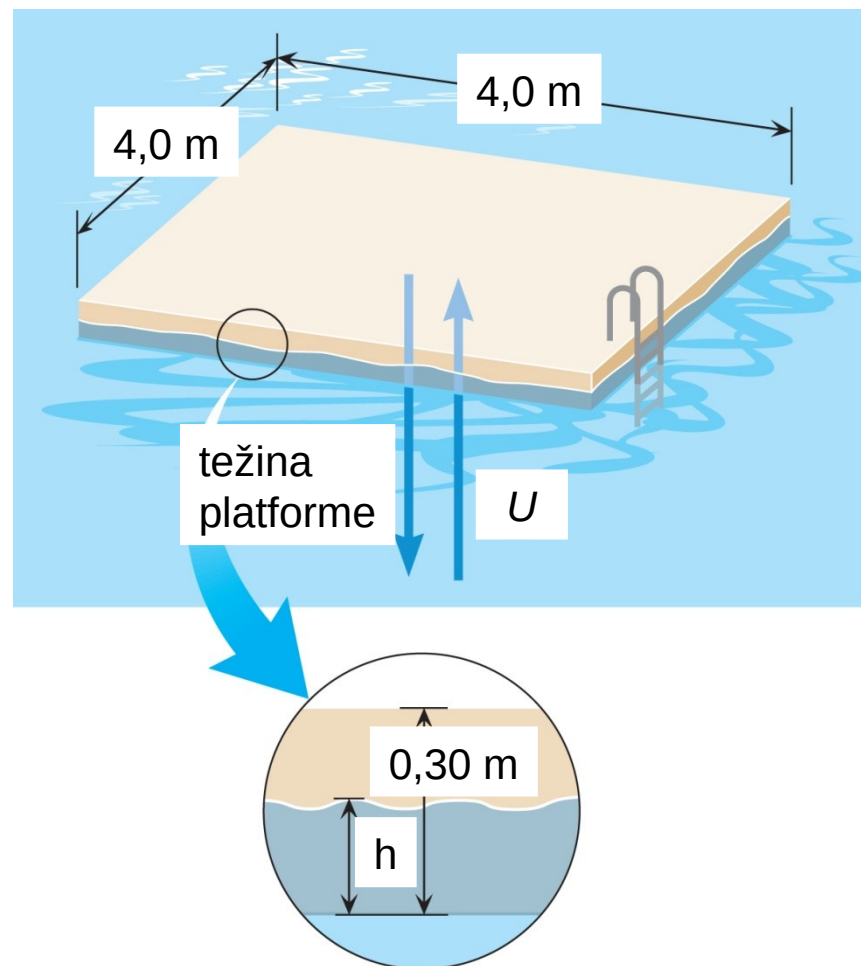
$$U = \rho_{\text{fluid}} g V_{\text{uronjen}}$$

$$U_{\text{max}} = \rho_{\text{fluid}} g V_{\text{ukupni}}$$

$$U_{\text{max}} = 1000 \text{ kg m}^{-3} \cdot 9,8 \text{ m s}^{-2} \cdot 4,8 \text{ m}^3$$

$$U_{\text{max}} = 47000 \text{ N}$$

kad bi cijela platforma bila uronjena
tad bi uzgon bio 47000 N



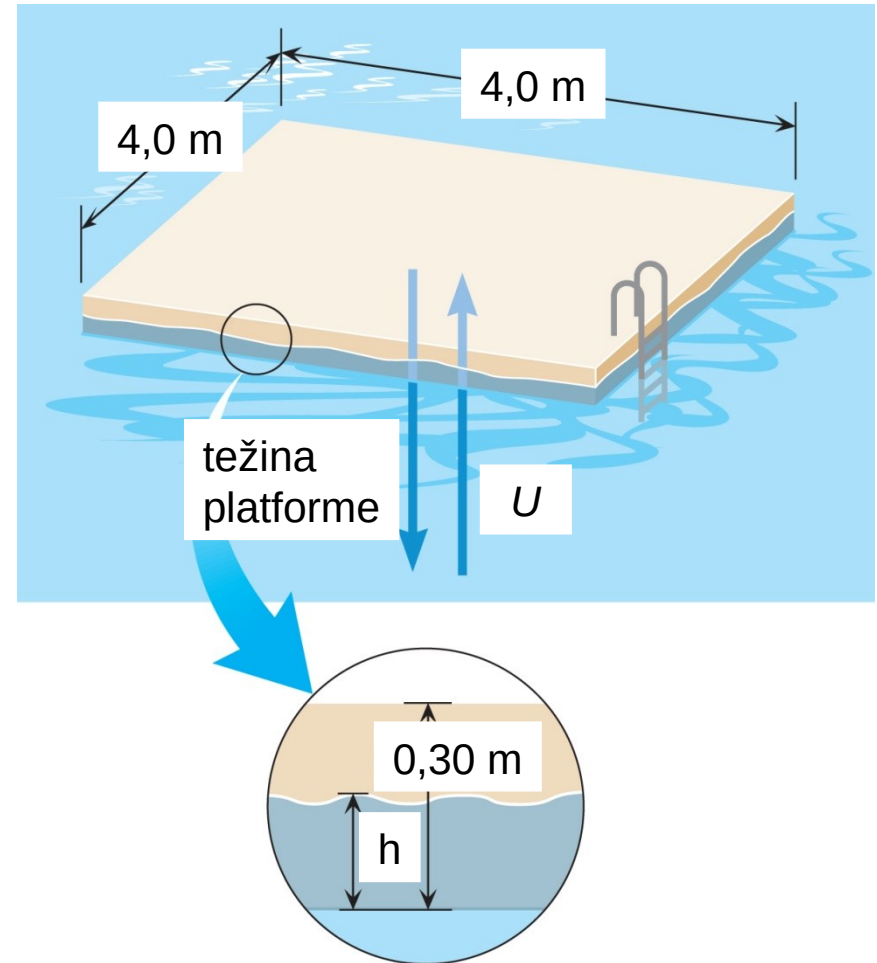
11.6 Arhimedov zakon

$$G = m g = \rho_{\text{drvo}} V_{\text{ukupni}} g$$

$$G = 550 \text{ kg m}^{-3} \cdot 4,8 \text{ m}^3 \cdot 9,8 \text{ m s}^{-2}$$

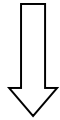
$$G = 26000 \text{ N} < 47000 \text{ N}$$

⇒ platforma pluta



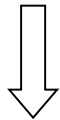
11.6 Arhimedov zakon

$$G = U$$

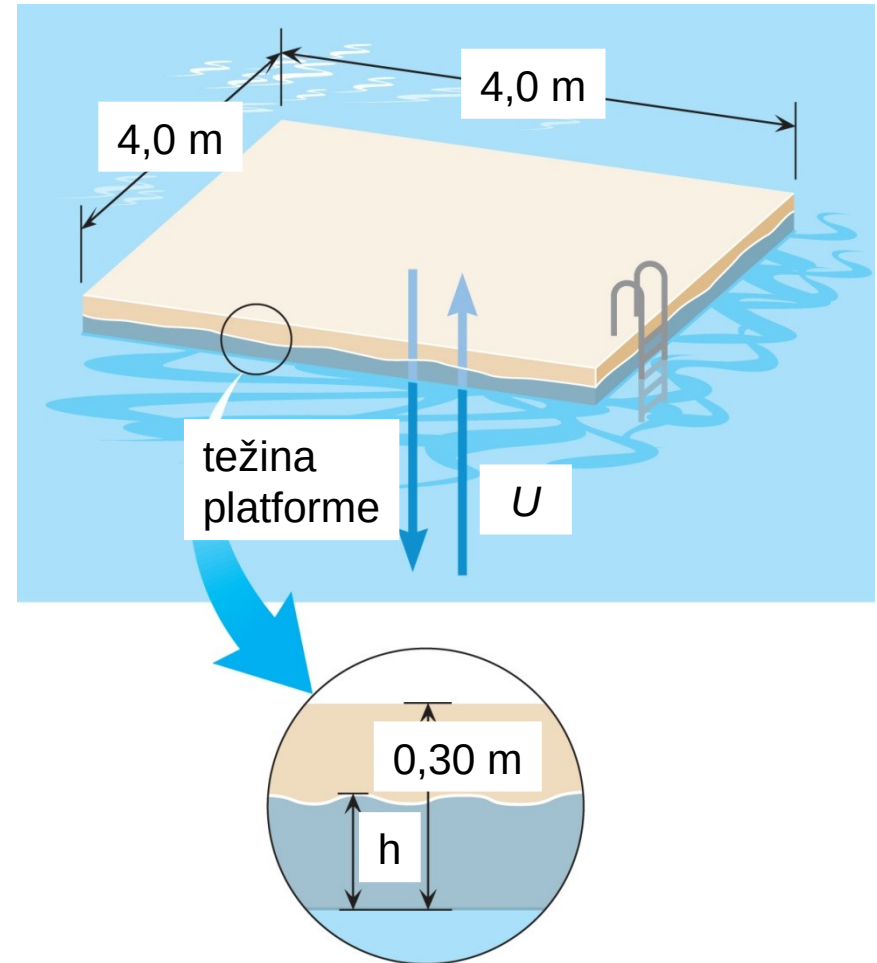


$$26000 \text{ N} = 1000 \text{ kg m}^{-3} \cdot 9,8 \text{ m s}^{-2} \cdot 4,0 \text{ m} \cdot 4,0 \text{ m} \cdot h$$

$$h = \frac{26000 \text{ N}}{1000 \text{ kg m}^{-3} \cdot 9,8 \text{ m s}^{-2} \cdot 4,0 \text{ m} \cdot 4,0 \text{ m}}$$



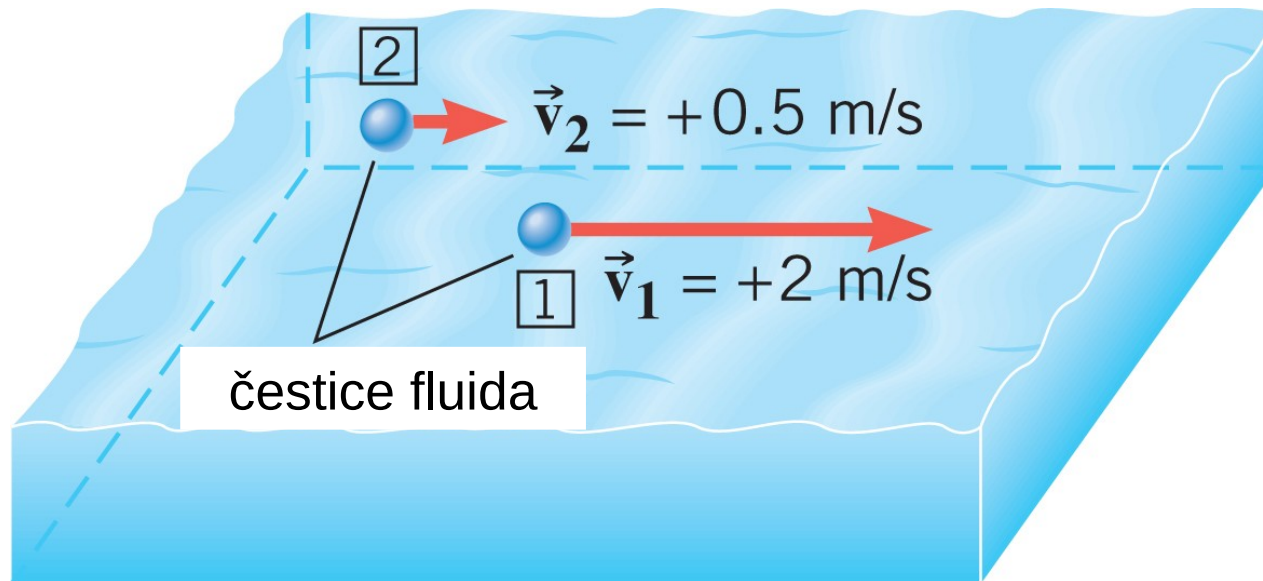
$$h = 0,17 \text{ m}$$



11.7 Gibanje fluida

Pri **stalnom toku** u svakoj je točki brzina čestica konstantna.

Nestalni tok znači da se brzina čestica, u svakoj točki, s vremenom mijenja.

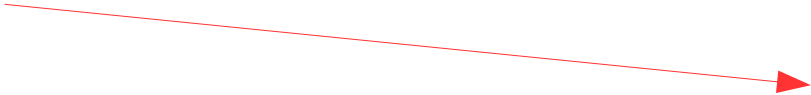


Turbulentni tok je ekstremni oblik nestalnog toka u kojem se brzina čestica u svakoj točki fluida nepredvidljivo mijenja i po iznosu i po smjeru.

11.7 Gibanje fluida

Fluidi mogu biti **stlačivi** ili **nestlačivi**. Većina tekućina su gotovo nestlačive.

Fluidi mogu biti **viskozni** ili **neviskozni**.

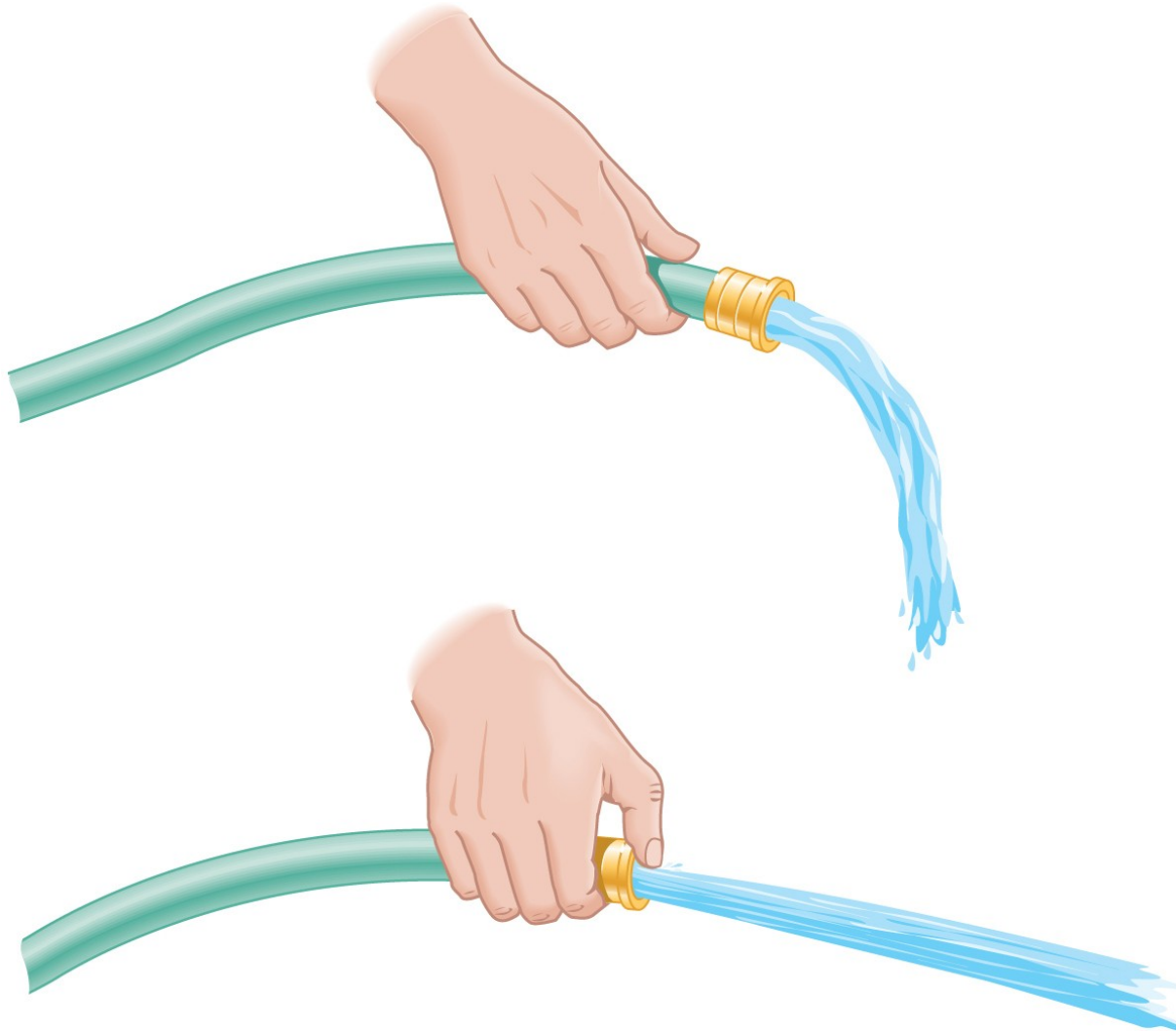


ljepljiv,
gusto tekući

Nestlačiv i neviskozni fluid nazivamo **idealnim fluidom**.

11.8 *Jednadžba kontinuiteta*

Maseni protok je masa fluida koja, u jedinici vremena, prođe kroz cijev.

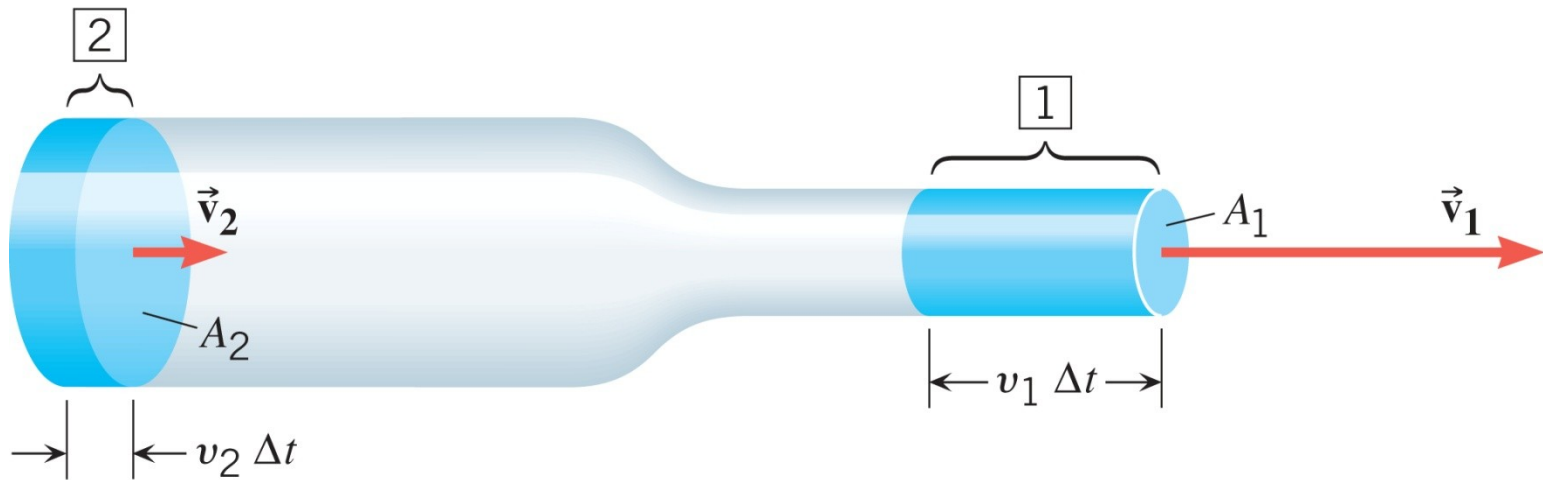


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11.8 Jednadžba kontinuiteta

udaljenost

$$\Delta m = \rho V = \rho A \overset{\text{udaljenost}}{v \Delta t}$$



$$\frac{\Delta m_2}{\Delta t} = \rho_2 A_2 v_2$$

$$\frac{\Delta m_1}{\Delta t} = \rho_1 A_1 v_1$$

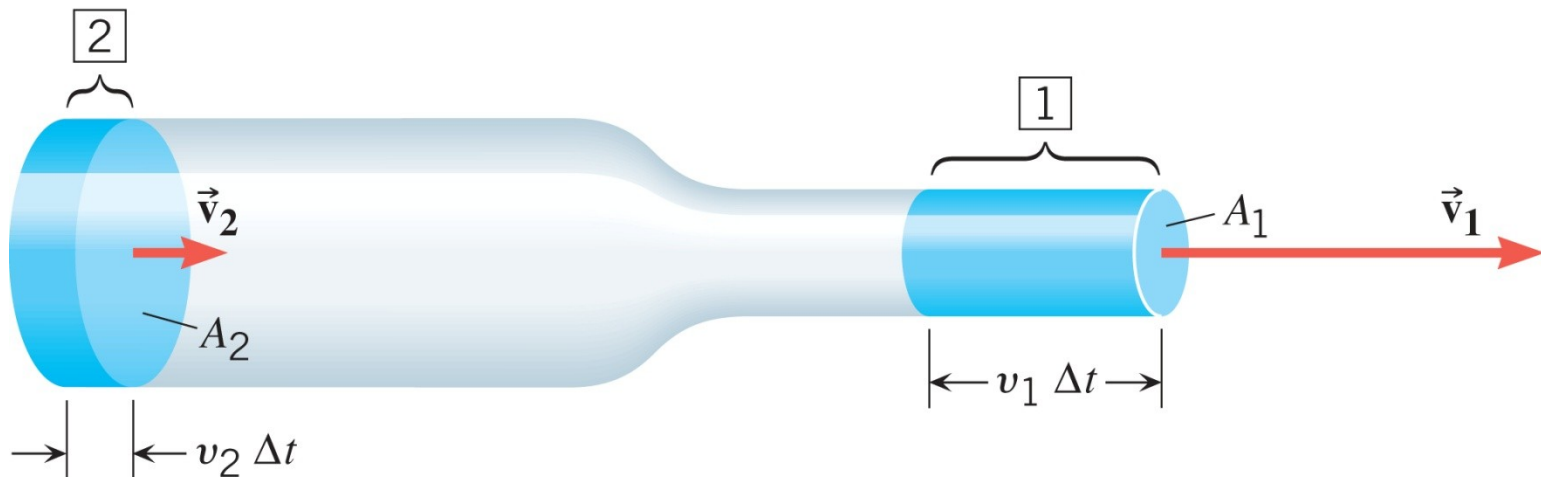
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JEDNADŽBA KONTINUITETA

Maseni protok isti je na svakom položaju uzduž cijevi koja ima jedan ulaz i jedan izlaz za tok fluida.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

jedinica SI za maseni protok: kg/s



11.8 Jednadžba kontinuiteta

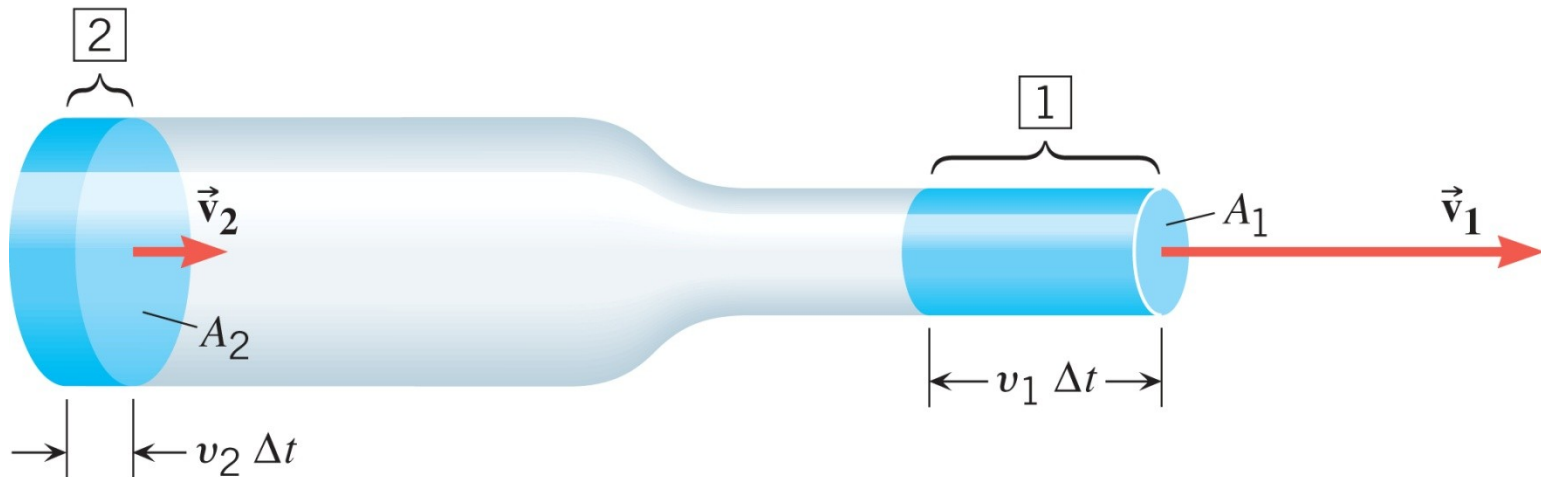
Za nestlačivi fluid:

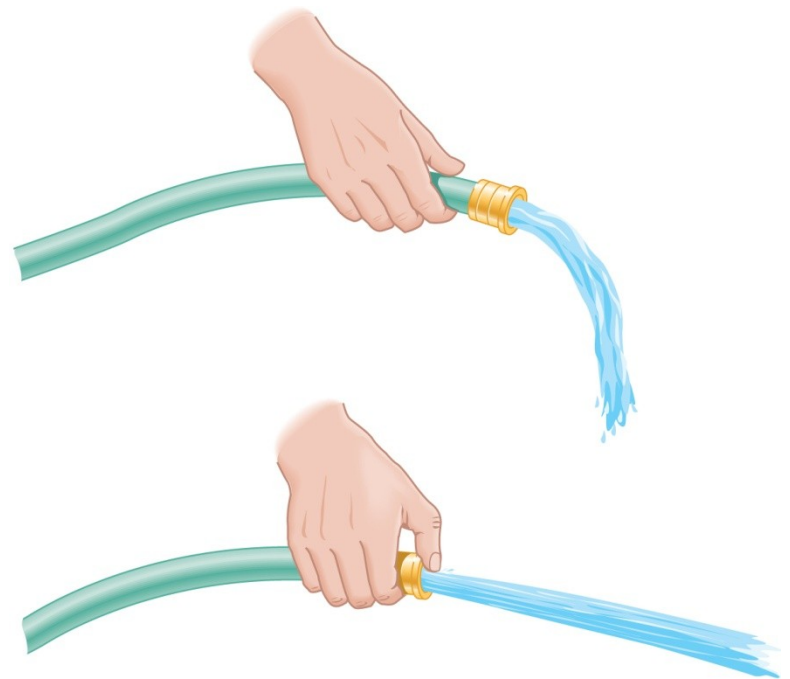
$$\cancel{\rho}_1 A_1 v_1 = \cancel{\rho}_2 A_2 v_2$$

$$A_1 v_1 = A_2 v_2$$

Volumni protok Q :

$$Q = A v$$





Primjer 12 Crijervo za polijevanje

Crijervo za polijevanje ima otvor poprečnog presjeka $2,85 \cdot 10^{-4} \text{ m}^2$. Tim se crijevom kanta volumena 8,00 L napuni za 30 s.

Kolikom brzinom voda: (a) nesmetano izlazi kroz crijevo?
(b) izlazi kroz crijevo čiji je izlazni poprečni presjek smanjen na polovicu?

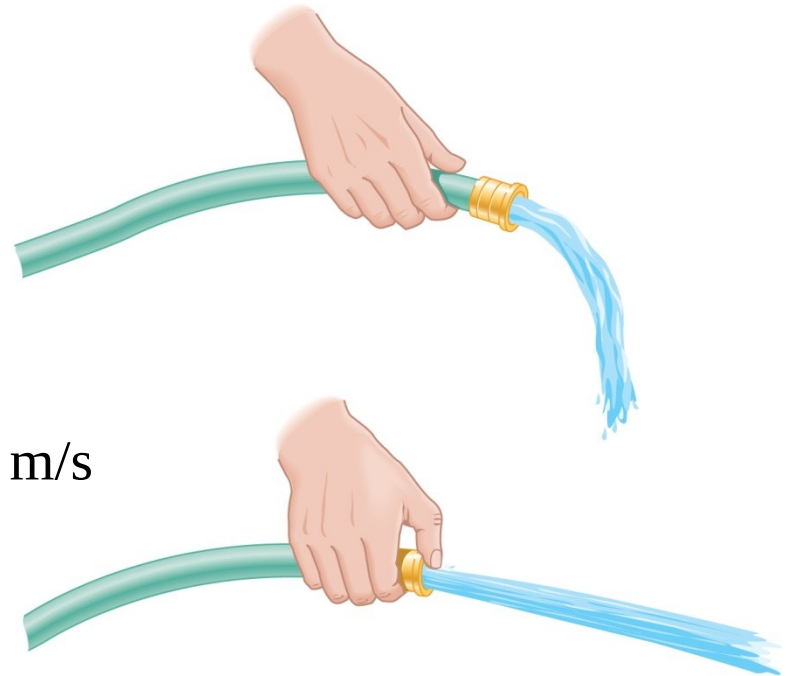
11.8 Jednadžba kontinuiteta

$$(a) \quad Q = A v$$

$$v = \frac{Q}{A} = \frac{(8,00 \cdot 10^{-3} \text{ m}^3) / (30,0 \text{ s})}{2,85 \cdot 10^{-4} \text{ m}^2} = 0,936 \text{ m/s}$$

$$(b) \quad A_1 v_1 = A_2 v_2$$

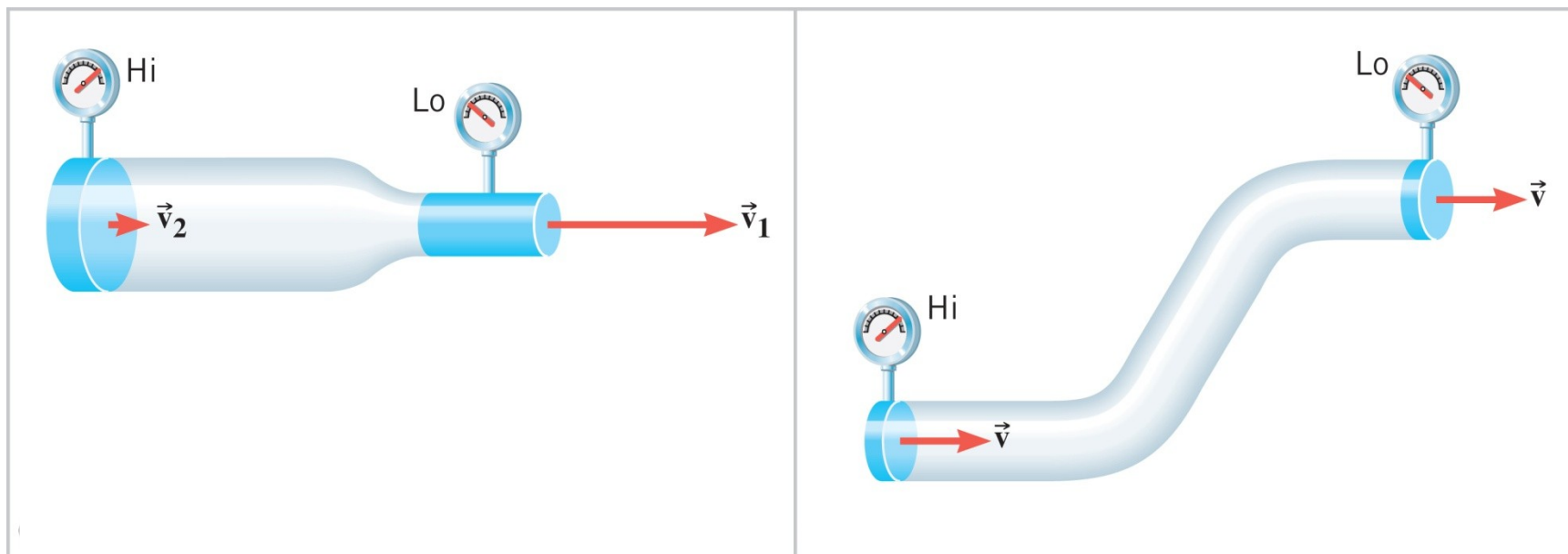
$$v_2 = v_1 \frac{A_1}{A_2} = 0,936 \text{ m/s} \cdot \frac{A_1}{0.5 \cdot A_2} = 0,936 \text{ m/s} \cdot 2 = 1,87 \text{ m/s}$$



11.9 Bernoullijeva jednadžba

Fluid ubrzava prema području nižeg tlaka.

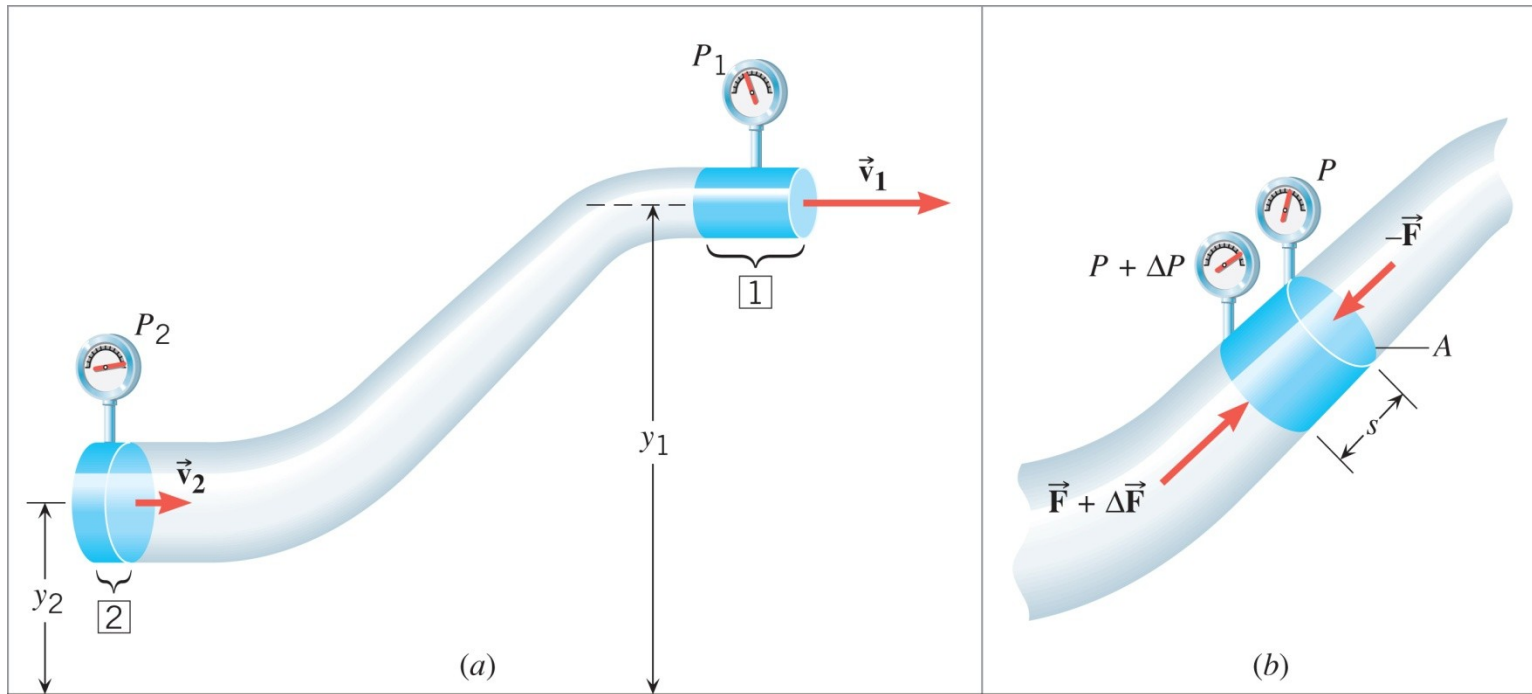
Iz odnosa tlaka i dubine, tlak je niži na višem nivou (ako se presjek cijevi ne mijenja).



11.9 Bernoullijeva jednadžba

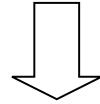
$$\Delta W = \Delta F \cdot s = \Delta p \cdot A \cdot s = (p_2 - p_1) \cdot V$$

$$\Delta W = \left(\frac{1}{2} m v_1^2 + m g y_1 \right) - \left(\frac{1}{2} m v_2^2 + m g y_2 \right)$$



11.9 Bernoullijska jednačina

$$(p_2 - p_1) \cdot V = \left(\frac{1}{2} m v_1^2 + m g y_1 \right) - \left(\frac{1}{2} m v_2^2 + m g y_2 \right)$$



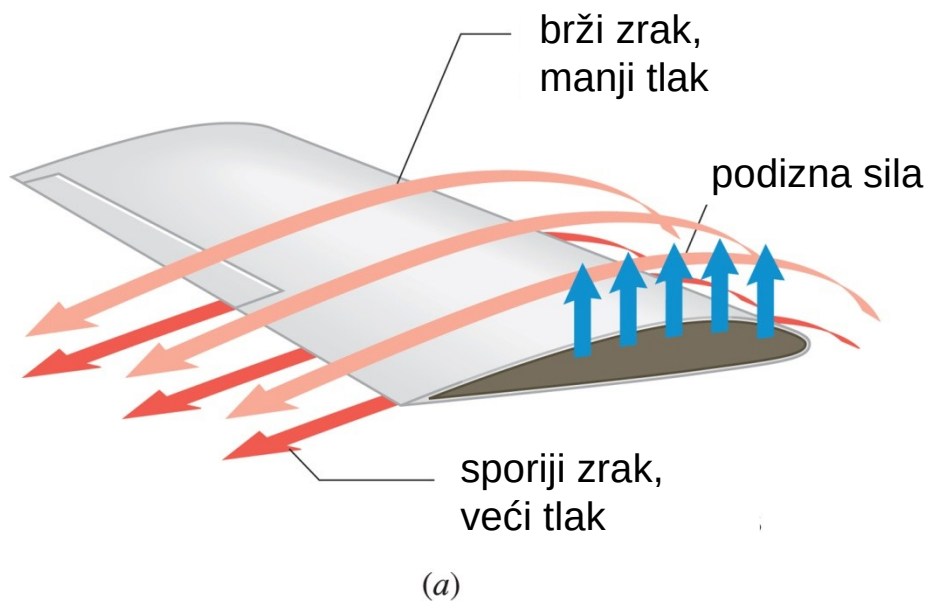
$$p_2 - p_1 = \frac{1}{2} \rho v_1^2 + \rho g y_1 - \frac{1}{2} \rho v_2^2 - \rho g y_2$$

BERNOULLIJEVA JEDNADŽBA

Pri stalnom toku neviskoznog, nestlačivog fluida, tlak, brzina i visina povezani su ovako:

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

11.10 Primjene Bernoullijeve jednačbe



Joe McBride/Getty Images

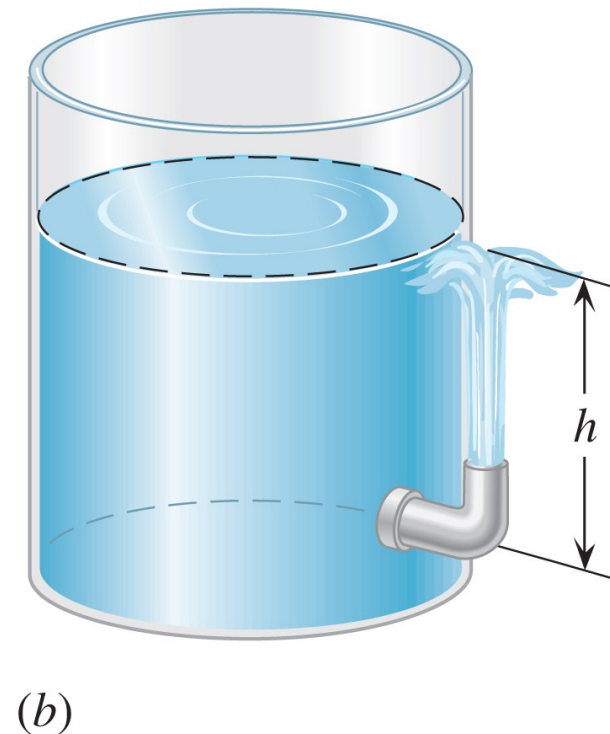
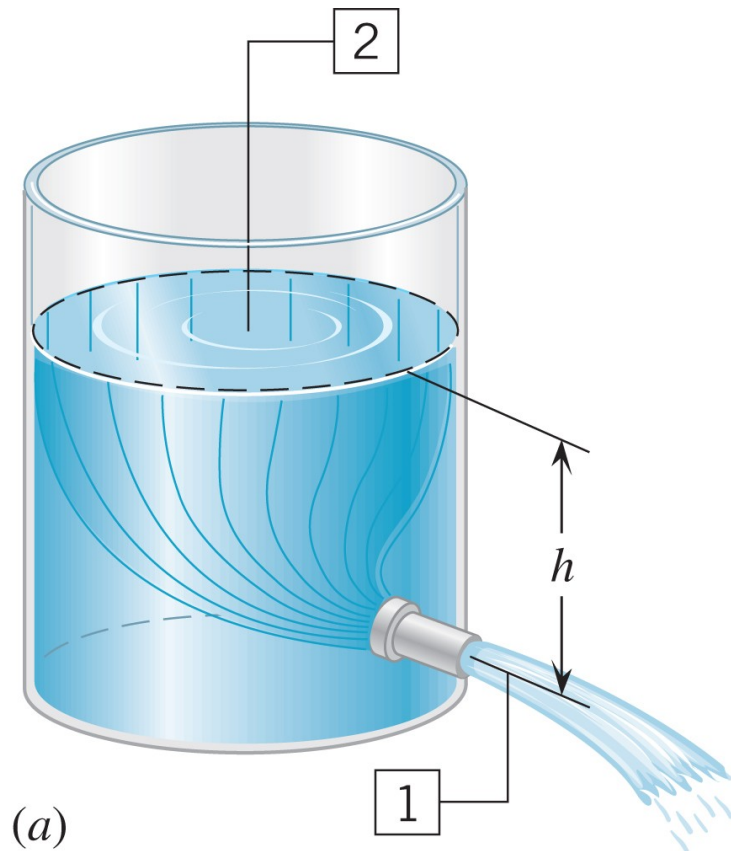
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Primjer 16 Brzina istjecanja

Rezervoar je s gornje strane otvoren.

Izvedite izraz za brzinu kojom tekućina izlazi iz cijevi na dnu.



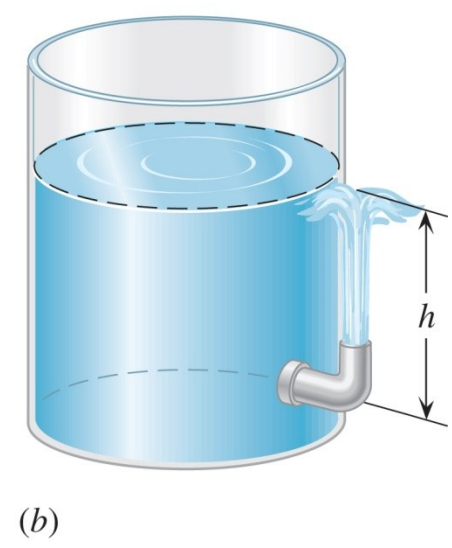
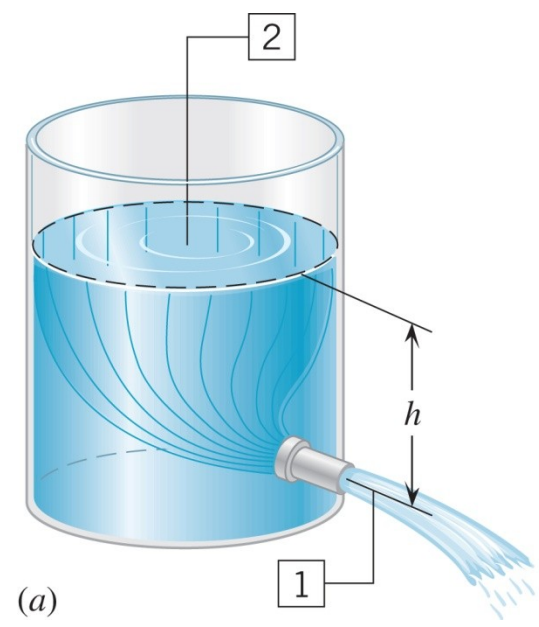
11.10 *Primjene Bernoullijeve jednačbe*

$$\overset{p_{\text{atm}}}{\cancel{p_1}} + \frac{1}{2} \rho v_1^2 + \rho g y_1 = \overset{p_{\text{atm}}}{\cancel{p_2}} + \frac{1}{2} \rho \overset{v_2 \approx 0}{\cancel{v_2^2}} + \rho g y_2$$

$$\frac{1}{2} \rho v_1^2 + \rho g y_1 = \rho g y_2$$

$$\frac{1}{2} \cancel{\rho} v_1^2 = \cancel{\rho} g h$$

$$v_1 = \sqrt{2 g h}$$



ZADACI ZA VJEŽBU

1. Jedan kraj žice pričvršćen je za strop, a na drugom kraju visi puna mjedena kugla. Napetost žice je 120 N. Odredite polumjer kugle.

RJEŠENJE: 7,0 cm

2. Hipotetski sferni planet građen je samo od željeza. Odredite period satelita koji oko tog planeta orbitira uz samu površinu.

RJEŠENJE: 71 min

3. Osoba težine 625 N vozi bicikl težine 98 N. Pretpostavite da je težina jednako raspoređena na oba kotača. Kolika je dodirna površina svake gume s tlom ako je tlak u svakoj gumi $7,60 \cdot 10^5$ Pa?

RJEŠENJE: $4,76 \cdot 10^{-4} \text{ m}^2$

4. Skijaš mase 58 kg spušta se niz padinu nagiba 35° . Dodirna površina svake skije sa snijegom je $0,13 \text{ m}^2$. Odredite tlak kojim svaka skija pritišće snijeg.

RJEŠENJE: 1800 Pa

ZADACI ZA VJEŽBU

5. Pliva patka preko Save. Izračunajte prosječnu gustoću patke ako je 25% njezinog obujma stalno pod vodom?

RJEŠENJE: 250 kg/m^3

6. Gustoća leda je 917 kg/m^3 , a gustoća morske vode 1025 kg/m^3 . Polarni medvjed sjedi na santi leda obujma $5,2 \text{ m}^3$. Odredite masu najtežeg medvjeda koji može sjediti na santi, a da santa ne bude potpuno pod vodom.

RJEŠENJE: 560 kg

7. Pacijent koji se oporavlja od operacije dobiva infuziju. Svaki šest sati primi $9,5 \cdot 10^{-4} \text{ m}^3$ tekućine gustoće 1030 kg/m^3 . Odredite maseni tok u kg/s .

RJEŠENJE: $4,5 \cdot 10^{-5} \text{ kg/s}$

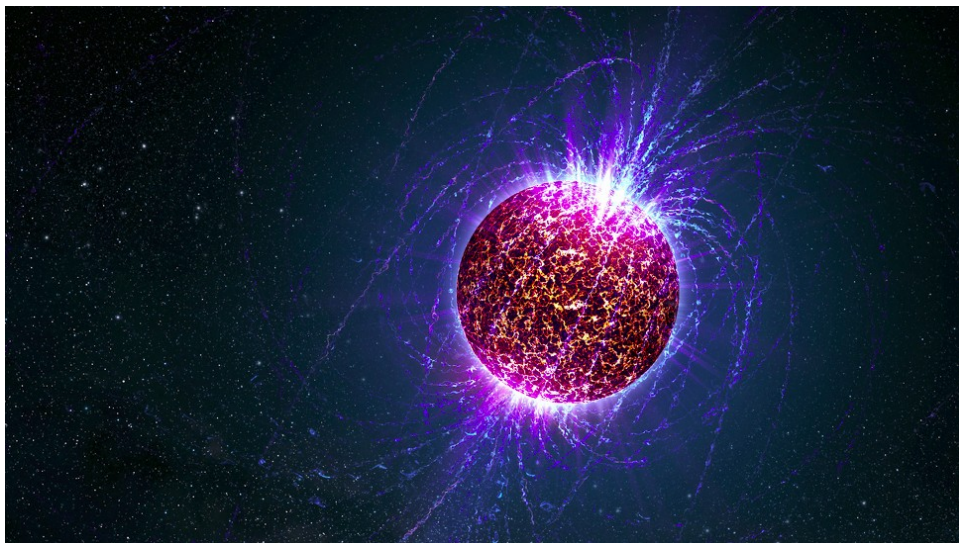
8. Avionsko krilo dizajnirano je tako da brzina zraka iznad krila bude 251 m/s onda kad je brzina zraka ispod krila 225 m/s . Gustoća zraka je $1,29 \text{ kg/m}^3$. Kolika podizna sila djeluje na krilo površine $24,0 \text{ m}^2$?

RJEŠENJE: $1,92 \cdot 10^5 \text{ N}$

ZADACI ZA VJEŽBU

9. Neutronske zvijezde sastoje se samo od neutrona i nevjerojatno su guste. Tipična masa i polumjer neutronske zvijezde mogu biti $2,7 \cdot 10^{28}$ kg i $1,2 \cdot 10^3$ m.
- a) Odredite gustoću te zvijezde.
 - b) Kolika bi bila masa novčića (obujma $2,0 \cdot 10^{-7}$ m³) kad bi bio napravljen od istog materijala?

RJEŠENJE: $3,7 \cdot 10^{18}$ kg/m³; $7,5 \cdot 10^{11}$ kg (750 milijuna tona)



10. Tok vode kroz cijev je 1,50 m³/s. Odredite brzinu vode kroz onaj dio cijevi koji ima polumjer 0,500 m.

RJEŠENJE: 1,91 m/s

WILEY

PITANJA ZA PONAVLJANJE

1. Fluid
2. Gustoća
3. Tlak
4. Atmosferski tlak
5. Pacalovo načelo
6. Uzgon
7. Arhimedov zakon
8. Idealni fluid
9. Jednadžba kontinuiteta
10. Bernoullijeva jednadžba