NAŠE SUNCE ROTIRA OKO SREDIŠTA MLIJEČNE STAZE, NAŠE GALAKSIJE. POLUMJER ROTACIJE IZNOSI $2{,}2 \cdot 10^{20}$ m, A KUTNA BRZINA SUNCA $1{,}1 \cdot 10^{-15}$ rad/s. KOLIKO MUJE ROTACIJA SUNCA OKO CENTRA GALAKSIJE, IZRAŽENO U GOD.?

$r = 2{,}2 \cdot 10^{20}$ m

$\omega = 1{,}1 \cdot 10^{-15}$ rad/s

$T_{god} = ?$

$$\omega = \frac{\Delta\theta}{\Delta t} \Rightarrow \Delta t = \frac{\Delta\theta}{\omega}$$

$$\Delta\theta = 2\pi \text{ rad}$$

$$\Delta t = \frac{2\pi \text{ rad}}{1{,}1 \cdot 10^{-15} \text{ rad/s}} \cdot \frac{1\text{h}}{3600\text{s}} \cdot \frac{1\text{d}}{24\text{h}} \cdot \frac{1\text{god}}{365{,}25\text{d}}$$

$$\Delta t = 1{,}8 \cdot 10^{8} \text{ god.}$$

PROPELER ROTIRA OKO OSI KOJA JE OKOMITA NA CENTRALNU PARALELNU SA ZEMLJOM, KAO STO JE PRIKAZANO NA SLICI. PREMA PROPELERU USMERENO STRELICU, KOJA SE GIBA PARALELNO S OSI ROTACIJE I PROLAZI KROZ SREDINU IZMEDU LOPATICA. KUT IZMEDU LOPATICA IZNOSI 60°. VERTIKALNI POMAK STRELICE ZANEMARUJEMO. NADITE MAX. VRIJEDNOST ZA KUTNU BRZINU PROPELERA T.D. NE UHVATI STRELICU DULJINE L I BRZINE v PRIKAZANE U TABLICI.

	L	v
a)	0,71 m	25,0 m/s
b)	0,71 m	91,0 m/s
c)	0,81 m	91,0 m/s

$t_s = t_p$

$t_s = \frac{L}{v}$ - cijela duljina mora proći kroz propeler brzinom v

$t_p = \frac{\Theta}{\omega} = \frac{60°}{\omega} = \frac{\frac{\pi}{3}}{\omega} = \frac{\pi}{3\omega}$

$t_s = t_p$

$\frac{L}{v} = \frac{\pi}{3\omega} \Rightarrow \omega = \frac{\pi \cdot v}{3L}$

a) 111 rad/s
b) 134 rad/s
c) 118 rad/s

SUSILICA ROBLJA RUKA NAKON ODJEĆU KONSTANTNOM BRZINOM 5,2 rad/s. KRENUVŠI IZ MIROVANJA, ZA KOJE VRIJEME SUSILICA POSTIGNE TU BRZINU, AKO SE UBRZAVA SREDNJOM KUTNOM AKC. OD $4,0\ rad/s^2$?

$\omega_0 = 0\ rad/s$

$\omega = 5,2\ rad/s$

$\alpha = 4,0\ rad/s^2$

$t = ?$

$\omega = \omega_0 \pm \alpha \cdot t$

$\omega = 0 + \alpha \cdot t$

$\alpha t = \omega \ / : \alpha$

$t = \frac{\omega}{\alpha} = \underline{\underline{1,3\ s}}$

KUTNA BRZINA ROTORA CENTRIFUGE SE POVEĆAVA SA 420 rad/s NA 1420 rad/s TIJEKOM 5,00 s.

a) ODREDITE KOLIKI KUT PRIJEĐE ROTOR ZA TO VRIJEME.

b) KOLIKO IZNOSI KUTNA AKC. ROTORA?

$\omega_1 = 420\ rad/s$

$\omega_2 = 1420\ rad/s$

$t = 5\ s$

a) $\varphi = ?$

b) $\alpha = ?$

$\varphi = \varphi_0 + \omega_0 \cdot t \pm \frac{\alpha}{2} t^2$

$\omega = \omega_0 \pm \alpha \cdot t$

b) $1420 = 420 + \alpha \cdot 5$

$1000 = 5\alpha \ / : 5$

$\alpha = 200\ rad/s^2 = \underline{\underline{2,00 \cdot 10^2\ rad/s^2}}$

a) $\varphi = 420 \cdot 5 + \frac{200}{2} \cdot 5^2$

$\varphi = \underline{\underline{4,6 \cdot 10^3\ rad}}$

UJEDNAČENA SE NA POČETKU GIBANJA GIBA STALNOM KUTNOM BRZ. TAKO UJEDNAČENO POVEĆAVA SVOJU JAČOST, TURBINA SE POČINJE UBRZAVATI STALNOM KUTNOM AKC. OD $0,140\ rad/s^2$. NAKON 2870 okretaja, POSTIGNE BRZINU (KUTNU) OD $137\ rad/s$.

a) KOLIKO IZNOSI POČ. KUTNA BRZ.?

b) KOLIKO DUGO SE TURBINA UBRZAVA?

$\alpha = 0,140\ rad/s^2$

$n = 2870\ obr. \Rightarrow \Theta = 2\pi \cdot n = 2\pi \cdot 2870$

$\Theta = 1,80 \cdot 10^4\ rad = \varphi$

$\omega = 137\ rad/s$

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a) $\omega_0 = ?$

b) $t = ?$

$\omega = \omega_0 \pm \alpha \cdot t$

$\omega^2 = \omega_0^2 \pm 2\alpha \cdot \varphi$

$\omega_0 = \sqrt{\omega^2 - 2\alpha \cdot \varphi} = \underline{117\ rad/s}$

$\omega = \omega_0 + \alpha \cdot t$

$t = \frac{\omega - \omega_0}{\alpha} = \underline{140\ s}$

ZEMLJA SE OKRENE OKO SUNCA JEDNOM GODIŠNJE ($3{,}16 \cdot 10^7$ s) PO PRIBLIŽNO KRUŽNOJ PUTANJI POLUMJERA $1{,}50 \cdot 10^{11}$ m. U ODNOSU NA SUNCE, IZRAČUNAJTE: a) KUTNU BRZ ZEMLJE

b) TANG. BRZ. ZEMLJE

c) VELIČINU I SMJER a_{cp} ZEMLJE

$T = 3{,}16 \cdot 10^7$ s

$r = 1{,}50 \cdot 10^{11}$ m

$\omega_z, v_{T_z}, a_{cp_z} = ?$

a) $\omega_z = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{T}$

$\omega = 1{,}99 \cdot 10^{-7}$ rad/s

b) $v_{T_z} = r \cdot \omega_z = 2{,}98 \cdot 10^4$ m/s

c) $a_{cp_z} = r \cdot \omega^2 = 5{,}94 \cdot 10^{-3}$ m/s^2

↳ usmjereno u centar buduće putanje - orbite

~~ROSTLINE~~ BORER EL. BUŠILICE IZ MIROVANJA ~~BRZINU~~ UBRZAMO STALNOM AKC. NAKON ŠTO OPIŠE ODREĐENI KUT, VELIČINA a_{cp} TOČKE NA BORERU IZNOSI DVOSTRUKO VIŠE OD a_T. KOLIKI JE TO KUT?

$$a_{cp} = 2a_T \longleftarrow \begin{cases} \alpha = \frac{a_T}{r} \Rightarrow a_T = \alpha \cdot r \\ a_{cp} = \frac{v^2}{r} = \frac{(\omega \cdot r)^2}{r} = \omega^2 \cdot r \end{cases}$$

$$\omega^2 \cdot \cancel{r} = \alpha \cdot \cancel{r} \cdot 2$$

$$\alpha = \frac{\omega^2}{2}$$

$$\omega^2 = \cancel{\omega_0^2}^{\,0} \pm 2\alpha\varphi \qquad (v^2 = v_0^2 \pm 2as)$$

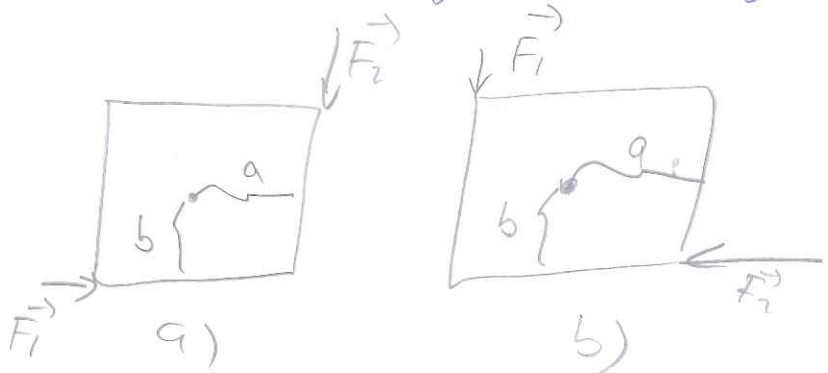
$$\cancel{\omega^2} = \cancel{2} \cdot \frac{\cancel{\omega^2}}{\cancel{2}} \cdot \varphi \Rightarrow \underline{\varphi = 1 \text{rad}}$$

OS ROTACIJE JE OKOMITA NA RAVNINU U kojoj se NALAZI ČVADRAT DUŽINE STR. 1m, TE JE SMJEŠTENA KAKO JE PRIKAZANO NA SLICI. DVIJE SILE, $\vec{F}_1$, $\vec{F}_2$, $|\vec{F}_2| = 3|\vec{F}_1|$, DJELUJU U NASUPROTNIM VRHOVIMA ČVADRATA, KAKO JE PRIKAZANO NA SLICI. U OBA SLUČAJA, UKUPNI MOMENT JEDNAK JE 0.
KOLIKO IZNOSE a I b, KOJE LOCIRAJU OS ROTACIJE?

$|\vec{F}_2| = 3|\vec{F}_1|$

$\ell = 1m$

$a, b = ?$



a) $\Sigma M = 0$

$F_1 \cdot b - F_2 \cdot a = 0$

$F_1 b - 3F_1 a = 0$

$b - 3a = 0$

$b = 3a$

b) $\Sigma M = 0$

$F_1 \cdot (\ell - a) - F_2 b = 0$

$F_1 (1 - a) - 3F_1 b = 0$

$1 - a - 3b = 0$

$1 - a - 3 \cdot 3a = 0$

$1 - 10a = 0$

$1 = 10a$

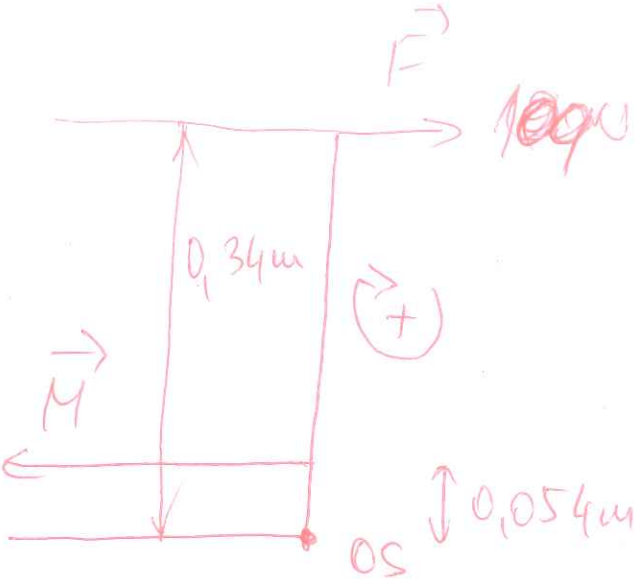
$a = \frac{1}{10} m = \underline{0,1m}$

$b = 3a = \underline{0,3m}$

(4) $F = 100N$

$\vec{M}, |\vec{M}| = ?$

$\Sigma M = 0$



$0 = F \cdot 0,34 - M \cdot 0,054$

$M \cdot 0,054 = F \cdot 0,34$

$M = \frac{F \cdot 0,34}{0,054} = \underline{1196,3N}$

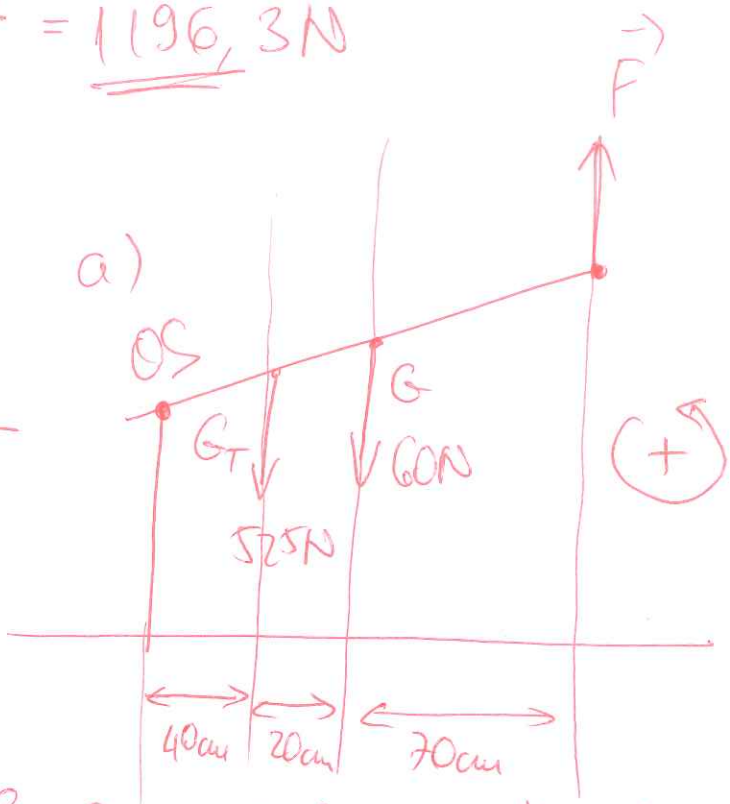
(5) $G = 60,0N$

$G_T = 525N$

$F = ?$

$\Sigma M = 0$

a)

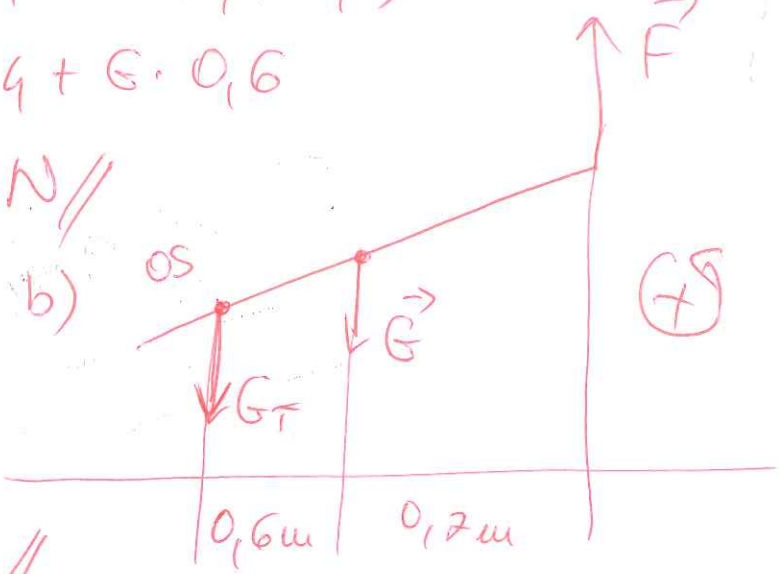


$F \cdot (0,4+0,2+0,7) - G_T \cdot 0,4 - G \cdot (0,4+0,2) = 0$

$F \cdot (1,3) = G_T \cdot 0,4 + G \cdot 0,6$

$F = 189,23N //$

b) $\Sigma M = 0$



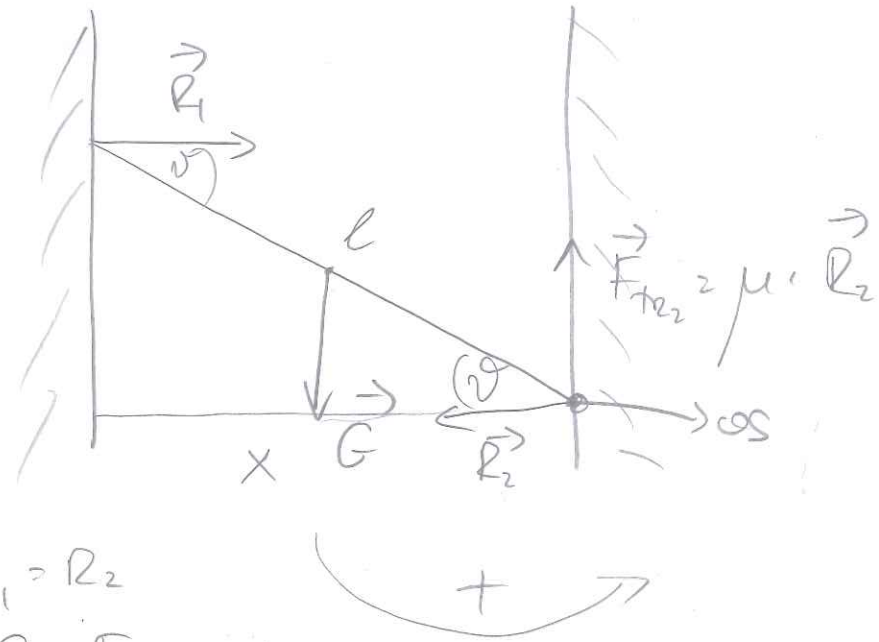
$F \cdot 1,3 - G \cdot 0,6 = 0$

$F = \frac{G \cdot 0,6}{1,3} = 27,7N //$

8. $x = 1{,}5\,m$

$\mu_2 = 0{,}98$

$l_D = ?$



$\Sigma F = 0$

$\Sigma M = 0$

$x: \quad R_1 - R_2 = 0 \Rightarrow R_1 = R_2$

$y: \quad F_{tr_2} - G = 0 \Rightarrow G = F_{tr_2}$

$$G \cdot \frac{l}{2} \cdot \cos\vartheta - R_1 l \sin\vartheta = 0 \Rightarrow \operatorname{tg}\vartheta = \frac{G}{2R_1} = \frac{\mu R_2}{2R_1}$$

$$\cos\vartheta = \frac{x}{l} \Rightarrow l = \frac{x}{\cos\vartheta} = 1{,}7\,m \qquad \operatorname{tg}\vartheta = \frac{\mu}{2} \Rightarrow \vartheta = 26°$$

8) $\omega_0 = 0\,rad/s$

$t_A = 3,00\,s$

$\varphi = \varphi_0 + \omega_0 \cdot t + \frac{\alpha}{2} t^2$ ($\varphi_0 = 0$, $\omega_0 = 0$)

$\varphi_1 = \varphi_2 \Rightarrow t_B = ?$

(1) $\varphi = \frac{\alpha}{2} \cdot t^2$

$F_A = F_B$

(2) $M = I \cdot \alpha = F \cdot r \Rightarrow \alpha = \frac{F \cdot r}{I}$

$\varphi = \frac{1}{2} \cdot \frac{F \cdot r}{I} \cdot t^2 \Rightarrow t = \sqrt{\frac{2\varphi I}{F \cdot r}}$

$$\frac{t_B}{t_A} = \frac{\sqrt{\frac{2\varphi_B I_B}{F_B \cdot r_B}}}{\sqrt{\frac{2\varphi_A I_A}{F_A \cdot r_A}}} = \sqrt{\frac{I_B r_A}{I_A r_B}}$$

$I_A = \frac{1}{3} M L^2$

$I_B = \frac{1}{12} M L^2$

$r_A = L, \quad r_B = \frac{1}{2} L$

$$\frac{t_B}{t_A} = \sqrt{\frac{\frac{1}{12} M L^2 \cdot L}{\frac{1}{3} M L^2 \cdot \frac{1}{2} L}} = \sqrt{\frac{1}{2}} \Rightarrow t_B = 3 \cdot \frac{1}{\sqrt{2}} = \underline{2,12\,s}$$

9) $l = 0,75\,m$

$\omega = 4,2\,rad/s$

$m_A = 0\,kg, \quad m_{\check{C}A} = 0,66\,kg$

$m_B = 0,66\,kg$

$E_{kA}, E_{kB} = ?$

$E_{kl} = \frac{1}{2} I \cdot \omega^2$

$I_A = 0 + m_{\check{C}A} \cdot l^2$

$E_{klA} = \frac{1}{2} \cdot m_{\check{C}A} \cdot l^2 \cdot \omega^2 = \underline{3,3\,J}$

$I_B = \frac{1}{3} m_B \cdot l^2$

$E_{klB} = \frac{1}{2} \cdot \frac{1}{3} m_B \cdot l^2 \cdot \omega^2 = \underline{1,1\,J}$

10) $m_e = 240\,kg$

$l_e = 6,7\,m$

$\omega = 44\,rad/s$

a) $I = I_1 + I_2 = 2I = 2 \cdot \frac{1}{3} m_e l_e^2 = \underline{7200\,kg \cdot m^2}$

b) $E_k = \frac{1}{2} I \cdot \omega^2 = \underline{7 \cdot 10^6\,J}$

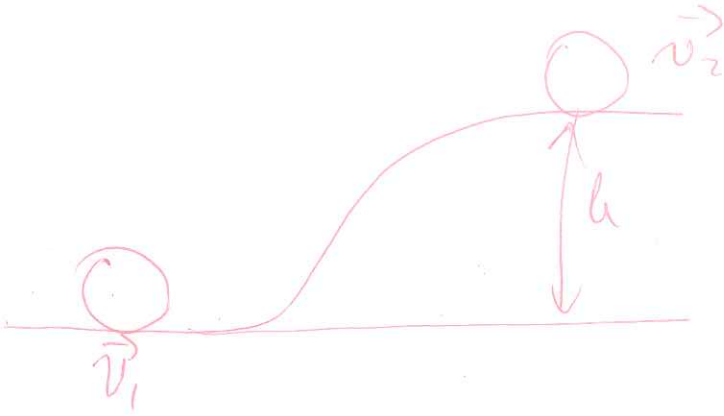
a) $I = ?$

b) $E_k = ?$

22. $h = 0{,}760\,m$

22 $v_1 = 3{,}50\,m/s$

$v_2 = ?$



ZOE:

$$E_{kR_1} + E_{kT_1} = E_{kR_2} + E_{kT_2} + E_{gp}$$

$$\frac{1}{2} m v_1^2 + \frac{1}{2} I \omega_1^2 = \frac{1}{2} I \omega_2^2 + \frac{1}{2} m v_2^2 + mgh$$

$$\frac{1}{2} m v_1^2 + \frac{1}{2} \cdot \frac{2}{5} m r^2 \cdot \frac{v_1^2}{r^2} = \frac{1}{2} \cdot \frac{2}{5} m r^2 \cdot \frac{v_2^2}{r^2} + \frac{1}{2} m v_2^2 + mgh$$

$$\frac{1}{2} v_1^2 + \frac{1}{5} v_1^2 = \frac{1}{5} v_2^2 + \frac{1}{2} v_2^2 + gh$$

$$\frac{7}{10} v_1^2 = \frac{7}{10} v_2^2 + gh \;/ \cdot \frac{10}{7}$$

$$v_1^2 = v_2^2 + \frac{10}{7} gh$$

$$v_2 = \sqrt{v_1^2 - \frac{10}{7} gh} = 1{,}26\,m/s$$

22

5.) $h = 0{,}760\,m$

$v_{T_1} = 3{,}5\,m/s$

$v_{T_2} = ?$

$I = \frac{2}{5} m r^2$

ZOE:

$$E_{kT_1} + E_{kr_1} = E_{kT_2} + E_{kr_2} + E_{gp}$$

$$\frac{1}{2} m v_{T_1}^2 + \frac{1}{2} I \omega_1^2 = \frac{1}{2} m v_{T_2}^2 + \frac{1}{2} I \omega_2^2 + mgh$$

$$\frac{1}{2} m \cdot (3{,}5)^2 + \frac{1}{2} \cdot \frac{2}{5} m r^2 \cdot \left(\frac{v_{T_1}}{r}\right)^2 = \frac{1}{2} m \cdot v_{T_2}^2 + \frac{1}{2} \cdot \frac{2}{5} m r^2 \cdot \left(\frac{v_{T_2}}{r}\right)^2 + mgh$$

$$\frac{1}{2} \cdot 3{,}5^2 + \frac{1}{5} \cdot (3{,}5)^2 = \frac{1}{2} v_{T_2}^2 + \frac{1}{5} v_{T_2}^2 + g \cdot h$$

$$8{,}575 - 2{,}4556 = 0{,}7\, v_{T_2}^2$$

$$v_{T_2} = 1{,}26\,m/s$$

6.) $\alpha = 35^\circ$

$h = 1{,}5\,m$

$x_D = ?$

ZOE:

$$\frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 = mgh$$

$$\frac{1}{2} m v_0^2 + \frac{1}{2} \cdot \frac{2}{3} m r^2 \cdot \frac{v_0^2}{r^2} = mgh$$

$$\frac{5}{6} v_0^2 = g \cdot h$$

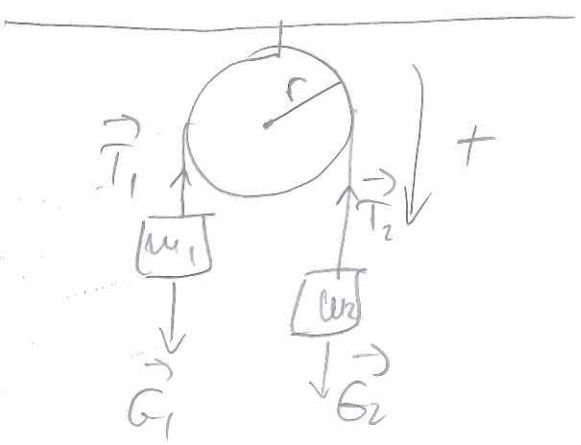
$$v_0 = \sqrt{\frac{6gh}{5}} = 4{,}6\,m/s$$

$$x = \frac{v_0^2 \sin 2\alpha}{g} = 2{,}03\,m$$

7.) $m_1 = 11{,}0\,kg$ $T_1 \neq T_2$

$m_2 = 44{,}0\,kg$ $a_2 = \frac{a}{2}$

$\omega_k = ?$



m_1:

$$T_1 - G_1 = m_1 \cdot a \Rightarrow T_1 = m_1 a + m_1 g = \frac{3}{2} m_1 g$$

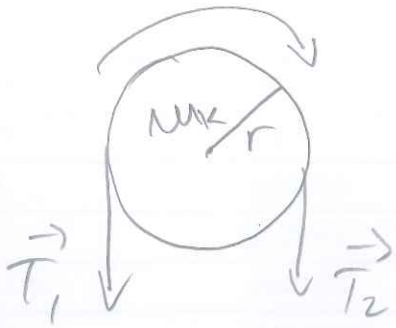
$$T_1 = 161{,}87\ N$$

m_2:

$$G_2 - T_2 = m_2 a \Rightarrow T_2 = m_2 g - m_2 a = \frac{1}{2} m_2 g$$

$$T_2 = 215{,}82\ N$$

m_k:



$$M = M_2 - M_1 = I(-\alpha)$$

$$T_2 r - T_1 r = I \cdot \alpha = \frac{1}{2} m_k r^{\not 2} \left(\frac{a}{r^{\not 2}}\right)$$

$$T_2 - T_1 = +\frac{1}{2} m_k \cdot a \;/\cdot \left(+\frac{2}{a}\right)$$

$$m_k = \frac{2(T_2 - T_1)}{a} = 22\ kg$$